TEACHING
Arithmetic We Need

MANUAL
★
FACSIMILE OF TEXT
★
ANSWERS

CURRICULUM

6

BUSWELL
BROWNELL
SAUBLE

Placement of Topics in *A*

	Grade 3	Grade 4
Number System	Meaning, reading, writing 1-, 2-, and 3-place numbers Money numbers to $9.99; meaning of 0 Ordinals through "twentieth"; Roman numerals through XII	Meaning, reading, writing 4-, 5-, and 6-place numbers 0, a number and a place-holder; rounding to nearest 10 Ordinals through "thirtieth"; Roman numerals through L
Addition of Whole Numbers	Meaning and maintenance (facts with sums to 18) Ways to express addition; order of addends; adding 0 A. of 2- and 3-place numbers without and with carrying Column A.; check; table of 81 A. facts Relation of sum to addend; term "sum"	Reteaching meaning of addition (sums to 18) Maintenance through oral and written practice Meaning of, and practice in, addition of 4-place numbers Recognizing and using term "addend" Estimating sums
Subtraction of Whole Numbers	Meanings and maintenance (facts with minuends to 18) Ways to express subtraction; subtracting 0; 0 as remainder S. of 2- and 3-place numbers without and with borrowing S. when 0 is in ten's and one's places; term "remainder" Relation of remainder to minuend; check; table of 81 S. facts	Reteaching meanings of subtraction (minuends to 18) Maintenance through oral and written practice Meaning of, and practice in, subtraction of 4-place numbers Terms: "minuend," "subtrahend," "remainder," "difference" Estimating remainders
Multiplication of Whole Numbers	Meaning of multiplication; relation with addition Ways to express multiplication; term "product" Facts for 2's and 2; 3's and 3; 4's and 4; M. by 1 or 0 Pairs of M. facts; whole stories (M. and D.) M. of 2- and 3-place by 1-place number with carrying Money numbers; check	Reteaching meaning of multiplication; maintenance Multiplication facts involving 4, 5, 6, 7, 8 and 9 Terms: "multiplicand," "multiplier," "partial product" Relation of product to multiplicand; estimating products M. of 2-place by 2-place number; of 4-place by 1-place Shortening partial products; check; table of 81 M. facts
Division of Whole Numbers	Meanings of division; relation with subtraction Ways to express division; term "quotient" Facts with divisor or quotient 2, 3, 4, 1; check Pairs of D. facts; whole stories (M. and D.) D. of 2- and 3-place by 1-place number; money numbers	Reteaching meanings of division; maintenance Division facts with divisor or quotient 4, 5, 6, 7, 8, 9 D. with remainder; use of R; table numbers for even D. D. of 2-, 3-, and 4-place numbers by 1-place; 0's in division Terms: "divisor," "dividend"; table of 81 D. facts Relation of quotient to dividend; estimating quotients
Fractions and Mixed Numbers	Meaning, recognition, reproduction of one equal part of object, group, or number Definition of a fraction as a number Fractions used with measures ($\frac{1}{2}$ in. and $\frac{1}{4}$ in. only) Meaning of number below fraction line Relative sizes of fractions	Reteaching meanings of unit fraction Meaning and use of multiple fractions Terms: "numerator" and "denominator" Comparing by using a fraction Comparing sizes of fractions Equivalents; meaning only of A. and S. of fractions
Geometry	Recognizing and reproducing simple geometric forms: line, circle, rectangle, square	Maintenance of ability to recognize and reproduce simple geometric forms
Use of Literal Numbers	Use of letter *n* to stand for missing sum, addend, remainder, minuend, or subtrahend	Maintenance; use of letter *n* to stand for missing product, multiplier, multiplicand, quotient, dividend, or divisor
Measures	Meaning of measures: linear, time, weight, dozen, calendar, liquid Reference measures: finger inch, arm foot, arm yard or body yard, local mile, book pound	Reteaching kinds of measure and tables; maintenance Extending reference measures: ton, ounce Dry measure; temperature Changing one unit of measure to another
Problem-Solving	Meanings of A., S., M., D.; likeness of groups Use of dots, number line, and pictures Differentiating processes; drawing pictures for helpers Telling why problems are a certain kind Making problems: from pictures; from a story; for a process Recognizing tricky words; telling story in own words Detecting absurdities; problems with missing data Testing answers; relation of answer to numbers given Problems without numbers; using measures; oral problems	Maintenance of techniques already taught Estimating sums, remainders, products, quotients Summary of helps in problem-solving Meaning and use of average; two-step problems Detecting mistakes in problem-solving Problems with extra numbers; checking answers Using a sales slip; using a diagram Using simple charted material

Placement of Topics in *Arithmetic We Need*

	Grade 5	Grade 6
Number System	Meaning, reading, writing numbers through millions Rounding whole numbers to tens, hundreds, thousands Extending meaning of number system to right of one's place Meaning, reading, writing tenths and hundredths Counting by tenths; by hundredths Roman numerals through M	Meaning, reading, writing numbers through billions Extending meaning of decimals through millionths Counting by tenths; by hundredths; by thousandths Rounding large whole numbers Rounding mixed numbers and decimals Roman numerals through thousands
Addition of Whole Numbers	Reteaching meaning of addition Maintenance through oral and written practice Addition of 5-place numbers	Reteaching meaning of addition Maintenance; more addends in column addition Oral and written practice
Subtraction of Whole Numbers	Reteaching meanings of subtraction; oral and written practice Subtraction of 5-place numbers	Reteaching meanings of subtraction Maintenance through oral and written practice
Multiplication of Whole Numbers	Reteaching meaning of multiplication; maintenance M. of 3-place and 4-place by 2-place number M. of 3-place by 3-place number Multiplicand-multiplier-product relationship	Reteaching meaning of multiplication; maintenance Larger products
Division of Whole Numbers	Reteaching meanings of division; maintenance D. of 5-place by 1-place number, 4- or 5-place quotient D. by 2-place divisor, 1-, 2-, 3-place quotient Steps in division Use of term "partial dividend"; relation between M. and D.	Reteaching meanings of division; maintenance D. by 3-place divisor getting 3-place quotient Short division
Fractions and Mixed Numbers	Reteaching meanings of fractions; maintenance Fractional unit; proper, improper fractions; mixed numbers Equivalents: halves, fourths, eighths, sixteenths; halves, thirds, sixths, twelfths; halves, fifths, tenths A. and S. of like- and unlike-fractions; estimating answers Remainder in D. shown in fraction; 3 meanings of a fraction	Reteaching meanings of fractions; maintenance M. with fractions and mixed numbers; cancellation Relation of product to multiplier and multiplicand D. with fractions and mixed numbers Relation of dividend to divisor: how many times; what part of Three relationships in M. and D. with fractions
Decimals and Per Cents	Meaning, reading, writing tenths and hundredths Equivalent common and decimal fractions A. and S. of tenths; of hundredths Decimals on a number line	Reteaching meaning of decimals; maintenance Meaning, reading, writing smaller decimals; rounding A. and S. of thousandths and ten-thousandths M. of decimals D. with decimals; short division with decimals Three relationships in M. and D. with decimals
Graphs and Scale Drawing	Meaning of scale drawing; use to find perimeter and area Meaning of graph: pictograph; bar graph; line graph Reading graphs; scale; terms	Reteaching meaning of scale drawing; of graph Reading and making picture, bar, line graphs Use of scale on maps
Geometry	Maintenance and use of simple geometric forms Finding perimeter and area of rectangle; of square	Maintenance and use of simple geometric forms Maintenance of finding perimeter and area
Use of Literal Numbers	Maintenance and use of letter ***n*** to stand for missing term in processes with whole numbers Use of ***n*** to stand for missing sum or minuend with fractions	Maintenance and use of letter ***n*** to stand for missing term Use of ***n*** to stand for missing divisor or quotient with fractions or decimals
Measures	Reteaching kinds of measures and tables; maintenance Extending reference measures: 30 inches; 7 feet; 30 feet Extending measures of time to century Perimeter and area of rectangle; of square	Reteaching kinds of measures and tables; maintenance Extending reference measures: 5 lb.; 10 lb.; 100 lb. Exact and measuring numbers; cooking measures A., S., M., D. with denominate numbers
Problem-Solving	Maintenance of techniques already taught Using fractions and decimals in problems; using a table Using well-known relationships Telling which answer must be wrong	Maintenance of techniques already taught Checking decimal problems by common fractions Three relationships in M. and D. for problems with fractions; with decimals

Placement of Topics in *Arithmetic We Need*

	Grade 7	Grade 8
Number System	Maintenance of previous number meanings and practices for whole numbers, fractions, decimals Meaning of per cents Maintenance of Roman numerals	Maintenance of previous number meanings and practices Notation systems: Egyptian; Greek; Roman; Hindu-Arabic Decimal system and others; positive and negative numbers introduced by use of thermometer and number line Some laws of our system: Order; Grouping
Addition of Whole Numbers	Reteaching meaning of addition Maintenance; longer and wider columns	Reteaching meaning of addition Maintenance through oral and written practice
Subtraction of Whole Numbers	Reteaching meanings of subtraction Maintenance; subtraction of larger numbers	Reteaching meanings of subtraction Maintenance through oral and written practice
Multiplication of Whole Numbers	Reteaching meaning of multiplication Maintenance through oral and written practice	Reteaching meaning of multiplication Maintenance through oral and written practice
Division of Whole Numbers	Reteaching meanings of division Maintenance through oral and written practice	Reteaching meanings of division Maintenance through oral and written practice
Fractions and Mixed Numbers	Reteaching meanings and processes with fractions Maintenance through oral and written practice	Reteaching meanings and processes with fractions Maintenance through oral and written practice
Decimals and Per Cents	Reteaching decimal meanings and processes; maintenance Meaning, reading, writing per cents Equivalent common fractions, decimals, per cents Three types of per-cent examples and applications in discount, commission, interest, increase, decrease Rounding and estimating; three relationships in M. and D.	Reteaching decimal meanings and processes; maintenance Reteaching meanings, types of examples, and applications in per cents Maintenance of per cents through practice
Graphs and Scale Drawing	Reteaching meaning of scale drawing; of graph Reteaching reading and making graphs (picture, bar, line) Harder graphs (fraction symbols); map graph Divided-bar graph; circle graph More than one graph on same axes; per cents and graphs	Reteaching meaning of scale drawing; of graph Maintenance through practice
Geometry	Recognizing, reproducing, and measuring lines, angles, and geometric forms: circle, triangle, quadrilaterals, pentagon, hexagon, octagon Measuring with instruments: ruler, compass, protractor Developing relationships; developing formulas	Reteaching work with lines, angles, circles, polygons; formulas and relationships for plane figures Constructions; similar and congruent figures Solids and formulas for finding: volume of prism, cylinder, pyramid, cone; surface area of rectangular prism, cylinder
Business Usage	Buying through mail-order catalogue; budgets Quantity and quality buying; cash or delayed payment Checking account; interest on savings; safe-deposit box Savings bonds; sending money by mail Operating a business; terms used in business	Money and its uses; business ownership Investments: banks; stocks and bonds Borrowing money: promissory notes; mortgages Insurance; social security Taxes
Use of Literal Numbers	Maintenance and use of letter *n* to stand for missing number Use of letters in formulas	Maintenance and use of letter *n* to stand for missing number Making equations Solving formulas; solving equations
Measures	Reteaching kinds of measures and tables; maintenance Extending reference measures: time for riding one mile Finding area: parallelogram; rectangle; square; triangle Measuring electrical energy; gas; energy value of food Meaning of volume; time zones; tolerance	Reteaching kinds of measures and tables; maintenance Extending reference measures: 100 ft.; 1 sec.; 1 min. Majority, median, frequency, mean, plurality Metric system: length; liquid, dry, square, weight Ratio and proportion; hypotenuse and square root
Problem-Solving	Maintenance of techniques already taught Using tabulated material and maps Illustrations of errors in figuring Three relationships in M. and D. for per-cent problems	Maintenance of techniques already taught; 3-step problems Problems using ratios; using relationships Solving problems by using formulas; by using equations Problems involving business usage

MANUAL
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FACSIMILE OF TEXT
★
ANSWERS

TEACHING Arithmetic We Need

GRADE 6

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Cartoons by CHARLES WILTON

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Courtesy Wilmington Public Schools

Dramatizing the operation of a bank provides an opportunity for utilizing skill in arithmetic and for demonstrating its practical value.

PART I · YOUR MANUAL

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Introduction

You have never before seen an arithmetic manual like this one. Leaf quickly through the Manual and see if this statement is not true. Note the large amount of help and the variety in the forms of help you are given. But note this also: the Manual is no mere book of recipes for teaching arithmetic. It offers practical suggestions, yes, but it goes beyond these suggestions to explain the reasons for them.

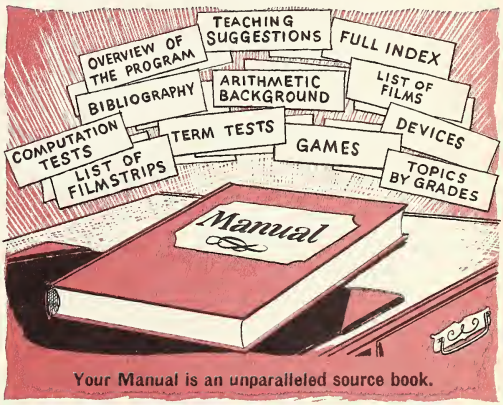
There was no need for a manual of this kind twenty years ago. At that time arithmetic was generally viewed as a tool subject or as a drill subject, and the teaching process was not very complicated. The children's books carried the burden of instruction. These books told the children what to do, showed them how to do it, and gave them plenty of abstract examples as practice to develop their skills. Teachers had only to make sure that their pupils could follow directions, and then assign textbook lessons and correct papers. Perhaps this description oversimplifies what went on in the typical arithmetic class, but not by very much. Be that as it may, the description is quite unacceptable as an account of what is to be done with arithmetic at the present time.

Arithmetic is no longer regarded as a textbook subject, as something to be taught intensively for thirty or forty-five minutes a day and then abandoned for some other unrelated subject. Instead, arithmetic is thought of as something which pervades the whole curriculum, as something that influences thought and activity throughout the day, both in school and out of school. As a consequence, arithmetic is too extensive a subject to be put within the covers of a book, no matter how good the book is—and the demands upon the teacher have expanded accordingly.

Most teachers recognize that they need help in dealing effectively with the new arithmetic program. This is so because theory has advanced so rapidly. This Manual then provides a means for bringing to you the newer knowledge that has become available concerning the teaching and learning of arithmetic.

Helps Provided in the Manual

Organization of the Manual



Your Manual is an unparalleled source book.

To ensure the most efficient use of the Manual, and to help in your orientation to it, let us begin by noting how the Manual is set up.

Part II, which follows, is intended to give you an overview of your arithmetic program. It tells you how to get started and discusses some of the topics you will be teaching. It lists some of the learning outcomes to be achieved, but the listing is no mere catalogue of items arranged in some logical or chronological sequence. The Manual does much more than this. It attempts to elaborate upon the

modern conception of arithmetic, the conception according to which *Arithmetic We Need* has been written and according to which it may be taught. Even on the first reading, however, you will see many chances to enrich the program from your own experiences and from the numerous activities of your pupils.

A good deal of attention is paid in the Manual to the nature of the learning process in arithmetic. Through numerous illustrations of what is involved as children learn specific ideas, skills, and attitudes, the Manual helps you see how to utilize the best psychological procedures in guiding their learning. Constant attention to the details of the skills and an appreciation of the obstacles which the pupil must overcome will enable you to guide each child's progress carefully.

Part III consists of seven sections, one for each chapter of the text, which provide the specifics of instruction. Each of these seven sections has a brief introduction summarizing the learning outcomes of the chapter and, where necessary, explaining in some detail what they mean. The introduction each time is followed by subsections written to help you teach the separate lessons as you come to them. Here you will find specific suggestions for activities to be engaged in before, during, and after each lesson in the text. Included are ideas for enriching and individualizing instruction which may be adapted to your arithmetic program, or which will suggest to you ideas of your own. Also included is background information to help you understand the nature of the lesson's learning task.

Late in the Manual is Part IV. Here you will find: (*a*) a series of seven computation tests to be used as alternate chapter tests, if you choose to administer the corresponding tests in the text for pre-testing or practice purposes; (*b*) two batteries of achievement tests for use following Chapters 4 and 7 to evaluate rather thoroughly the learning over each term or semester; (*c*) a section devoted to instructional aids which includes many suggestions for constructing or obtaining devices, games, and equipment useful in the teaching of arithmetic, as well as a list of commercial aids covering films, filmstrips, charts and flash cards, devices, and games; (*d*) a bibliography of books and pamphlets for teachers and pupils; and (*e*) a table of measures. You will notice also that further convenience is offered by the complete index provided for the Manual.

Types of Help in the Manual

In addition to the helps already mentioned (listing of outcomes chapter by chapter, supplementary tests, and so on), the Manual gives you other helps. A very important one is that every page in the text is reproduced in the Manual in reduced size. Consequently as you read the Manual you have before you the content of each lesson and suggestions for teaching that lesson. As a matter of fact, you may *teach* the lesson from the Manual, if you so desire, and thus eliminate the bothersome necessity of referring to both a manual and a text.

In order to understand and to appreciate more fully the advantages of having the reproduced pages of the text in the Manual, turn now to pages 24–27 in the Manual and study the matter relating to text pages 1 and 2.

1. Since the reproduction contains everything on the corresponding page of the text, you can make full use of various symbols.

a. The clue caption "Reading numbers for meaning" in the upper right corner of reproduced page 1 tells you the arithmetical purpose of the lesson. (Sometimes the title of the lesson, as for text page 2, imparts similar information.)

b. The symbol [O] at the top right of page 1 means that the first part of the lesson is to be taught orally. (Note that this oral development represents another way in which the authors help you in guiding the classroom instruction.) The symbol [W] at the right near the bottom of page 2 means that the lesson there is to be assigned for written work.

2. In order further to aid classroom instruction, answers for examples and problems are printed in red

on the reproduced page. (A red star will be found beside each question for which there is not enough room on the reproduced page to show the answer. Another red star near the bottom of the Manual page marks the section in which the overflow of answers will be found.)

3. The teaching suggestions accompanying the reproduced pages are usually organized for convenient reference under these headings:

- Pupil's Objectives
- Background
- Teacher's Preparation
- Pre-book Lesson
- Book Lesson
- Differentiations and Extensions

What is written under these headings amounts to a complete lesson plan. To help in your orientation to the lesson, the Manual indicates objectives from the pupil's point of view, and also provides essential background and preparation activities. Then, through the Pre-book Lesson, provision is made for the new idea or skill to appear first in natural or prearranged classroom situations quite apart from the text. When these situations have been explored and when as much as possible about the idea or skill has been gained from the exploration, a shift is made to the text in order to profit from the systematic development provided there.

The needs, both of the *slower learner* and of the *more capable child*, are recognized and are served through use of the materials furnished in the Manual. Because it is often desirable to carry the idea or skill beyond the limits of the lesson in the text, specific suggestions are also provided for this purpose.

Every effort has been made to provide you with effective and efficient help, but in the end it is the use you make of the Manual that counts. If your approach to a particular lesson is better than any you find mentioned, jot it down in one of the spaces left blank for your notes. The Manual was designed for you, now make it work for you.

Using the Manual

Our sole motive in preparing this Manual has been to give you all the help we can. As authors, we cannot talk to you in the confines of the text, but in the Manual we have the chance to tell you about the arithmetic program you will teach and to suggest ways of teaching it.

Please regard the detailed lesson plans merely as models. They show you how particular ideas or skills *may* be taught, *not* how they *must* be taught. Conceivably, some of our lesson plans may be just what you want, and in these cases you will adopt them. Just as likely, special conditions in your situation or in your experience may make our plans inappropriate. In such cases you will want to modify our plans or to reject them entirely, substituting your own. Any system of teaching that enables children to learn arithmetic economically, intelligently, and happily is a good one.

All teaching, whether of arithmetic or of anything else, should be stimulating, imaginative, creative, and artistic. It can be so only when teachers understand clearly what they are about. In the Manual we have tried to help you gain this perspective. This is the reason for the sections where we explain rather fully what is involved in an arithmetical idea or skill and in learning it for functional mastery. These discussions have a serious purpose, and merit careful study.

A good place to commence this careful study is with Part II, which follows immediately. Read it slowly and thoughtfully. The more you know about the over-all picture of the subject, the better you can teach arithmetic.

This teaching team will produce results.

PART II · OVERVIEW OF THE PROGRAM FOR GRADE 6

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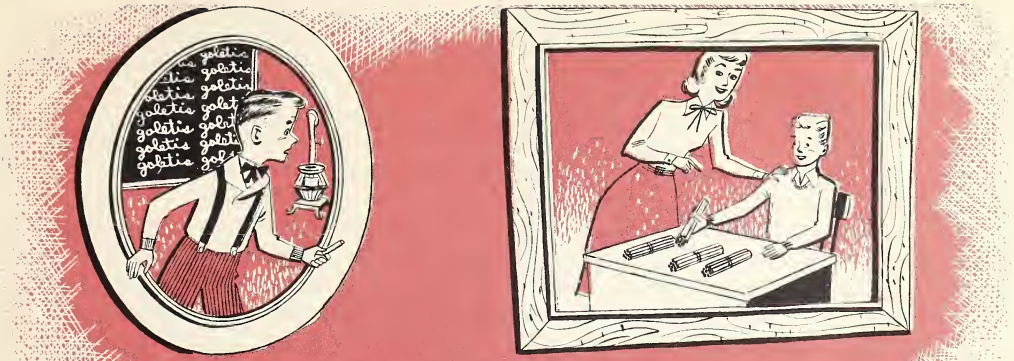
The Conception of Arithmetic Basic to This Series

Why do we teach arithmetic at all? We teach arithmetic because it contributes directly to efficient, intelligent, and enriched living. Our culture is a quantitative one, as no other culture has been previously, and it is becoming steadily and rapidly even more quantitative.

Consider the growing need for careful scheduling and programing and how this must inevitably involve quantitative relationships and arithmetic. Consider also the increasing practice in science and industry of identifying things by number (we now have a tin can for food numbered 303), of using more precise measures (the fraction 0.0005 inch has some significance today), and of employing larger and larger numbers for sizes and dimensions that must be understood (astronomical distances, for example), and how these practices demand of your pupils arithmetic competency never before necessary. True, the invention and use of calculating machines may save us from more or less routine computations, but only at times, and even then there must be someone to do the crucial quantitative thinking first or to interpret and to apply the results of the machine computation.

The basic reason that arithmetic is given an important place in the curriculum is a *social* reason. It is expected that the arithmetic we teach will make real differences in the kinds of lives we lead, and not merely as adults either. The arithmetic we teach must function in the life of every child *as he learns it*. Arithmetic programs which do not meet this test are entirely without justification. The arithmetic program represented in *Arithmetic We Need* does meet this test.

Arithmetic, then, has a dominating social purpose; but so do geography, history, and reading. Although arithmetic is related to these as well as other school subjects, it has its differences. These differences may be summed up by saying that arithmetic is a phase of mathematics. Arithmetic has to do with numbers and with their various relationships; so arithmetic comes to have a second purpose or aim, namely, a *mathematical* purpose. It is designed to make children and adults competent in the mathematical aspect of the subject. This is far from saying that we teach arithmetic in order to develop mathematicians, for the mathematical competence we seek is limited by the social purpose of the subject. We strive to establish the mathematical ability necessary to deal effectively with the quantitative situations of practical living.



Meaningless drill must give way to varied practice that results in understanding.

The social aim and the mathematical aim of arithmetic complement each other. This rather simple illustration reveals their mutual relationship. Although Peter may have learned the correct steps for obtaining the quotient in the box when it occurs in a division example, he may not know how to proceed when confronted with the following problem:

$$\begin{array}{r} 8 \\ 36\overline{)288} \\ \underline{288} \end{array}$$

How many months will it take Mrs. Harris to pay for a TV set costing $288 if she pays $36 a month?

Peter's difficulty is due to the fact that he does not realize that $288 is the product of 36 and another factor, which he must find by dividing. He needs guidance in discovering the relationship between multiplication and division and in understanding the mathematical generalization that "If we know a product and one of its two factors, we can find the other factor by dividing." Until Peter learns more than the mechanics of division, his ability to use it in the solution of problems arising in life situations is extremely limited.

To sum up and to conclude this discussion, each book in this series is built upon a conception of arithmetic that involves two aims, the *social aim* and the *mathematical aim*. Adherence to the latter aim requires that children *see sense* in what they learn. Their learning becomes sensible in direct proportion to their understanding of arithmetic. They acquire this understanding as they explore, discover, identify, and organize mathematical ideas and relationships which together make arithmetic a systematic body of knowledge. The social aim they achieve partly through the mathematical aim, partly in other ways. The social aim requires that children see arithmetic as *useful*, and that they make the most productive use of it.

In other words, children are guided to appreciate the value of what they learn *as* they learn it. They encounter arithmetical ideas and skills initially and repeatedly thereafter in the processes of living both in and out of school—encounter them and employ them in realizing ends of significance to them.

Special Features of the Text

I. The Promotion of Understanding

Teaching Concepts and Technical Terms and Symbols

We commonly confuse words with ideas, thinking that if we know words we also know the ideas they symbolize. It is an easy matter to demonstrate this type of error. Pronounce this word: goletis (go-le′-tis). Say it again. Say it three times more. Now what does it mean? Which, if any, of these sentences is correct? "The goletis is worn out"; "Her coat is goletis"; "I watched a mechanic goletis an automobile." You cannot be sure, can you? Why not? You have learned the word, have you not? But do you have the idea? At all costs we must avoid this kind of "goletis business" in teaching arithmetic, and *Arithmetic We Need* will give you real help at this point. No topic is introduced without a careful development of its meaning, and drill is not called for *until* meanings have been clearly established.

To return again to our example above, where you "learned" goletis by saying it to yourself five times: you could also have written it fifty times with no gain in meaning. *Repetitive practice* like this, doing the same thing over and over again, yields no increase in meaning. It simply makes us more proficient in doing what we have been doing all along. Obviously, when we strive to increase proficiency, then, and then alone, do we need to provide repetitive practice. To advance meaning we must engage in *varied practice.* That is, while we work on a thing we want to learn (an idea or concept or technical term), we must do so in different ways and in different settings.

Let us see what this implies in arithmetic in Grades 5 and 6 for the concept of decimal fractions. Pupils can memorize the place-value names —tenths, hundredths, thousandths, etc., and they can read decimal fractions by memorizing the rule that we say the name of the last place to the right of the decimal point when we read a decimal fraction. For example, 0.38 is read "38 hundredths" because it is a 2-place decimal and the name of the last place on the right is hundredths while 0.256 is read "256 thousandths" because it is a 3-place decimal and the name of the last place on the right is thousandths.

The fact that a pupil has memorized words and rules does not guarantee that he has genuine understanding which will enable him to interpret correctly situations involving decimals. How often have you observed pupils inserting decimal points in products or quotients without regard for the reasonableness of the answer? Such a practice indicates that pupils have little or no real understanding to guide their performance when memory has failed them. It is only when pupils understand that they can really succeed in arithmetic.

A basic aim of this series is to provide the kinds of experience that develop truly meaningful and functional concepts. In *Arithmetic We Need* you will find experiences such as these:

1. Manipulative experiences with real or natural objects.

2. Manipulative experiences with representative objects (smaller objects, such as pegs or buttons or marks on paper, that are intrinsically less distracting and at the same time more readily manageable)

3. Experiences in identifying the idea in pictures of real or natural objects, and/or in reproducing the idea in the same medium

4. Experiences in identifying the idea in semi-concrete pictures (regular patterns of dots or crosses), and/or in reproducing the idea in the same ways

5. Experiences in identifying the idea in abstract form (in examples with numbers) or in verbally described situations (word problems), and/or in reproducing the idea in these same ways

In each type of activity the child encounters the idea in a variety of forms, different objects, different charts, different words, different abstract numbers, and in a wide variety of social situations. The purpose is to carry the child by easy stages from the concrete to the abstract and back again. Also, the purpose is to encourage the child himself to explore and to discover, to identify the idea in question, to relate the new idea to previously-known ideas and thus to construct a meaning for it that can function in true quantitative thinking.

It is not necessary, nor even desirable, to present every one of the activities for each new learning in the exact sequence described above. The nature of some pupils and some arithmetic learnings will be such that many of the steps may be omitted. Other pupils, or other learnings, will require many different activities at each level in the sequence. To see how the Manual and the text present some of these activities for lessons involving decimal fractions, turn to pages 231–234 in the Manual and look at reproduced text pages 216–219 along with the accompanying teaching suggestions. Note particularly these points:

a. The need for the use of decimal fractions in a typical life-situation is established on pages 216 and 217 in the text with the picture of a stop watch and an explanation of its functions in relation to a real situation. Moreover, the Manual suggests other instruments such as a pedometer, thermometer, and the like, which can be used to establish the need for decimals.

b. The Manual also suggests that pupils reproduce tenths using real objects such as pie plates, circles cut from cardboard, and the like.

c. For experiences involving representative objects, the Manual suggests the use of calculating strips. (See pages 312–313 in the Manual for a more complete explanation of the construction and use of these strips.)

d. The picture of the stop watch on page 217 in the text presents also an opportunity for children to identify tenths using a picture of a real object.

e. For experiences in identifying tenths through the use of semi-concrete pictures, the Manual suggests the use of graph paper. If a sheet is cut to show ten rows of ten squares each, one row may be colored to show 0.1, two rows to show 0.2, and so on. The use of number lines showing tenths is also suggested, for this represents a form of semi-concrete picture.

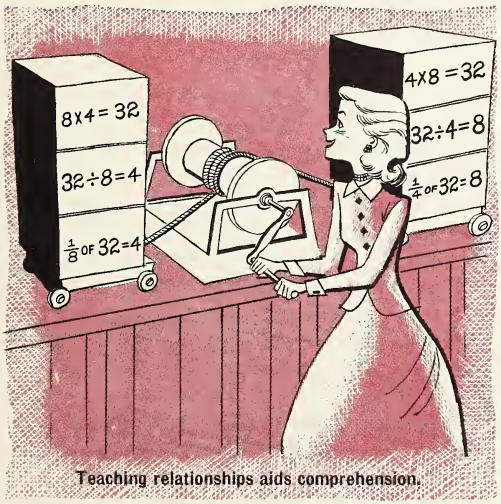
f. The box in Ex. 3 on page 217 in the text, along with all the written work on pages 218 and 219, gives practice in identifying and/or reproducing tenths in abstract form, while Ex. 1–7 on page 218 gives decimals in verbally-described situations.

Teaching Relationships and Generalizations

When we use the words "relationships" and "generalizations," we are referring to ideas that involve two or more concepts. "Three," when it means a group of 3 items, is a *concept*, whereas "three and two are five" is a *generalization*.

The program for teaching relationships and generalizations is a special feature of this series. Let us illustrate for Grade 6 by referring in the three sections that follow to the teaching of common fractions, the teaching of "strands of meaning," and the development of problem-solving ability.

The teaching of common fractions



Teaching relationships aids comprehension.

A few of the many provisions for teaching relationships and generalizations are illustrated in the following outline and brief discussion of the teaching of common fractions:

1. Early in the work with common fractions in Grade 6, the concept of a fractional unit to designate units smaller than 1 whole is reviewed. In dividing a whole into equal parts, we may think of the size of one of the equal parts as the *part unit* or the *fractional unit* by which the whole is measured. In the equal circles A and B below, three parts of each are colored. However, the fractional units are different sizes, so we designate the colored part of A as $\frac{3}{5}$ (three $\frac{1}{5}$'s) and the colored part of B as $\frac{3}{8}$ (three $\frac{1}{8}$'s). Like-fractions are defined as fractions with the same fractional unit as shown by the denominator.

A

B

2. The sequence of presentation and the many opportunities afforded for manipulation and visualization of fractional parts make it possible for pupils to discover such relationships as the following:

a. The more equal parts into which a whole is divided, the smaller each equal part will be.

b. Denominators tell the size of the fractional unit and numerators tell the number of equal parts in the fraction.

3. Practice in counting by fractional parts ($\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{5}$'s, etc.) on a ruler, on a number line, or on a fraction chart helps pupils understand the equivalence of improper fractions and whole or mixed numbers, as in counting by thirds in the sequence given below.

$\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$ (or 1), $\frac{4}{3}$ (or $1\frac{1}{3}$), $\frac{5}{3}$ (or $1\frac{2}{3}$), $\frac{6}{3}$ (or 2), etc.

4. Pupils study and construct for themselves fraction diagrams like the one below to show different fraction families.

One whole											
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$			
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

From these activities, pupils discover many relationships, such as these:

a. Changing a fraction to an equal fraction in higher terms means making more parts and smaller parts ($\frac{5}{6} = \frac{10}{12}$).

b. Making a denominator twice as large makes the fractional unit half as large ($\frac{1}{12}$ is half as large as $\frac{1}{6}$).

c. Making a denominator 4 times as large makes the fractional unit $\frac{1}{4}$ as large ($\frac{1}{8}$ is $\frac{1}{4}$ as large as $\frac{1}{2}$).

d. When we divide both terms of a fraction by 3, we get an equal fraction because dividing the denominator by 3 makes the fractional unit 3 times as large and dividing the numerator by 3 makes $\frac{1}{3}$ as many parts ($\frac{6}{9} = \frac{2}{3}$).

5. After pupils have had wide experience in studying and visualizing relationships between the numerators and denominators of equal fractions, they are ready for the "Golden Rule of Fractions":

Multiplying or dividing both terms of a fraction by the same number changes the form but not the value of a fraction.

6. In adding and subtracting fractions, pupils find that carrying and borrowing are necessary. Carrying when the sum of the fraction column is an improper fraction necessitates regrouping parts in terms of the size of the fractional unit. It takes 16 sixteenths to regroup as 1 whole, while it takes only 4 fourths to regroup 1 whole. This can be contrasted with carrying in whole numbers and in decimal fractions when the regrouping is done in terms of the standard-sized 10 group.

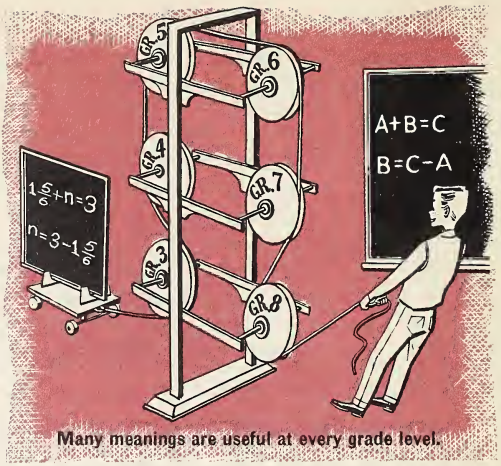
Borrowing from the whole number when the fractions cannot be subtracted necessitates breaking 1 whole into parts of the size designated by the fractional unit. This can be contrasted with borrowing in whole numbers when 1 ten, 1 hundred, or 1 of any order is regrouped as 10 of the next smaller order.

Teaching "strands of meaning"

You may remember that the first part of this section emphasized the fact that arithmetic is a system. This means that arithmetic is tied and bound together by various logical relationships. Some of these relationships we call "strands of meaning," and these strands are laced through each grade level of instruction from Grade 3 through Grade 8. Some examples of this pattern in *Arithmetic We Need* are given below.

a. The order of addends does not affect the sum.

b. Only like-numbers can be added or subtracted.



Many meanings are useful at every grade level.

c. To find the difference between two numbers, subtract.

d. If you know the product of two factors and one of the factors, you can then find the other by dividing.

e. Multiplying or dividing the dividend and the divisor by the same number does not change the quotient.

Note that these "strands of meaning" operate not alone in arithmetic, but throughout the whole range of mathematics. (This fact is one reason why arithmetic *is* mathematics and why the study of arithmetic is fundamental to a comprehension of the mathematics studied in later grades.)

Example *a* results, in arithmetic, in the same sum for $3 + 2 + 5$ as for $5 + 3 + 2$ and for $2 + 5 + 3$; in algebra, for $6a + 2a + 3a$ and for $2a + 3a + 6a$ (or for any other order of the three addends).

Example *b* tells us that we cannot directly add 3 girls and 6 cabbages. Instead, we must first change "girls" and "cabbages" to make them somehow alike, perhaps as "objects." Likewise, we cannot add $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ until we make them like-fractions by changing them to equivalent fractions with 8 as the denominator, $\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$. So, in algebra we can add the a terms in $4a + 3c + 5a$, but we cannot combine $3c$ directly with $9a$ because the terms are unlike.

Example *c* appears in arithmetic in such guises as, "How much larger is 45 than 9?" and in algebra as, "What is the difference between 46 *xyz* and 15 *xyz*?"

Example *d* occurs in arithmetic as, "If a man walked 12 miles in 3 hours, what was his average

rate of speed?" In algebra, "If $c = ab$, then $a = \frac{c}{b}$.

Example *e* is helpful in arithmetic in dividing by 2-place numbers. Pupils discover that $630 \div 90 = 7$, just as $63 \div 9 = 7$. In algebra, the same principle applies in simplifying the radical $2b\sqrt{\frac{a}{b}}$, since multiplying both the numerator (dividend) and denominator (divisor) of $\frac{a}{b}$ by b yields the equivalent fraction $\frac{ab}{b^2}$, which makes it possible to express $2b\sqrt{\frac{a}{b}}$ as $2\sqrt{ab}$.

There are many such strands of meaning in arithmetic. They are not new discoveries, for their existence has long been known. Indeed, they are implicit in arithmetic, whether we identify them or not. Certain of these "strands" *have* been identified and have been taught in arithmetic, but in our opinion, not enough of them. As a consequence, arithmetic has been less a system than it might be. Hence, we have increased the usual number of strands of meaning in the program in *Arithmetic We Need.*

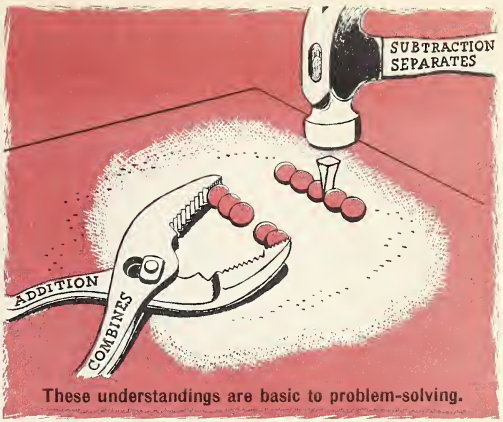
Strands of meaning might be justified as learning outcomes in arithmetic because of their usefulness in later mathematical study; but no such remote reason need be cited. These broadly applicable relationships contribute in a major sense to understanding, to flexibility in quantitative thinking, and, perhaps more practically, to the quick and resourceful solution of many kinds of common problems in daily living.

As they are introduced in the text, strands of meaning are not difficult to learn. They appear first in the simplest of situations where they seem to be almost self-evident truths. They appear later in the grade and in subsequent grades, each time in new situations and in forms a bit more complicated, but always within the grasp of children.

The development of problem-solving ability

A comparison of a problem with its corresponding abstract example is enlightening. *Problem:* If John earns an average of $2.75 a month, how much will he earn in a year? *Example:* $12 \times \$2.75 = ?$

In the example, the sign "×" tells the children to multiply; it is one of a few direct cues for this process, others being "product" and "multiply." None of these cues occur in the problem, however.



These understandings are basic to problem-solving.

Somehow children must find in the problem the need for multiplication if they are to solve it correctly and economically. But how does one do this? If we can answer this question, we shall be successful in teaching our children to solve problems.

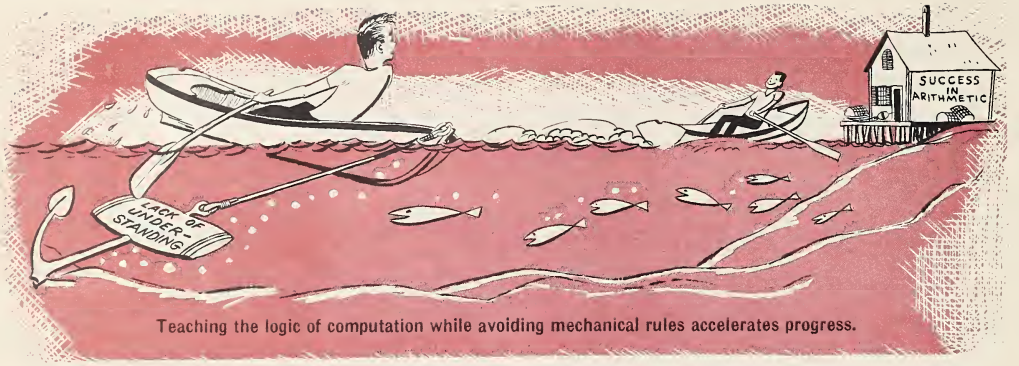
The answer to this question has two facets: one relates to the meanings of the basic operations; the other, to the translation of the language in the problems into these meanings. Let us start with the second facet.

The problem given above would involve the same computational skills and pupils would find it much easier to solve if it were stated thus: If John earns $2.75 each month, what total amount would he earn in 12 months? The original problem necessitates the correct interpretation of the term "average" and a knowledge of the fact that 1 year equals 12 months. When the word "total" is included we imply that numbers are to be combined. Because the numbers to be combined are equal, we can multiply.

Word problems in Grade 6 provide a wide variety of language patterns so that pupils may have extensive practice in identifying the problem requirements and in selecting the correct process. Problems range in complexity from those in which the process can readily be selected to those in which pupils must read between the lines.

Competence in the first-mentioned facet is attained by teaching pupils the basic meanings, or the purposes, of the four fundamental operations.

It becomes obvious, then, that a careful teaching of the meanings of addition, subtraction, multiplication, and division processes is fundamental to the



Teaching the logic of computation while avoiding mechanical rules accelerates progress.

problem-solving program. You can quickly discern how well the Grade 6 pupil's book and this Manual provide for this by glancing at the development in the text of the meaning of addition on page 7; of subtraction on pages 13 and 14; of multiplication on pages 28 and 29; and of division on pages 46 and 47.

Note also the many different kinds of problem-solving activities provided in the text. So that children will have experience in dealing with those parts of the language which carry the key to the process to be used in solving problems, they are asked to select processes without solving problems, and to defend their choices; they are directed to write questions to complete unfinished problems in the different operations; they draw pictures and tell the story in their own words; they write original problems for this or that operation by using given facts; they work problems which contain facts unnecessary for the solution of the problem; and they obtain data for problems from pictures, tables, maps, and graphs.

Developing Rational Computational Skills

There is no reason in the world why computation and computational forms (algorisms) cannot be taught so that they make sense to children, and there is every reason why they should be so taught. In their initial experiences with new skills and subskills in each book in this series, children are led to see the logic of what they are learning. Explanations that really explain are employed in place of mechanical rules. Where helpful in making a record of what is being done, crutches are used without apology. At the outset, understanding is the first consideration. When this understanding is achieved, the proper time has been reached for eliminating uneconomical thought patterns (full explanations) and written devices (crutches).

We cannot at this point outline the program of instruction for every computational skill, so suppose we select the reteaching of multiplying by a 2-place number and by a 3-place number and see what is done and why.

The developmental steps which provide readiness and meaning have their beginnings in Grades 3, 4, and 5.

1. To multiply 2- and 3-place multiplicands by 1-place numbers, we may multiply the multiplicand in parts: ones, tens, then hundreds; or we may multiply in the reverse order if we use the "long way" and place each product correctly. For the "short way" we use carrying (regrouping in terms of the ten's system).

A

Long way	Short way
54	54
× 3	× 3
12	162
150	
162	

2. When the multiplier is an even ten's number, as in box B, pupils learn that we multiply by even tens just as we multiply by ones. We write a zero in one's place in the product to show that the product means tens.

B

56	8	21
× 10	× 30	× 40
560	240	840

3. When the multiplier means tens and ones (i.e., 24 × 16), pupils may first find the product by making three separate examples as in box C.

C

16	16	64
× 4	× 20	+ 320
64	320	384

4. Next, the consolidated form is presented. Pupils are now ready for the generalization that the right-hand figure of each partial product is written in the same column as the figure by which we multiply.

D

$$\begin{array}{r} 16 \\ \times\ 24 \\ \hline 64 \\ 32 \\ \hline 384 \end{array}$$

As in box E, *slower learners* may need to use zeros for place holders in the ten's and hundred's partial products if these aids (or crutches) are necessary for their success with 3-place or 4-place multipliers. However, when understanding of the procedure has been established, most pupils should be encouraged to shorten the work and to utilize the generalization stated above for the placement of partial products.

E

$$\begin{array}{r} 286 \\ \times\ 324 \\ \hline 1{,}144 \\ 5{,}720 \\ 85{,}800 \\ \hline 92{,}664 \end{array} \qquad \begin{array}{r} 286 \\ \times\ 324 \\ \hline 1{,}144 \\ 5{,}72 \\ 85{,}8 \\ \hline 92{,}664 \end{array}$$

This one illustration of rational computational skill development may be used to make several points:

a. Care is exercised to use language expressions that carry appropriate meanings in an immediately sensible way.

b. The reasons for placing figures where we write them in computation are fully explained, and these reasons are found in the meanings of numbers and in the logic of place value. (Here, note the reasons for the placement of the ten's and the hundred's partial products.)

c. Crutches are employed when they help children to see the nature of computational procedures. (Here, zeros are written to show each complete partial product.)

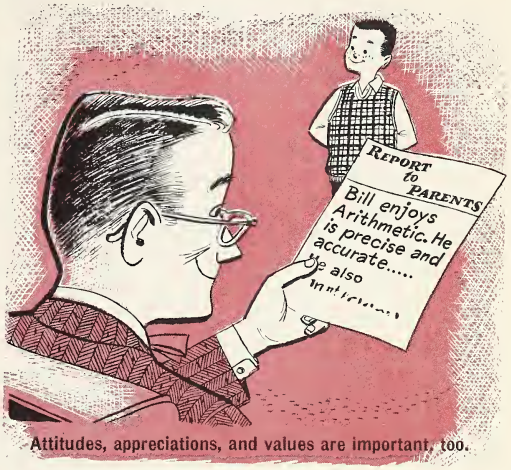
d. The pace of instruction is unhurried; understanding is established first, and then steps are taken to assure both accuracy and speed in performance.

What has been said about the reteaching of rational computation for 2-place and 3-place multipliers holds for other skills as well. In division with 2-place divisors, pupils estimate the number of 62's in 253 (as in box F) by using the short cut, "6's in 25," only after they have used the 60 (in 62) and the 250 (in 253) as guide figures and have found that the quotient is the same if dividend and divisor are both divided by the same number (10 in this case).

F

	Estimate
$62\overline{)253}$	$60\overline{)250}$ or $6\overline{)25}$

Only through such a program will children attain the mathematical aim of arithmetic. Indeed, only in this way can they get very far with the social aim. Our ability to use skills intelligently is dependent upon the degree to which we have employed intelligence in learning the processes which precede the skill development.



Attitudes, appreciations, and values are important, too.

II. The Cultivation of Desirable Attitudes, Appreciations, and Values

About the only time one hears about attitudes in relation to arithmetic is when the attitudes mentioned are bad, that is, when children are found to be indifferent or hostile to the subject. One might almost get the impression that, emotionally, arithmetic must necessarily be harmful rather than helpful. Such, of course, is not at all the case. Properly taught—and thus, properly "learned"—arithmetic makes a positive contribution to the personality and to wholesome, happy adjustment to life situations.

The chief reason why children may react negatively to arithmetic is that they do not understand it and so do not know how and when to use it. The program in each book in this series is planned to remove adverse reactions by preventing their occurrence. Desirable attitudes, appreciations, and values are just as important outcomes of arithmetic instruction as are any others; and they are just as attainable. Specifically we believe that:

1. Children should attach value to order, precision, and accuracy in quantitative matters.

2. Children should appreciate the logic of arithmetic and of arithmetical procedures.

3. Children should approach each successive learning task with the intent and the will to understand.

4. Children should acquire the disposition to use arithmetic and the habit of using it in their lives.

There is nothing mysterious about the manner in which we acquire desirable attitudes, appreciations, and values. Emotional ways of reacting are, like all other learning outcomes, the results of our total experience. If we want children to expect to understand arithmetic, we guide them so that they can understand, and we help them see the worth—the practical benefits—of understanding. If we want them to appreciate order and precision, then we encourage them to be orderly and precise in quantitative situations, rewarding them when they so behave—or, better, we let them find the reward for themselves in the consequences of their behavior. True, some of our attitudes and values emerge more or less accidentally and incidentally; but when those values and attitudes are truly important, we are wise to cultivate them as goals of planned instruction.

Other Features of the Text

I. Teachability

Extra pains have been taken to make each book in this series a teachable text. We have already alluded to the clue captions placed inconspicuously on the pages of the text to tell you at a glance the purpose of each lesson.

As a further aid, chiefly to the problem-solving program, crucial words and phrases are underscored in red to help you and your pupils realize quickly the important points that are being introduced, and whenever emphasis will aid learning, or relationships need to be brought out, red print is used. Moreover, as you teach you will increasingly appreciate the careful organization of the pages in the text, with their sequential order of learning activities to promote learning and facilitate teaching in the most economical way.

Simplicity and clarity of presentation which further aid the teaching of the concepts outlined above have been achieved through the use of short, uncomplicated sentences. Few paragraphs comprise more than three sentences or take more than four lines of print. Such a clear, concise presentation aids inestimably in achieving a ready understanding of the point at hand.

Reading interest has been assured primarily by the choice of subjects and topics from situations common in child life: pets, school happenings, parties, hobbies, shopping, visits and excursions, and the like. The interest inherent in these situations is enhanced by the lively and attractive illustrations and pictures with which *Arithmetic We Need* is replete.

Notice, also, the functional use of these illustrations. In some cases, the practical, social use of arithmetic is emphasized. In other cases, groups of real objects representing actual arithmetic computations are shown. At a more advanced learning level, graphs, diagrams, tables, and the like appear. Whatever the case, the illustrations and the use of color are designed to produce a maximum amount of learning.

II. Provisions for Oral Experiences

You will be impressed by the large amount of oral work provided in *Arithmetic We Need.* This is one of the features of the text which might have been included under "teachability" above, but which is given a separate heading and special discussion here. New ideas and skills are regularly introduced orally. Here are a few of the many possible illustrations:

The meaning of a fractional unit (page 99)

Changing fractions to equal fractions with larger or smaller fractional units (page 102)

Improper fractions and mixed numbers (pages 100 and 108)

Borrowing in subtracting mixed numbers (page 113)

Estimating the whole quotient (page 61)

Oral work reveals the pupil's thought processes.

Varied, challenging, and interesting oral exercises provide cumulative review of meanings, relationships, and generalizations. Such exercises are often identified by these titles:

Oral Exercises (pages 109, 201, 212)
So You Won't Forget! (pages 156, 241, 248, 275)
Oral Practice (page 274)

One purpose of the oral work is, of course, to provide you with as much help as possible in the guidance of class discussion. Another, and probably even more important purpose, is to provide material that will enable you to keep track of the thought procedures your pupils are using. Paper-and-pencil assignments are inferior to oral reports as means of revealing children's mental processes. When you have only the written record before you, you know the results of thinking, the *product* of thinking; but with this record alone you cannot with much confidence get behind the product to the *process* used.

There has been a tendency to neglect the matter of thought processes used by pupils as they learn arithmetic. You well know that correct answers may be arrived at by faulty methods, even by juggling numbers, and that incorrect answers may result from good thinking which has gone astray at some minor point. Both types of errors, perhaps not evident in written work, become unmistakably clear when children "think out loud"—hence, the prominence given oral work in this text.

If faulty mental processes can be checked early, much future trouble will be avoided. Most teachers have had the experience of discovering some slight misconception that has acted as a barrier to further learning. If these barriers are not removed, the learning process is impeded, confusion sets in, and the child despairs. Often proper use of oral work will rectify the misconception and set the child free to learn.

Frequently, pupils find oral work more stimulating than the teacher's explanation followed only by written practice. Not only in arithmetic, but in most other areas, pupils need numerous opportunities to develop confidence in the oral expression of ideas.

III. Practice Material

Once children understand what they are learning, they need practice to become more proficient in their skills and to fix those skills for long-time retention. You will find ample practice material for each new skill immediately following the initial development in the text and in mixed-practice exercises appearing thereafter. Further practice is provided at the end of the text, where there is a reservoir of "Extra Practice." This extra practice is to be drawn upon whenever the amount of practice in the text is less than enough to develop the desired degree of competence. References on the proper text pages serve as a guide to use of appropriate Extra Practice Sets. When so mentioned in the text, the corresponding teaching pages in the Manual will show the reproduced sets with answers in red. As a further aid, the Manual index lists practice sets by number and refers to the

Reteaching and practice are vital to instruction.

Manual page where the answers may be found in case you wish to adopt your own procedure for assigning the sets.

The Extra Practice material is also intended for use in providing for individual differences. Slower learners need extra practice, and on occasion so do the more capable children. This work offers the means for practice that will give the slower learners further impetus and the more capable learners greater facility as well.

IV. The Evaluation Program

True evaluation demands a complete testing program.

The unusually complete program for evaluation comprises six types of test:

1. Pre-tests to inventory retention of previously-learned skills (see pages 7, 10, 12, 16, 27, etc., in the text)
2. Chapter achievement tests in computation (see page 43 in the text)
3. Chapter diagnostic tests in computation (see page 42 in the text)
4. Chapter tests of information and meanings (see page 41 in the text)
5. Chapter tests in problem-solving (see pages 42 and 43 in the text)
6. Term tests (see pages 304–308 in the Manual)

The inventory tests, item 1, will help you to determine your pupils' backgrounds and readiness for learning.

Chapter tests, items 2–5 above, are intended to measure achievement and to diagnose shortages over fairly extensive periods of time. If you wish to use the computation tests in the text for practice purposes, you will find alternate tests in this Manual, pages 301–303.

Two optional term tests are provided in the Manual, one set for each term, on pages 304–308.

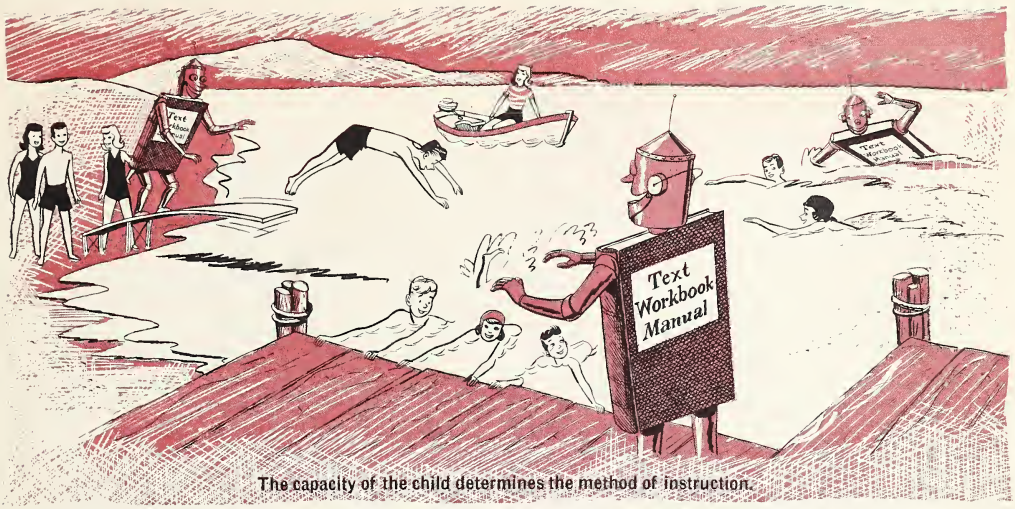
With this array of tests you will be able to determine, at any time, both the strengths and weaknesses of your pupils in most of the essential areas of arithmetical learning. More than that, your pupils will have corresponding information for their own enlightenment, stimulation, and help.

Another value of such a battery of tests is that the pupil develops a certain self-reliance. Informal tests in the number presented in this series tend to carry the pupil beyond his first fright into a more casual frame of mind in approaching tests; he then attacks the work with more objectivity. Also, the teacher gains a better knowledge of the areas of doubtful understanding. Situations which otherwise might go untouched for some time to the detriment of the pupil are readily located, analyzed, and corrected with frequent testing. Repeated practice on newly-learned and previously-mastered skills is provided. By dint of such a testing program, capability is kept at a uniform level and no one phase slips behind the other. Moreover, both pupil and teacher become aware of the growth in arithmetical competence.

Provisions for Individual Differences

That children differ both in capacity and in achievement—and that they differ over a wide range—is by now a trite observation. Just what is to be done about these differences is, however, another matter. Certainly the common practice of using symbols in arithmetic texts, some to designate exercises for slow learners, others for more capable children, does not meet the need. The designated activities are far too few and are too unevenly scattered throughout the books to deserve being called "a program to meet individual differences"; and the space devoted thereto may be much better employed for other purposes.

In *Arithmetic We Need*, each teaching lesson in the text is used for a full, complete development of arithmetical content, and a very complete program for differentiating instruction is provided in the accompanying Manual and Workbook.



The capacity of the child determines the method of instruction.

This does not mean that the text serves no purpose in the matter of individual differences. The tests provided are of value in locating children in difficulty and, when supplemented by interviews, in diagnosing the sources of their difficulty. Mention, too, has been made of the reservoir of practice material in the text as offering one kind of help in dealing with individual differences, but the Manual affords a better and a more comprehensive way of offering suggestions. Moreover, these suggestions can be made more effective through presenting them in the Manual, since they can be related directly to the teaching program, lesson by lesson.

A word should be added about the Workbook. The Workbook is not a duplicate of the textbook, nor is it a drill book. Instead, it has been designed with special attention to individual differences. Where understanding needs reinforcement, the Workbook enables the pupil to study out the idea again. Where skills seem to be unusually difficult, the Workbook provides extra practice.

The presentation in the Workbook does not repeat that in the text. Development proceeds along parallel lines, serving the pupil as a check to see whether he has understood the point at hand in the text. Each Workbook section contains only the vital points from the text with a few selected examples. Functional illustrations also aid in clarifying less easily learned concepts.

The combination (*a*) of a teacher well prepared in the mathematics of arithmetic and sensitive to the social uses of arithmetic and to the way children learn; (*b*) of a good textbook; (*c*) of a manual full of helpful ideas; (*d*) of a workbook written with the deviating child in mind—this combination is hard to equal in working out a real program for meeting learning differences among children.

Making Arithmetic Function

Up to this point we have dealt with topics relating chiefly to the mathematical aim of arithmetic. This fact, however, will not be construed to mean that we are indifferent to the social aim. We have mentioned the latter aim too often to warrant this idea, and we repeat once again our belief that arithmetic must be taught so that it functions in the lives of children as they learn it.

Nevertheless, it remains true that the ultimate achievement of the social aim is uniquely the responsibility of the teacher. As authors, we have gone as far as we can go. We have consistently pictured arithmetic as a normal, natural phase of life by showing the many situations in which it occurs and the many ways in which it is helpful. But after all, what we have had to say is *arithmetic in a book* and, while much of the program must come from books, there must also be provision for *arithmetic in life*.

On the next page we are suggesting ways to get part of the arithmetic program away from the book and away from the special arithmetic period and

Each aim of the program must carry equal weight.

into the daily activities of your pupils because only by so doing will they really comprehend its value. After all, this is the fundamental reason for studying arithmetic, and is certainly deserving of more than mere lip service.

1. Have your pupils cite real occurrences matching those described in the various problems of the textbook, thus relating what they read to their own experiences.

2. Encourage your pupils to extend sets of problems to include related experiences in their lives.

3. Take advantage of the opportunities available for using familiar arithmetic and for motivating new learning in the activity units organized in your grade or class.

4. Consider what can be done with arithmetic as you teach other subjects. Even the simple "history" and "geography" of Grade 6 contain much quantitative matter and many quantitative expressions.

5. Do not overlook the arithmetic that is part of many routine activities in the classroom (e.g., passing papers and estimating needed supplies) and in the direction of special projects (e.g., the holding of sales, the collecting of rummage and other things).

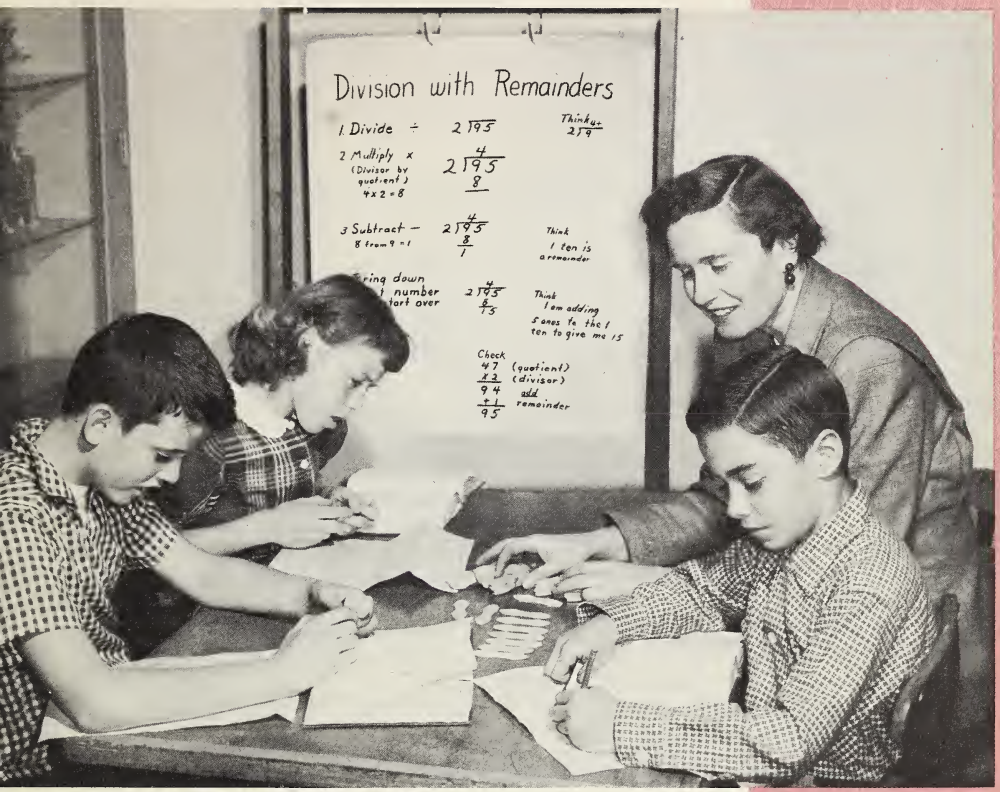
6. Look for arithmetic in the incidental happenings in the school and call it to the attention of your pupils. Children must come to school and be at other places at stated hours. How can they do these things without using numbers? How else than through arithmetic can they keep score in games, and, in some cases, play them?

7. Hold children to precision in the quantitative aspects of the reports they make.

8. Encourage children to keep arithmetic notebooks and to make arithmetic collections which will contain illustrations of arithmetical usages found in newspapers and magazines, advertisements, and the like.

9. In introducing a new arithmetical skill, relate it to some natural or contrived situation in the classroom to impress children with the usefulness of the thing they are about to learn.

The catalogue of nine items above does not exhaust the possibilities. Perhaps the most valuable and practical hint that can be given is this: Become steadily more sensitive yourself to the ways in which arithmetic contributes to the ease and richness of your living. You will then be in a better position to interpret these values to those you teach.


Division with Remainders
1. Divide ÷
2 Multiply x (Divisor by quotient) 4x2=8
3 Subtract — 8 from 9 = 1
Bring down 5 number start over
Think 2 into 9
Think 1 ten is a remainder
Think I am adding 5 ones to the 1 ten to give me 15
Check 47 (quotient) x2 (divisor) 94 add +1 remainder 95

Audio-Visual Department, Alhambra City Schools

Bundle numbers made up of wooden spoons or other representative objects are an invaluable aid in reteaching whole-number operations.

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Teaching Chapter 1 of Grade 6

A separate division with a heading similar to that above is provided in this Manual to accompany each chapter of the text. The general pattern is to provide first an overview of the chapter by presenting the learning outcomes expected and a discussion of certain of these outcomes. The remainder of each division is devoted to a detailed discussion of the teaching of each lesson.

Introduction

To develop confident, self-reliant learners of arithmetic in Grade 6, it is desirable to devote some time early in the year to an overview of the work of earlier grades. For some children, new learnings or relearnings will result, while other children will gain the satisfaction that comes from realizing that they have a firm foundation upon which to build new abilities, knowledge, and understandings. The principal purpose of Chapter 1 is to review what has been taught in the earlier grades about addition, subtraction, and multiplication.

I. Learning Outcomes in Chapter 1

The most convenient way to summarize the content of Chapter 1 is by means of the following list (similar lists preface each Teaching Chapter to focus your planning):

1. Understanding of and ability to read and write numbers through billions
2. Ability to read and write Roman numerals
3. Understanding of addition and knowledge of the addition facts and families
4. Skill in working harder addition examples with up to six 4- and 5-place addends
5. Understanding of subtraction and knowledge of the subtraction facts and families
6. Skill in working harder subtraction examples with up to 5-place minuends and subtrahends
7. Understanding of multiplication and knowledge of the multiplication facts
8. Skill in working harder multiplication examples with up to 8-place products
9. Ability to solve problems
10. Understanding of various specific concepts, generalizations, relationships, and skills
11. Desirable emotionalized responses (attitudes, appreciations, values)

II. The Nature of the Major Outcomes

Understanding of and Ability to Read and Write Numbers through Billions (pages 1–3, 6)

Pupils learn to read, write, and compute with large numbers intelligently only if they understand numbers as part of a highly logical system. In fulfilling this outcome, pupils will gain a deeper insight into the characteristics of the number system: its base is 10; it incorporates the principle of place value; zero is both a number and a placeholder. These characteristics make it possible to write any number however large by using only 0 and the symbols from 1 through 9.

Understanding of the logic of our number system is developed through such activities as the following:

a. Seeing numbers pictured as bundle numbers
b. Thinking of numbers as built up by multiplying and adding

c. Building numbers and taking them apart in a variety of ways (see page 2 in the text)

d. Reading numbers by seeing and thinking the place values and the period values

e. Noting the result when figures in a number are interchanged (358, 385, 835, 853, etc.)

f. Noting the change in a number if a figure is replaced by zero (as 308 instead of 358)

g. Showing numbers on a number line

Ability to Read and Write Roman Numerals (page 5)

In Grade 6 pupils extend this ability to include reading and writing thousands. Although Roman numerals are not used in computation, pupils do need some understanding of the meanings of the symbols and of the principles used in reading and writing them. There is also great value in presenting to pupils another kind of number system. They are led to discover that the Arabic notation uses ten figures (0 through 9) and incorporates the idea of place value while the less efficient Roman notational system has no place value and only has seven number-letters (I, V, X, L, C, D, M).

Understanding of Addition and Knowledge of the Addition Facts and Families (pages 7–9)

While most children in Grade 6 should be adept at using the addition facts, it is best not to take this for granted. That is the reason for the inventory test of page 7 and the reteaching pages in addition that follow.

Pupils should understand that addition is a "putting-together" process involving things that are alike in some way. They should also be able to read addition examples and addition facts using the terms "plus" or "and." If they are not proficient in using the addition facts, several helpers are given on page 8 and pupils are helped to use these facts in adding a 2-place and a 1-place number both when there is and is not bridging into the next decade.

The addition of columns of 1-place figures, as presented on page 9 in the text, demands a knowledge of the basic addition facts and ability to add a 2-place to a 1-place figure mentally. Since this skill is also required in adding columns of larger numbers, you will want to make sure that your pupils have a good strong foundation. Considerable reteaching may be necessary to ensure the later success of your pupils.

Skill in Working Harder Addition Examples (pages 10, 11, 22)

After pupils have reviewed the meaning and use of addition and have worked to gain automatic response for the basic addition facts and families, they take Inventory Test 2 to evaluate their ability to add larger numbers. If pupils have difficulty, they will be helped by a reteaching program designed to bring out some of the fundamental ideas used in adding larger numbers.

The basic concept for pupils to understand is that our numbers belong to a ten-system in which we add first ones, then tens, then hundreds, etc. (adding tens, hundreds, etc., just as ones are added), grouping ten of each smaller unit as one of the next larger-sized unit. Accordingly, 10 ones are regrouped and carried as 1 ten; 10 tens are regrouped and carried as 1 hundred; and so on. Carried tens are added with the given tens in the example; carried hundreds are added with the given hundreds; etc. Once this idea is clear, there is nothing new in adding larger numbers. Of course, when six addends of 4 or 5 places are to be added, there is more chance for error than in examples of less length, but there are no new concepts to be learned.

Understanding of Subtraction and Knowledge of the Subtraction Facts and Families (pages 12–15)

Inventory Test 3 on page 12 will help you discover which pupils have difficulty in understanding the meaning of subtraction and in using subtraction facts. Some of the important points made on page 13 in reteaching the meaning of subtraction are listed below.

a. Subtraction is related to addition. If we know one number and a sum, we subtract to find the other number.

b. We can write the subtraction in two ways (horizontally or vertically).

c. Subtraction examples may be read using the word "minus" or "from."

d. If the other number in subtraction is a number left or a number gone, we call it a *remainder*.

e. When we subtract to find the other number in a comparison situation, we call it a *difference*.

f. The number subtracted from is called the *minuend*, the number that is subtracted is called the *subtrahend*.

g. Subtraction may be checked by adding the remainder and the subtrahend to see that it equals the minuend.

It is no safer to assume that pupils know all of the subtraction facts than it is to assume that they know the addition facts. There are, however, as in addition, several helpers that pupils can use to help them learn the subtraction facts. These are presented on page 14 in the text.

Once the facts are learned, they are incorporated into families of facts (the family of facts for $8-5=3$ are $18-5=13$, $28-5=23$, $38-5=33$, etc., to $98-5=93$). In some families of subtraction facts the ten's figure in the remainder is the same as the ten's figure in the minuend, but in others the ten's figure in the remainder is one less than the ten's figure in the minuend. Pupils should work toward a high level of proficiency in using families of facts, for they are also important in the subtraction step in division.

Skill in Working Harder Subtraction Examples (pages 17–19, 22)

The Inventory Test on page 16 will help you decide which pupils have difficulty in subtracting larger numbers. If the test uncovers pupils who need help, work is provided so that pupils may be retaught some of the basic ideas for subtracting: that we subtract ones first, then tens, then hundreds, and so on; and that when we cannot subtract, we change the minuend (i.e., we "borrow" one of the next larger units to make ten of the smaller units).

Notice the "crutch" (small figures, some of which are circled) in the work on pages 18 and 19. Pupils can use this crutch, if necessary, in showing the changes in the minuend, but it should be eliminated as soon as possible, since it will cause confusion if used in the subtraction step of division. Notice, also, that special help is given when there are zeros in the minuend. When the minuend has several zeros, as in 3,005 minus 286, pupils first think of the number as being made up of 300 tens and 5 ones. They may then borrow a ten and show the 299 tens left as in box D on page 19.

Understanding of Multiplication and Knowledge of the Multiplication Facts (pages 27–29)

After pupils take Inventory Test 5 on page 27 in the text, help them gain a broader and more complete understanding of the nature and meaning of multiplication. These are some of the points that should be made:

a. Multiplication is the process we use to find totals when the groups or the numbers to be combined are equal.

b. In a multiplication example, one of the factors indicates the size of the equal groups and the other factor indicates the number of equal groups to be combined.

c. The product does not change if the factors are reversed ($6 \times 8 = 8 \times 6$).

d. If one factor is 0, the product is 0.

e. If the multiplier is 1, the product is the same as the multiplicand.

f. If the multiplier is larger than 1, the product is larger than the multiplicand.

Pupils should also be familiar with the terms *multiplicand*, *multiplier*, *product*, and *factor*, and should be able to multiply in either the horizontal or the vertical form using the language "five 8's are 40" and "5 times 8 is 40."

Skill in Working Harder Multiplication Examples (pages 30–38)

Once pupils understand the meaning of multiplication and have good control of the multiplication facts, they are given systematic redevelopment of the generalizations important in computation involving multiplication. This skill is also extended in Grade 6 to include multiplying two 4-place numbers that result in 7- and 8-place products.

The important concept in multiplication—that tens, hundreds, thousands, etc. may be multiplied just like ones—is presented on pages 30 and 31, both without and with carrying. Then an inventory test is provided so that you can tell which pupils need help in multiplying by 2- and 3-place numbers. As pupils learn how to multiply first by ones, then by tens, etc., and how to write partial products, they are given much computational practice and considerable help in multiplying when there are zeros in the multiplier or multiplicand (pages 33–38).

Ability to Solve Problems (pages 24–26, 40)

The problem-solving program cannot be limited to specific pages in the text. Almost everything pupils learn about arithmetic has a bearing on their ability to solve problems. Some of the features of the problem-solving program that occur in Chapter 1 are given at the top of the next page.

a. Development of the meanings and functions of addition, subtraction, and multiplication

b. Study of the various language forms in which the need for addition, subtraction, and multiplication may be indicated

c. Use of rounded numbers in estimating answers.

d. Finding missing addends and missing minuends represented by the letter *n*

e. Solution of problems in which pupils must differentiate between addition, subtraction, and multiplication

f. Solution of two-step problems involving hidden questions

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

add, addition (A.)
addend
Arabic notation
borrowing
carrying
checking
comparison
difference
doubles
estimating
even numbers
factor
facts (number facts)
families (of examples)
hidden question
like-numbers
minuend
minus
multiplicand
multiplier
multiply, multiplication (M.)
n (for missing number)
near doubles
number line
odd numbers
partial products
place value and place names (to hundred billion)
plus
product
remainder
reverse fact
Roman numerals
rounded numbers
subtract, subtraction (S.)
subtrahend
sum
teen-numbers
two-step problems
whole numbers

2. *Important relationships and generalizations.* All relationships and generalizations developed in the text appear in heavy type. The majority of these relate to logical principles that help to bind arithmetic into a rational system. The statement of each logical principle is withheld until its meaning and significance have been carefully and systematically developed. It is not intended that pupils be required to memorize these statements. They are merely the verbal formulations of important meanings and understandings, and it is the understandings the pupils must gain.

The generalizations appearing in Chapter I will be found on the following pages in the text and on the corresponding reproduced pages in this Manual: pages 2, 7, 9, 18, 23, 24, 29, 30, 33, 39.

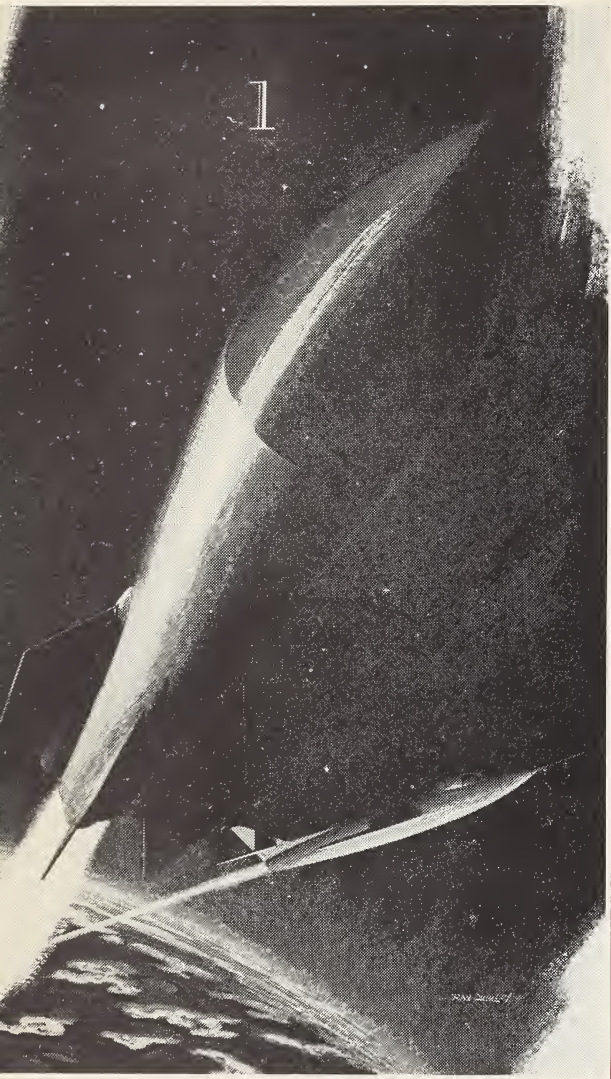
3. *Skills beyond those already listed:*

a. Ability to round numbers (page 4)
b. Ability to check sums and remainders (page 20)
c. Ability to estimate sums and remainders (page 21)
d. Ability to find *n* when it represents a missing addend or minuend (page 23)
e. Ability to estimate products (page 39)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

You may remember the emotionalized responses listed as outcomes on pages 13 and 14 of this Manual. If not, this would be a good time to reread them.

The emotionalized responses listed there should not be outcomes set for attainment in Chapter 1, nor, for that matter, for Grade 6 alone. Rather, they are goals for arithmetic instruction from kindergarten into high school. It is important to discuss emotionalized responses in Grade 6, and in each chapter in the Grade 6 text, because every experience with number can make its contribution toward fulfilling these outcomes. As objectives of instruction these particular attitudes, appreciations, and values deserve your attention as fully as do the more commonly mentioned objectives of skills and meanings.



Higher and Faster!

Reading numbers for meaning {O}

The picture shows a piloted rocket plane that can travel 1,238 miles an hour. It has reached an altitude of 79,494 feet. The guided rocket has gone as high as 1,320,000 feet.

These are days of big numbers. That makes it important for us to be able to read any number quickly and to understand the meaning of each of the figures that make it up.

1. The numbers in the paragraph about rockets are repeated in box A. Read each number. Think about place values if you need to.

2. The 1-place number 3 means this many: ∴ On the board, show the meaning of 4; of 5; of 7.

A

Place value	Hundred million	Ten million	Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	One
Number						1,	2	3	8
					7	9,	4	9	4
				1,	3	2	0,	0	0

3. The number 48 means forty-eight of anything. This 2-place number is built up by multiplying and adding, as shown in box B.

B

This is the multiplying. The 4 of 48 means 4 tens, or 4×10, or 40
The 8 of 48 means 8 ones, or 8×1, or 8
48
This is the adding.

4. Explain the meaning of the 3-place number 572, in box C.

C

This is the multiplying. The 5 of 572 means 5 hundreds, or 5×100, or 500
The 7 of 572 means 7 tens, or 7×10, or 70
The 2 of 572 means 2 ones, or 2×1, or 2
572
This is the adding.

5. Using box A to help you, write at the board the meaning of the 4-place number 1,238; of the 5-place number 79,494; of the 7-place number 1,320,000. (e.g., 1,238 means 1 thousand + 2 hundreds + 3 tens + 8 ones)

Teaching Page 1, the Page Opposite, and Page 2

Pupil's Objectives. (*a*) To develop increased understanding and appreciation of the logic of the number system; (*b*) to review the idea that larger numbers may be built up by combining smaller groups, and that one large number may signify various combinations of smaller groups.

Background. An understanding of the number system is basic to all work in mathematics. The beginning pages in this chapter review the following ideas: the figures 0 to 9 express quantities according to the value of the place that each figure

occupies; the meaning of large numbers may be better understood by observing the parts of which they are composed; a large number may represent not just one combination of smaller parts, but many different combinations (e.g., 1,000 may mean 1 group of a thousand, 10 groups of one hundred each, 100 groups of ten each, or 1,000 units).

Teacher's Preparation. Become familiar with the section headed "Instructional Aids" in Part IV of this Manual. Start to collect some of the devices and equipment suggested. You will find them extremely valuable in clarifying meanings and providing practice.

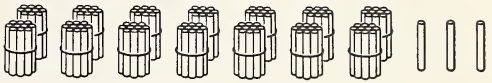
The following materials are particularly helpful at this time: an abacus; peg boards; pocket charts; dowels or cardboard strips in groups of thousands, hundreds, tens, and ones.

Pre-book Lesson

1. If you prefer, use some of the teaching aids suggested above in order to introduce this lesson. For example, use a pocket chart and bundle-numbers to show the several meanings of a number like 143. Start with a hundred-bundle made up of 10 groups of 10 sticks each; 4 ten-bundles, each composed of 10 groups of 10 sticks each; and 3 single sticks. Then break apart the hundred-bundle and show 143 as 14 tens and 3 ones. Finally, break apart each of the ten-bundles and show the number as 143 ones. Using pictures in place of bundle-numbers, these three meanings for 143 might be shown as below.

143 = 1 hundred + 4 tens + 3 ones

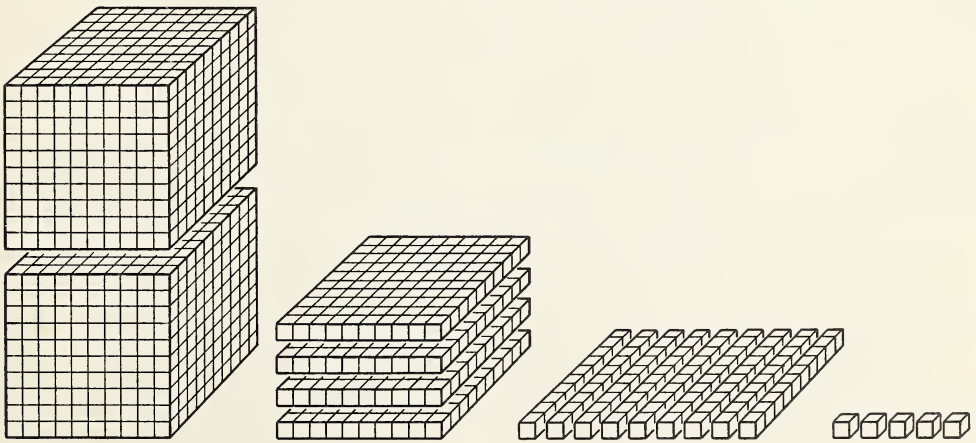
143 = 14 tens + 3 ones



143 = 143 ones

2. With a picture such as the one below, pupils should begin to develop a feeling for the size of larger numbers. Also, by means of questions such as "How many tens make up the number 2,495? how many hundreds? etc." you should be able to lead pupils to see how larger numbers may be thought of in many different ways.

3. If you are unable to provide a picture such as the one below, help pupils develop a feeling for larger numbers by having them try to compare quantities expressed as inches. For example, you might show them a one-inch cube and ask them to mention a distance they think would be about as



2 thousands	4 hundreds	9 tens	5 ones
2 x 1,000	4 x 100	9 x 10	5 x 1
2,000	400	90	5

long as a column containing 1,000 such cubes placed end to end ($83\frac{1}{3}$ feet). You will probably find that some will overestimate and some will underestimate the distance. The same procedure may be used for other numbers, such as those below.

a. 31,680 inches ($\frac{1}{2}$ mile)
b. 950,400 inches (15 miles)
c. 63,360,000 inches (1,000 miles)

Book Lesson (page 1). *Ex. 1–5:* Oral work.

The function of each lesson is described either by the page heading or by a clue caption (smaller, italicized type, usually found below and to the right of the title). Also, all work is clearly designated as either oral or written work by the page heading or by symbols ([O] for oral work, [W] for written work) located at the right side of the page. For example, the clue caption and the symbol on page 1 identify an oral lesson devoted to reading numbers for meaning, whereas the page heading and symbols on page 2 indicate that the lesson helps children learn how to take numbers apart in several ways, and that Ex. 1–8 constitute oral work while Ex. 9–13 are to be done as written work.

Notice, too, that for added convenience all answer material appears in red on the reproduced text pages.

Call attention to the full-page picture of the rocket ships. Tell pupils that on page 1 they will be able to find the speed of the piloted rocket plane and the altitude obtained by both rockets. Diagrams on the board, using arrows to signify speed and altitude, may clarify the terms and concepts. Notice that the guided rocket attained an altitude many times (actually over 16 times) greater than that of the piloted rocket plane. (Notice in the picture that the piloted rocket, distinguished by the Plexiglas cabin, is in the background.)

Book Lesson (page 2). *Ex. 1–8:* Oral work. *Ex. 9–13:* Written work.

1. Provide sufficient time for a discussion of the generalization in dark print at the middle of the page in the text.

2. Observation of reactions of individuals during the oral part of the lesson, as well as the results of written work, will help you to determine which children need more review.

Differentiations and Extensions

1. *More capable pupils* may make charts similar to boxes B and C on page 1, using the terms *product* and *sum* as substitutes for statements about multiplying and adding. When the charts are completed, select some to be put on the chalkboard or the

(*Continued on page 27*)

Taking Numbers Apart in Several Ways

[O]

We may show the meaning of a number of two or more places by taking it apart figure by figure.

1. $56 = 5$ tens and 6 ones, or $5 \times 10 + 6 \times 1$, or $50 + 6$. Is this 2-place number the sum of two products? Yes Explain.

2. On the board, show the meaning of 38; of 46; of 25. 38 = 3 tens and 8 ones, or $3 \times 10 + 8 \times 1$, or $30 + 8$.

3. $732 = 7 \times 100 + 3 \times 10 + 2 \times 1$, or $700 + 30 + 2$. This 3-place number is the sum of how many products? Three

4. On the board, show the meaning of 486; of 594. $486 = 4 \times 100 + 8 \times 10 + 6 \times 1$, or $400 + 80 + 6$

5. $8{,}427 = 8 \times 1{,}000 + 4 \times 100 + 2 \times 10 + 7 \times 1$, or $8{,}000 + 400 + 20 + 7$ This is the sum of how many products? Four

6. In the same way, tell the meaning of 3,426; of 7,694.

7. See if you can tell the meaning of these larger numbers:
a. 38,742 **b.** 209,654 **c.** 1,024,691 **d.** 30,976,507

A whole number is the sum of the products shown by its figures.

There are also other ways of taking a number apart to show its meaning. In the work below, two or more figures are used together to show new meanings.

57,126 = 57 thousands and 1 hundred and 2 tens and 6 ones,
or = 571 hundreds and 2 tens and 6 ones,
or = 571 hundreds and 26 ones,
or = 5,712 tens and 6 ones

(e.g., Row 1: $57{,}126 = 57 \times 1{,}000 + 1 \times 100 + 2 \times 10 + 6 \times 1$; $0 + 5{,}700 + 100 + 20 + 6$)

8. On the board show that each row in the box equals 57,126.

[W]

Write in three different ways the meaning of ★

9. 47,632. 10. 853,297. 11. 64,927,050. 12. 3,401,762.

13. For each number below, write what "6" means:
a. 128,604 hundreds **b.** 6,715,000 millions **c.** 596,287 thousands

2

★Answers Not on Reproduced Page 2

9–12. There are many more than three meanings for each of these numbers. Some of the meanings pupils might write for Ex. 9 are given below.

47,632 = 4 ten-thousands and 7 thousands and 6 hundreds and 3 tens and 2 ones
47,632 = $4 \times 10{,}000 + 7 \times 1{,}000 + 6 \times 100 + 3 \times 10 + 2 \times 1$, or $40{,}000 + 7{,}000 + 600 + 30 + 2$
47,632 = 47 thousands and 6 hundreds and 3 tens and 2 ones
47,632 = $47 \times 1{,}000 + 6 \times 100 + 3 \times 10 + 2 \times 1$, or $47{,}000 + 600 + 30 + 2$
47,632 = 476 hundreds and 3 tens and 2 ones
47,632 = $476 \times 100 + 3 \times 10 + 2 \times 1$, or $47{,}600 + 30 + 2$

bulletin board to be explained to the rest of the class.

2. *Slower learners*

a. Help them learn the several meanings a number may have by guiding them in the use of the objects and devices mentioned under the heading "Teacher's Preparation."

b. Have them "put together" large numbers from smaller parts. This is obviously the reverse of the "taking apart" procedure practiced on page 2.

Teaching Pages 3 and 4

Pupil's Objectives. (*a*) To extend knowledge of our number system to include billions; (*b*) to gain understanding and practice in using rounded numbers.

Background. The term billions is frequently used, particularly by children, to express an exaggeration. Here is an opportunity to show situations in which it expresses a real quantity.

Your pupils may need a definition of the term *period* as used here. A period consists of a group of three digits set off by commas. The first period, or that to the right, consists of units; the second period, thousands; the third period, millions; and the fourth period, billions.

Teacher's Preparation. If possible, secure bank statements, insurance-company statements, or other statistics that make use of figures in the billions.

Consider numbers from local situations that may be used as rounded numbers. These might include the number of pupils in the school, population of the town or city, distances to familiar points of interest, and the like.

Pre-book Lesson (page 3). Without using commas, write on the board a number such as \$4936825427. Notice those pupils who are willing to attempt reading it and whether the methods used are systematic or haphazard. Use this opportunity to clarify the meaning of the term *period.* Encourage pupils to express their opinions as to the importance of setting off long numbers in periods by the use of commas.

Book Lesson (page 3). *Ex. 1–5:* Oral work. *Ex. 6–8:* Written work. You may need to explain the term *estimated* in Ex. 7.

Billions of Things

Reading and writing billions [O]

Nobody knows how many stars there are, but people who study about them think that there are many billions.

Look at Ex. a to c in the box. In reading the number in Ex. a, we first say the number of billions, then the number of millions, and so on, this way: "5 billion, 837 million, 643 thousand, 842." Why do we not say, "842 ones"? "ones" is understood

1. Read Ex. b and c, thinking the place values and the period values in the box if you need to.

Value of Place	Hundred billion	Ten billion	Billion	Hundred million	Ten million	Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	One
a.			5,	8	3	7,	6	4	3,	8	4	2
b.		2	0,	6	4	7,	1	6	4,	9	1	4
c.	2	5	9,	1	0	5,	1	7	8,	7	8	5
Value of Period		Billion			Million			Thousand			One	

2. For Ex. c, tell the place value of each figure.

Read each of the following statements:

3. The cars made one year were valued at \$8,729,000,000. 8 billion 729 million

4. The amount of motor fuel supplied was 998,093,000 barrels. 998 million 93 thousand

5. The number of eggs produced one year was 60,046,000,000. 60 billion 46 million

[W]

Write in figures the numbers in these statements:

6. There are about 140 billion units of gas available. 140,000,000,000

7. It has been estimated that the fifteen billion barrels of oil in that section alone are worth \$40 billion. 15,000,000,000 $40,000,000,000

8. The expenses totaled more than ten billion dollars. $10,000,000,000

3

Pre-book Lesson (page 4). Use local situations that involve rounding numbers. Illustrate how rounding numbers may actually be more practical in making comparisons because of the simplified data. Point out that numbers ending in zeros are frequently only approximate numbers.

Book Lesson (page 4). *Ex. 1–5:* Oral work. *Ex. 6:* Written work.

Differentiations and Extensions

1. *All pupils*

a. With reference tables in social-studies texts, show the use of large numbers in tables giving populations and areas of countries. By rounding

Using Round Numbers

[O]

a	b	c	d	e
20	140	7,400	18,000	5,000,000
21	141	7,410	18,100	→5,100,000
22	142	7,420	18,200	5,200,000
→23	143	7,430	18,300	5,300,000
24	144	7,440	18,400	5,400,000
→25	→145	7,450	18,500	5,500,000
26	146	7,460	18,600	5,600,000
27	147	7,470	18,700	5,700,000
28	148	→7,480	→18,800	5,800,000
29	149	7,490	18,900	5,900,000
30	150	7,500	19,000	6,000,000

1. For many purposes, we use round numbers instead of exact numbers. In column a, is 23 nearer to (2 tens) or 3 tens? To the nearest ten, we round 23 to 2 tens, or 20.

2. In column a, 25 is halfway between 2 tens and 3 tens. When a number is just halfway, we generally round it to the next higher ten, so 25 rounded to tens is 30. In column b, will 145 be rounded to 140 or to (150)?

3. To round 7,486 to the nearest hundred, look at column c and think where 7,486 would come. Would you round it to 7,400 or to (7,500)? Why do you not need to think about the ones? the rounding is to 100's

4. Tell from column d why 18,897, to the nearest thousand, would be rounded to 19,000. In rounding to the nearest thousand, why do you not need to think about the tens or the ones? It is nearer 19,000 than 18,000. the rounding is to 1,000's

5. Rounded to the nearest million, 5,123,746 (column e) becomes 5,000,000. Why? It is nearer 5,000,000 than 6,000,000.

[W]

6. Round to the nearest million: a. 1,141,615,000 acres; b. 96,836,075 T.; c. 1,465,134,000 bu.; d. 26,567,374 children. 1,142,000,000 97,000,000 1,465,000,000 27,000,000

these numbers, show how they may be adapted to comparison situations.

b. Encourage children to report large numbers and rounded numbers found in newspapers, magazines, or other sources. Record some of these uses so that pupils will become increasingly aware of how they function in everyday affairs.

c. Give additional practice in rounding when the figure on which the rounding is based is 5, as below.

Round to thousands.

a. 6,500 *b.* 14,500 *c.* 95,500

Round to millions.

a. 3,500,000 *b.* 7,536,200 *c.* 500,900

2. *More capable pupils*

a. These pupils may try solving a few problems such as the following:

Illustration. If a pile of 16 coins = 1 inch, how high would a stack of a billion coins reach?

Have pupils convert the answer to feet or miles, using rounded quotients to simplify the work.

b. Have pupils tell the range within which specific rounded numbers would fall. For example, the number which was rounded to give 2,000,000 below must have been a number no smaller than 1,500,000, and no larger than 2,499,999.

Lower Limit		Rounded Number		Upper Limit
1,500,000	→	2,000,000	←	2,499,999
2,500,000	→	3,000,000	←	_ _?_ _
_ _?_ _	→	4,000,000	←	_ _?_ _

3. *Slower learners*

a. They may make and play card games for the purpose of matching large numbers with corresponding words. In Part IV of this Manual, see "Old Hat" (page 316), "Single Search" (page 316), or "Zooks" (page 317).

b. Rounding numbers is a basic skill in estimating and in mental computation. If necessary, provide additional help for *slower learners* by giving them practice of the type shown below.

2,176 is between _ _?_ _ thousand and _ _?_ _ thousand. It is nearer _ _?_ _ thousand.

4,500 would be rounded to _ _?_ _ thousand.
4,999 would be rounded to _ _?_ _ thousand.
5,499 would be rounded to _ _?_ _ thousand.

Teaching Pages 5 and 6

Pupil's Objectives. (*a*) To acquire understanding of the Arabic and Roman systems of number notation; (*b*) to recognize the important advantages of the Arabic system; (*c*) to review and practice certain learnings basic to the understanding of number relationships.

Background. Roman numerals are used today for recording purposes only (as for building dates, volume numbers, and the like). While computation with these numerals is possible, it is rarely attempted, since the lack of place value in the Roman notational system makes it quite difficult. This was not a serious drawback for those who knew no other system, for computation was generally done on an abacus or some other such device.

How the Romans Wrote Numbers

[O]

Our system of writing numbers is called Arabic notation. This system uses the idea of *place value;* that is, the value of a figure depends on its place in a number.

Roman notation does not have place value. Each of the seven number-letters in the Roman system always has the value shown in the box. To make other numbers, values are added.

I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

The Roman system also has six subtracting numbers. These are pairs of letters in which the values are subtracted. For example, IV means 5 − 1, or 4. In reading and writing Roman numbers, we use such a pair as a single number and add its value to the values of the other letters.

1. The six subtracting numbers are IV, IX, XL, XC, CD, and CM. Explain the meaning of each of these pairs.

2. In reading the number MCMLXXIV, first see the subtracting pairs CM and IV. Then add values. MCMLXXIV means 1,000 + 900 + 50 + 10 + 10 + 4, or 1,974.

3. To write 429 in Roman numerals, first *think,* "This number means 400 + 20 + 9, so I must use the subtracting pairs CD for 400 and IX for 9." Then the number is CDXXIX.

[W]

Write these in Arabic notation:

a	b	c	d	e	f
XVII	XXXIV	XIX	LVIII	CXI	XCV
DCX	MDC	MM	MCDL	MCM	MCMLIV

Write these in Roman notation:

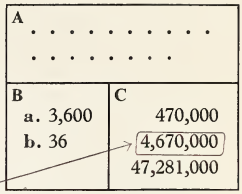
36 XXXVI 44 XLIV 19 XIX 27 XXVII 63 LXIII 48 XLVIII

150 CL 274 CCLXXIV 610 DCX 1,368 MCCCLXVIII 1,456 MCDLVI 1,948 MCMXLVIII

8. List ways in which we use Roman numbers.

Thinking about Whole Numbers

[W]



1. For the dots in box A, write the Arabic number; the Roman numeral. XVIII

2. In box B, number a is how many times number b? 100

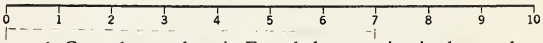
3. Copy the number in box C which you can round to 5 million.

The numbers 2, 4, 6, and 8 and all numbers ending in 2, 4, 6, 8, or 0 can be divided evenly by 2. They are called even numbers. Numbers that are not even are called odd numbers.

4. From the following, write the even numbers in a column headed "even" and the odd numbers in a column headed "odd."

11 (odd) 62 (even) 48 (even) 97 (odd) 260 (even) 509 (odd) 7,124 (even) 6,258 (even) 12,485 (odd)

5. Numbers may be represented by distances on a number line like the one below. Copy this number line and below it draw a dotted line to show the distance for the number 7.



6. Copy the numbers in Ex. a below, putting in the number that belongs where you see n. Do the same for Ex. b to f.

a. 2, 4, 6, 8 b. 5, 10, 15, 20 c. 5, 7, 9, 11

d. 80, 60, 40, 20 e. 26, 23, 20, 17 f. 15, 24, 33, 42

7. Write the smallest number you can round to 7,000,000. 6,500,000

8. Write the number which is 1 less than 1 billion. 999,999,999

9. Using only the figures 1, 2, and 3 each once, write the largest possible whole number, 321; then the smallest. 123

10. What number is 10 more than 482? 492 than 391? 401 than 990? 1,000

11. How many 1,000's of people are there in a population of 160,000,000? of 13,845,000? of 6,470,000?

There are good reasons for believing that this use of the abacus led eventually to the adoption of zero and the development of a place-value notational system.

Comparing our number system to the Roman system, noting the latter's differences and disadvantages, as well as its historical significance, may be helpful in generating in your pupils greater understanding and appreciation for our present number system.

Teacher's Preparation. Consider Ex. 8 on page 5, so that you may elaborate upon the current use of Roman numbers (e.g., to designate chapters, for clock figures, in making outlines, etc.).

Pre-book Lesson. To create interest and to establish a feeling for the need of Roman numerals, call attention to materials which use them.

Book Lesson (page 5). *Ex. 1–3:* Oral work. *Ex. 4–8:* Written work.

1. In discussing the first paragraph, identify Arabic notation as being a decimal system. Point out that the word decimal means ten, and that it is related to words such as December and decade.

2. After Ex. 4–8 have been completed, have each pupil correct his own or a partner's paper. Correct forms on the board, accompanied by oral explanations, will provide further opportunities for learning.

Book Lesson (page 6). *Ex. 1–11:* Written work. Provide opportunity for discussion of directions. A brief review of even and odd numbers, and an explanation of the use of *n* in place of missing numbers may be advantageous for all.

Differentiations and Extensions

1. *All children.* You may suggest that they try some computation using Roman numbers and then try the same computation with Arabic numbers.

2. *More capable pupils* may look up the history of numbers in several children's encyclopedias. They may report to the class on the way in which Roman numerals developed and compare them to Arabic numbers.

3. *Slower learners* may follow directions such as these:

Write Roman numbers

a. from 1 through 10.
b. from 10 through 100 by tens.
c. from 100 through 1,000 by hundreds.

These may be corrected by comparing them with the table of numbers usually found in school dictionaries.

Teaching Pages 7 and 8

Pupil's Objectives. (*a*) To take Inventory Test 1; (*b*) to recall the meaning of addition; (*c*) to relearn certain terms, generalizations, and relationships related to addition.

Background. Pages 7–11 in the text constitute a unit devoted to the reteaching of addition. Inventory Test 1 occurs on page 7 and should be helpful in identifying pupils in need of further reteaching.

Do not overlook the fact that a good solid background in addition (and the other basic processes, too) is a fundamental prerequisite for the new work in Grade 6. To aid both the computation and problem-solving programs, make sure that all pupils grasp firmly the idea that addition is a combining process involving like-things. This is a basic understanding and its importance cannot be overemphasized. On pages 7 and 8 are also some specific items that deserve close attention. Be sure, for example, that the ways of reading an addition example are known; that pupils know or can devise helpers to use when they have trouble remembering addition facts; and that they can add by endings (Ex. 6–12, page 8) both when the figure in ten's place in the sum does not change and when it must be increased by one.

Teacher's Preparation. Assemble materials, such as an abacus, bead board, discs (see Part IV pages 310–315, in this Manual), which can be manipulated by pupils, or which will be useful in demonstrating and verifying meanings in addition. For pupil manipulation, avoid materials frequently used in (and, hence, associated with) previous grades. Be adroit in using even transitory interests that provide practice material (e.g., substitute airplane cards for cardboard tickets).

Recommended at this time are checkers and a checkerboard, nails and partitioned boxes, balance scale for use with calculating blocks (see page 312 in this Manual), school equipment like paper clips (for other quantity objects, see page 319 in this Manual), and items from hobby or science collections. Remember, too, to keep mathematical aims constantly in view in practice periods with materials.

Book Lesson (page 7). *Ex. 1–5:* Written work. *Ex. 6–8:* Oral work. Give Inventory Test 1 to all. As you guide the discussion in the oral work on the page, keep the principles stated in dark print at the bottom of the page clearly in mind. It is recommended that a permanent chart of arithmetical terms encountered be kept in a conspicuous place in the classroom. Here you would record *addend*, *sum*, *total*, and the symbol +, for plus.

Book Lesson (page 8). *Ex. 1–9:* Oral work. *Rows 10–12:* Written work. Be sure that terms such as *doubles*, *near doubles*, *teen numbers*, *reverse fact*, and *families of addition examples* are understood, and that pupils know how to fold paper for writing sums in horizontal addition, as in rows 10–12 in the text.

Differentiations and Extensions

1. *More capable pupils* may check answers by casting out 9's in addends and sums. Casting out 9's may be accomplished using either of the methods in the next column. Method B deviates only slightly from the procedure for Method A, but in both cases, the remainders (the circled 2's) are the same after casting 9's from both the addends and the sum.

When We Add

Inventory Test 1 [W]

1. When all the birds shown in the picture above are together on the wire, how many birds will be there? 13

2. Miss Pike put 6 history books, 9 spelling books, 8 geographies, and 7 arithmetics on the table. How many books did she put on the table in all? 30

3. 16 cows, 3 horses, and 1 dog are how many animals? 20

4. Class committees sold 25 tickets, 9 tickets, 6 tickets, and 8 tickets. How many in all were sold by the committees? 48

5. Jack brought 18¢ for the record fund, Sue brought 9¢, Bill 6¢, and Jean 8¢. How much did all four pupils bring? 41¢

Helps for adding; meaning [O]

6. How many groups are put together in Ex. 1? 2 in Ex. 2? 4 in Ex. 3? 3 in Ex. 4? 4 in Ex. 5? 4

7. In which problems are the things in the groups exactly the same kind? In which are the things alike only in some ways? Ex. 2, 3 Ex. 1, 4, 5

Putting together like-groups, to find how many in all, is adding. Only like-things can be added.

Both Ex. a and Ex. b are read "15 plus 2 is 17" or "15 and 2 are 17."

8. For Ex. b tell the addends; the sum. 15 and 2; 17

a. 15 (addend) + 2 (addend) = 17 (sum)

b. 15 + 2 = 17

Addition (A.) Facts and Families

Helpers [O]

1. The 81 basic addition facts are given on page 291. Turn to that page, cover the answers, and say the facts. Then say the 19 facts that have zero as one of the addends. The sum of any number and 0 is the number itself.

2. Make a rule for facts that have zero as an addend, that have 1 as an addend. The sum of any number and 1 is 1 more than the number.

3. How can you use doubles to help you find the sums for near doubles (facts that are on either side of the doubles)? by going "up one" or "down one" from the sum of the near double.

4. For an example like 9 + 6, whose sum is a teen number, explain how to find the sum by first making a 10. Take away enough from one number to make the other a 10. Then add 10 and the smaller number.

5. What helper can you use for a fact which is the reverse of a fact that you know? the fact you know: e.g., for 6+9, use the helper 9+6=15

Each basic A. fact gives its name to a family of examples in which a 1-place number and a 2-place number are added. If you know the fact, you can say all the sums in its family.

6. Two examples in the 7 + 2 addition family are 47 + 2 and 7 + 22. Give two more examples in this family. 27+2, 7+12, etc.

7. Give three examples in the 6 + 7 family. 16+7, 6+17, 26+17, etc.

8. In examples like 56 + 2 and 3 + 24, the ten's figure in the sum is the same as the ten's figure in the 2-place addend. Explain. The sum of the ones is less than 10, so there is no 10 to carry.

9. In examples like 58 + 3 and 9 + 35, the ten's figure in the sum is how much larger than the ten's figure in the 2-place addend? / Explain. The sum of one's is 10 or more, so there is a 10 to carry.

[W]

Write just the sums for these:

	a	b	c	d	e
10.	84 + 3 87	6 + 34 40	49 + 9 58	68 + 4 72	21 + 8 29
11.	66 + 7 73	77 + 2 79	8 + 39 47	45 + 6 51	2 + 49 51
12.	8 + 45 53	9 + 88 97	75 + 7 82	4 + 15 19	38 + 7 45

Method A

568 → 5+6+8 = 19; 19 ÷ 9 = 2+R1
23 → 2+3 = 5; 5 ÷ 9 = 0+R5
+347 → 3+4+7 = 14; 14 ÷ 9 = 1+R5
938 → 938 ÷ 9 = 104+R2 (11 ÷ 9 = 1+R2)

Method B

568 → 5+6+8 = 19; 1+9 = 10; 1+0 = 1
23 → 2+3 = 5
+347 → 3+4+7 = 14; 1+4 = 5
938 → 9+3+8 = 20; 2+0 = 2 (11; 1+1 = 2)

2. *Slower learners*

a. They may make individual charts of addition facts organized under headings indicating the type of helper they might use. Except for the zero facts and the facts for one, only those facts with sums of ten or more are included in the illustrative chart on the next page. Other simpler facts should cause no difficulty at this grade level. Remind pupils that there may be several helpers for any one difficult fact. If, for example, the sum is a teen number, making a ten is a good helper. A reverse fact is often used as a helper.

0 facts *Facts for 1*

4	0	8		7	1	
+0	+1	+0	etc.	+1	+7	etc.
4	1	8		8	8	

Doubles

5	6	7	8	9
+5	+6	+7	+8	+9

Near-doubles

5	6	7	6	8	7	9	8
+6	+5	+6	+7	+7	+8	+8	+9

Making a ten

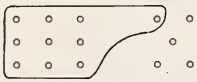
9	2	9	3	9	4	9	5
+2	+9	+3	+9	+4	+9	+5	+9
9	6	9	7	8	3	8	4
+6	+9	+7	+9	+3	+8	+4	+8
8	5	8	6				
+5	+8	+6	+8				

Using a reverse fact

6	4	7	3	8	2
+4	+6	+3	+7	+2	+8
7	4	7	5		
+4	+7	+5	+7		

b. Also have *slower learners* practice facts by making a 10 as shown in the box below.

$9 + 5 = ?$



$9 + 1 = 10;\ 10 + 4 = 14.\ 9 + 5 = 14$

c. Ask some pupils to demonstrate troublesome facts with the equipment that has been suggested previously. Others may draw sketches using ×'s, O's, or other representative symbols.

d. Be aware of situations constantly occurring in the classroom in which addition facts are needed. Call attention to each situation as it arises.

e. When pupils are ready for drill work, see Part IV of this Manual for games such as "Single Search," "Zooks," "Mads," and the like, which may be used in motivating the practice.

Teaching Pages 9 and 10

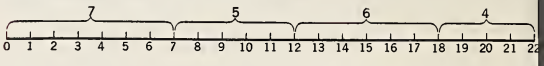
Pupil's Objectives. (*a*) To review the understanding that the order of the addends does not affect the sum; (*b*) to review adding by endings in column addition; (*c*) to take Inventory Test 2.

Helps in Column Addition

1-place addends [O]

1. This house has 7 windows on the north side, 5 on the east, 6 on the south, and 4 on the west. How many windows are there in all? *22*

On the board, make a number line and count off 7, 5, 6, and 4 in that order (see below). On the number line, the sum of $7 + 5 + 6 + 4 = 22$



2. On the board, make a number line and find the sum of $5 + 6 + 4 + 7$. Make another number line and find the sum of $6 + 4 + 7 + 5$. Why is the sum 22 each time? *Changing the order of addends does not change the sum.*

We may add groups or numbers in any order.

Usually we add downward, as in the box. Look at 7 and 5 and *think*, "12." With 12 in mind, look at 6 and *think*, "18." With 18 in mind, look at 4 and *think*, "22." So we *think*, "12, 18, 22."

If you add downward, then check by adding upward. *Think*, "10, 15, 22."

Add downward

7
5
6
+4
22 Check

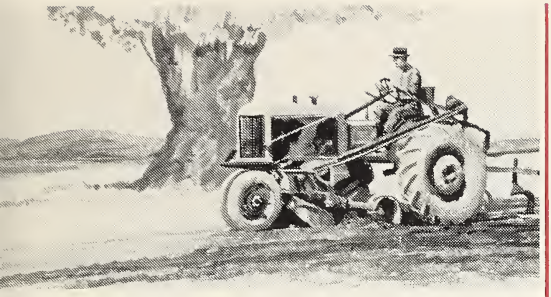
[W]

Find the sums. Check your work.

	a	b	c	d	e	f	g	h	i
3.	4	7	2	6	8	3	4	9	8
	9	8	9	5	4	8	9	2	1
	7	6	9	8	6	8	5	7	4
	+2	+5	+7	+4	+9	+7	7	6	9
	22	*26*	*27*	*23*	*27*	*26*	6	4	8
							+7	+6	+4
							38	*34*	*34*

Background. On page 9, pupils review the idea that changing the order of the addends does not affect the sum. This idea is the basis for checking addition by adding from the bottom upward after an example has been added from the top down. It also occurred on page 8 in work with reverse facts.

Notice in the work opposite the box on page 9 that the idea of adding by family endings is immediately put to work in column addition. Notice, also, that pupils receive further practice in this kind of addition when they check column addition by adding upward.



Addition on the Farm

Inventory Test 2 [W]

If you can find these sums quickly and correctly, you should be able to add whole numbers wherever you find them.

1. For each year listed, find Mr. Hall's total number of acres.
1910: 92 acres of grazing land and 29 acres of crop land. *121*
1930: 93 acres of grazing land and 37 acres of crop land. *130*
1950: 99 acres of grazing land and 40 acres of crop land. *139*

2. Find for each year his total number of bushels of grain.
1910: 94 bushels of wheat and 74 bushels of corn. *168*
1930: 143 bushels of wheat and 79 bushels of corn. *222*
1950: 294 bushels of wheat and 149 bushels of corn. *443*

3. In different years, Mr. Jackson bought for his farm a new tractor for $2,700, a potato harvester for $4,865, and a baler costing $2,299. This machinery cost how much in all? *$9,864*

4. What did Mr. Jackson's neighbor, Mr. Chalmers, spend in all for a sugar-beet harvester costing $5,700, a corn picker costing $840, and a grain combine costing $5,075? *$11,615*

5. Find the total cost of the farm machinery purchased by Mr. Jackson and Mr. Chalmers. *$21,479*

6. In three months, Mr. Hill's hens produced 4,926 eggs, 5,405 eggs, and 8,264 eggs. How many eggs in all was that? *18,595*

10

If it develops from Inventory Test 2 that some pupils need further reteaching in adding larger numbers without and with carrying, the help and practice required will be found on page 11 in the text.

Teacher's Preparation. You may wish to mount a tape measure on pasteboard for permanent use. Calculating strips (page 312 in this Manual) may then be fastened to the tape measure with paper clips to depict column addition in the manner shown on the number line at the top of page 9 in the text.

Book Lesson (page 9). *Ex. 1 and 2:* Oral work. *Row 3:* Written work. See the Differentiations and Extensions sections for more examples which may be used with a number line.

Book Lesson (page 10). *Ex. 1–6:* Written work. Inventory Test 2. Let pupils read all the way through the page either silently or orally. Be sure that there are no difficulties caused by vocabulary, nor any confusion as to procedure. After the work has been completed and corrected, have the computation done at the board.

Differentiations and Extensions

1. *All pupils*

a. Write three numbers on the board and let pupils discover all the ways of rearranging the numbers which will result in the same sum. Do the same with four numbers and five numbers.

b. Ask your pupils to check the "magic square" below to make sure that the same sum results when the figures are added by rows, by columns, or diagonally from corner to corner.

2	9	4
7	5	3
6	1	8

c. Have pupils complete "magic squares" in which one figure has been omitted in each row, as below (the figures should be added horizontally and vertically only).

n	3	4	9
9	4	*n*	3
3	7	9	*n*
4	*n*	3	7

Call attention to the fact that the above box uses only four different figures arranged in miscellaneous order. Encourage pupils to make original squares of this kind.

d. Other examples to use with a number line, or for further practice in column addition, are given below.

$9+5+4$	$4+8+7$	$5+9+6$
$7+4+9+5$	$9+6+7+9$	$8+9+8+4$

2. *Slower learners*

a. See Part IV of this Manual for a game ("Clover," page 317) and a device ("bead line," page 312) which might be helpful when used by these pupils.

b. Use a number line and calculating strips to show the bridging of a decade number. Be sure that the decade numbers 10, 20, 30, etc. are distinctive on the number line so pupils will realize when one of them has been bridged in addition (e.g., 29 + 6 bridges 30 resulting in a sum, 35, in the next higher decade).

c. They may use an intermediate step temporarily in higher-decade addition. For example, instead of giving the complete sum immediately in adding 38 + 5, these pupils may first think the sum of the one's column (13).

A few pupils may still need to write the number carried at the top of the column as a crutch. Some may find, with your encouragement, that they no longer need to use this crutch.

d. These pupils may do examples like these:

Write the missing figure.

23	17	46	37	496	899
+ 6	+ 8	+ 4	42	205	367
?9	?5	??	9	33	426
			?8	??4	???

3. *More capable pupils*

a. Suggest that they bring in actual or typical problems in column addition as used in their homes.

b. Have them figure the number of miles from one city to another by adding numbers given on road maps.

Adding Larger Numbers

Without and with carrying [O]

1. 31 + 26 = 57 In box A we break 31 and 26 into two parts (tens and ones) and add ones to ones and tens to tens. When we add the sums of the parts, 50 and 7, the final sum is 57.

A	B
30 and 1	Short way
+20 and 6	31
50 and 7,	+26
or 57	57

2. Explain why in box B the short way of adding is also adding by parts. Which part do we add first? The ones

We add 1's and 10's separately.

3. Tell how we add by parts in box C.

4. In box D, the sum of the ones is 17. Why must we carry 1 ten to ten's column? 17 ones = 1 ten + 7 ones 2 hundreds to hundred's column? 24 tens = 2 hundreds + 4 tens 1 thousand to thousand's column? 13 hundreds = 1 thousand + 3 hundreds Explain all the adding in box D.

C				D			
Thousands	Hundreds	Tens	Ones	Thousands	Hundreds	Tens	Ones
				1	2	1	
5,	1	2	4	3,	2	9	5
	5	3	0		6	8	4
+8,	2	1	3	+	5	6	8
13,	8	6	7	4,	5	4	7

[W]

Write sums on folded paper. Check by adding upward.

	a	b	c	d	e	f
5.	64	26	$4.61	306	$24.71	3,076
	+32	+81	+3.25	+82	+2.81	+2,813
	96	107	$ 7.86	388	$27.52	5,889
6.	28	48	$2.97	309	$53.82	3,418
	66	56	0.48	508	63.95	629
	+13	64	+7.06	605	+72.84	5,085
	107	+18	$10.51	+208	$190.61	+7,697
		186		1,630		16,829

Extra Practice. For extra practice in addition, turn to pages 295 and 296 and work Sets 1–8.

Teaching Page 11

Pupil's Objectives. (*a*) To review addition without and with carrying; (*b*) to review the generalization that numbers can be added by parts.

Background. It is unlikely that pupils will have serious difficulty in adding larger numbers. Devices may be used (as suggested in the Book Lesson below), but it may be possible to start immediately with figures as shown in box A in the text. It is, however, important that pupils eventually learn how to add the "short way" shown in box B. Somewhat the same procedure may be used for harder, or at least longer, examples involving carrying, as shown in box D. The important idea to get across is that we can add by parts, first ones, then tens, and so on, and that when we carry we are simply taking ten of a smaller group to make a larger group that will be added in the next column to the left.

Book Lesson (page 11). *Ex. 1–4:* Oral work. *Rows 5 and 6:* Written work.

1. If necessary, use a pocket chart (page 315 in the Manual), peg board (page 314), or an abacus (page 310) concurrently with Ex. 1–4.

2. In the written work (rows 5 and 6) you may find that some pupils have difficulty because they do not know where to write carried figures when

using folded paper. Show them how this may be done, but try to lead them away from the need for writing the carried figures.

Differentiations and Extensions. For your convenience, **Extra Practice Sets 1–8** (pages 295 and 296 in the text) are reproduced below with answers in red. As each set is called for in the text, it will be reproduced in this way. For easy reference, all sets are indexed in the Manual under the heading **Practice**.

Discuss with your pupils the purpose of the material on pages 295–328, indicating how they may select sets for practice on specific skills. Encourage them to work these sets whenever they feel it necessary.

Set 1. Higher-decade addition; no bridging

	a	b	c	d	e	f	g	h	i	j
1.	16	5	64	42	21	67	5	3	85	41
	+ 3	+ 50	+ 4	+ 3	+ 4	+ 2	+ 84	+ 21	+ 3	+ 2
	19	55	68	45	25	69	89	24	88	43
2.	43	2	35	4	51	62	16	3	11	4
	+ 5	+ 31	+ 2	+ 73	+ 6	+ 5	+ 2	+ 23	+ 7	+ 15
	48	33	37	77	57	67	18	26	18	19
3.	74	3	3	22	0	3	4	1	82	54
	+ 1	+ 12	+ 36	+ 7	+ 16	+ 94	+ 22	+ 18	+ 6	+ 4
	75	15	39	29	16	97	26	19	88	58

Set 2. Higher-decade addition; with bridging

1.	59	8	16	8	69	84	3	56	77	2
	+ 5	+ 47	+ 5	+ 23	+ 9	+ 8	+ 37	+ 8	+ 4	+ 19
	64	55	21	31	78	92	40	64	81	21
2.	46	18	39	6	58	79	7	75	14	45
	+ 7	+ 9	+ 4	+ 59	+ 6	+ 3	+ 26	+ 7	+ 6	+ 8
	53	27	43	65	64	82	33	82	20	53
3.	38	6	65	87	17	7	49	19	67	9
	+ 5	+ 36	+ 9	+ 7	+ 9	+ 28	+ 2	+ 7	+ 5	+ 86
	43	42	74	94	26	35	51	26	72	95

Set 3. Column addition involving bridging

	a	b	c	d	e	f	g	h	i	j	k	l	m
1.	8	7	9	6	8	6	8	6	6	9	8	9	7
	7	4	3	5	5	6	6	7	9	8	9	5	9
	+ 6	+ 9	+ 8	+ 9	+ 7	+ 9	+ 7	+ 8	+ 8	+ 8	+ 6	+ 6	+ 4
	21	20	20	20	20	21	21	21	23	25	23	20	20
2.	7	9	9	9	8	7	4	9	5	8	7	9	7
	7	6	9	4	4	5	9	7	7	6	8	9	6
	+ 8	+ 9	+ 3	+ 9	+ 8	+ 9	+ 8	+ 7	+ 9	+ 9	+ 8	+ 7	+ 9
	22	24	21	22	20	21	21	23	21	23	23	25	22

Set 4. Column addition involving bridging

	5	7	9	7	8	9	9	8	9	8	8	9	9
	6	6	7	9	8	9	8	7	6	9	8	9	7
	9	9	8	9	7	9	9	9	6	9	5	6	8
	+ 9	+ 8	8	8	+ 7	8	9	+ 6	+ 9	9	+ 9	9	8
	29	30	+ 9	+ 8	30	+ 7	+ 7	30	30	8	30	9	+ 9
			41	41		42	42			+ 8		+ 8	41
										51		50	

Set 5. Column addition; no carrying

	a	b	c	d	e	f	g
1.	41	76	34	22	25	64	15
	+ 18	+ 23	+ 65	62	10	12	11
	59	99	99	+ 2	+ 54	+ 22	+ 83
				86	89	98	109
2.	453	332	213	4,522	2,311	1,352	4,150
	303	245	132	+ 4,134	71	6,135	5,241
	+ 22	+ 301	+ 312	8,656	+ 7,616	+ 401	+ 504
	778	878	657		9,998	7,888	9,895

Set 6. Column addition; no carrying

12	24	23	921	113	1,431	2,010
24	22	31	203	310	6,030	4,113
42	30	14	51	512	5,335	1,235
+ 11	+ 23	+ 40	+ 2	+ 63	+ 201	+ 4,641
89	99	108	1,177	998	12,997	11,999

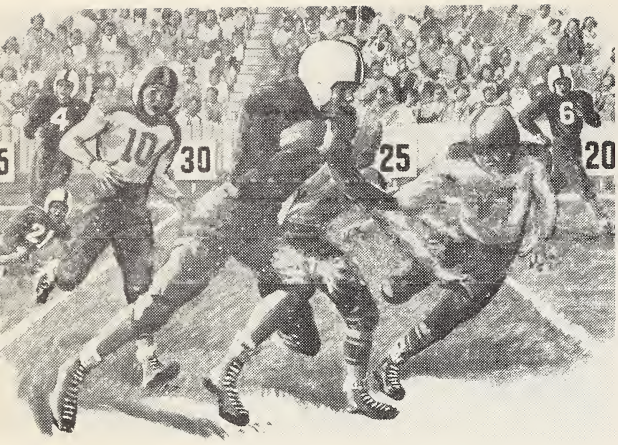
Set 7. Column addition; with carrying

1.	85	67	67	926	618	357	9,419
	+ 95	+ 56	+ 95	+ 95	+ 896	+ 883	+ 986
	180	123	162	1,021	1,514	1,240	10,405
2.	54	242	363	8,784	3,856	2,351	8,673
	21	732	312	+ 4,979	+ 7,867	4,430	1,703
	+ 58	+ 36	+ 406	13,763	11,723	+ 2,829	+ 6,699
	133	1,010	1,081			9,610	17,075

Set 8. Column addition; with carrying

1.	31	43	69	68	160	654	172
	52	25	81	32	943	30	254
	33	24	97	95	83	751	762
	+ 64	+ 97	+ 35	+ 41	+ 78	+ 49	+ 23
	180	189	282	236	1,264	1,484	1,211
2.	431	724	827	4,513	9,218	1,148	5,085
	420	904	684	9,754	2,370	9,386	8,375
	813	861	758	267	567	9,989	7,668
	+ 196	+ 354	+ 866	+ 755	+ 9,068	+ 4,579	+ 9,737
	1,860	2,843	3,135	15,289	21,223	25,102	30,865

NOTES



When We Subtract

Inventory Test 3 [W]

1. Tom Greene played end on the Clark School team. In one play he ran with the ball from the opponents' 35-yard line to their 20-yard line before he was downed. How far did the ball travel? *15 yd.*

2. Rules require the ball to be in play again after 25 seconds. Jack centered the ball for the next play just 6 seconds before the time limit was up. How long was the ball not in motion? *19 sec.*

3. Clark School won by a score of 36 to 7. What was the difference between the scores? *29*

4. After the game, Tom and some of the boys went to his house for sandwiches and pop. The boys drank 9 of the 14 big bottles of pop which Tom's mother had left in the refrigerator for them. How many bottles of pop were unopened when the boys left Tom's house? *5*

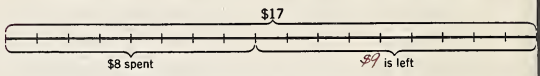
5. The boys left only 7 of the 40 sandwiches Tom's mother had made for them. How many sandwiches had the boys eaten? *33*

What Subtraction Does

Meaning [O]

1. How much was left if Bill had $17 and spent $8? *$9*

2. Explain how to work Ex. 1 by using the number line.



In Ex. 1 we can think of the $17 as the sum of two numbers. We know one of these numbers, $8. To find the other number, we subtract.

We can write subtraction in two ways, as in the box, but we read both ways the same:

"$17 minus $8 is $9," or
"$8 from $17 is $9."

$17	*minuend*	
−8	*subtrahend*	↑ Check
$9	*remainder*	

$17 − $8 = $9

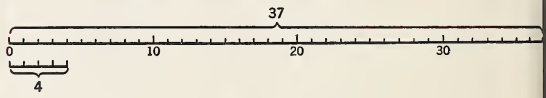
3. The "other number," or the **remainder**, in Ex. 1 is $ *9*. The remainder may be the number left or the number gone. Which is it in Ex. 1? *✓*

4. Tell from the box the name of the number from which we subtract, the name of the number that is subtracted. *minuend, subtrahend*

5. In the box add the remainder and the subtrahend. Why do you get the minuend? *8 and 9 are the parts of 17* Is this a good way to check a subtraction example? *Yes*

6. Ann has 37 stamps and Gail only 4. Ann has how many more stamps than Gail? *33* Gail has how many fewer than Ann? *33*

In this problem we use subtraction to compare two numbers. 37 − 4 = *33* The "other number," *33*, is the **difference** between the numbers that are given. Explain how the comparison is shown by using the number lines below. *From 4 to 37 is 33.*



Teaching Pages 12 and 13

Pupil's Objectives. (*a*) To take Inventory Test 3; (*b*) to review the meaning of the subtraction process.

Background. The inventory test on page 12 in the text ought to give you a good indication of the amount of reteaching you will need to do in relation to the meaning of subtraction, learning subtraction facts, and higher-decade subtraction. Pages 13–15 are designed to provide this reteaching if it is found to be necessary.

The lesson on page 13 teaches the basic meaning of subtraction and other skills and abilities, such as reading subtraction facts, using a number line, checking subtraction, and using proper arithmetical terms (*minuend*, *subtrahend*, *remainder*, *difference*).

In essence, subtraction is a separating process, as opposed to addition, which is a combining process. In subtraction we always know the size of the larger group and the task is to separate it into two smaller groups, one of which we also know. Sometimes this "other number" that we

are to find is called the *remainder* because it refers to that part of the larger group which is "left" or "gone" as far as the problem situation is concerned. Sometimes it is called the *difference* because it shows in a comparison situation how much larger or how much smaller one number is than another.

Book Lesson (page 12). *Ex. 1–5:* Written work. Assign Inventory Test 3 to all. After correcting tests, determine by an analysis of each item the particular types of examples in which members of the class are weak.

Book Lesson (page 13). *Ex. 1–6:* Oral work.

1. Call attention of the pupils to help provided by number lines.

2. This is a good time to assure yourself that pupils are familiar with the terms *remainder*, *difference*, *comparison*, *minuend*, and *subtrahend*.

Differentiations and Extensions

1. Have *more capable pupils* write subtraction problems which include some of the following phrases: *amount spent*, *amount saved*, *distance gone*, *more needed*, *in another place*, *amount lost*, *how many fewer*, *how many used*, *how much larger*. Study the problems, and if any are not subtraction problems present them to the class anonymously for discussion. Select some of the other problems and let pupils try to determine which type of subtraction example is represented.

2. *Slower learners*

a. Help them to use number lines such as those on page 13 in the text to differentiate between comparison problems (double line) and other subtraction problems (single line).

b. If necessary, use objects (such as buttons, pegs, and the like), a bead line, peg board, or other suitable devices which will help introduce the meaning of subtraction at a concrete level. Remember, however, that ability to work with objects must be accompanied by ability to work with number symbols if subtraction is to be useful in the everyday affairs of the child.

Subtraction (S.) Facts; Related Facts

Helpers [O]

1. Turn to page 292. Cover the answers and see if you can say correctly all 81 of the basic subtraction facts.

2. Now say all the subtraction facts that have 0 as the subtrahend. Make a rule for subtracting 0 from any number. *Subtracting 0 from any number gives the number itself.*

3. Say all the facts that have a remainder of 0. Then make a rule for subtracting any number from an equal number. *Subtracting any number from an equal number gives 0.*

4. Make a rule for facts that have 1 as the subtrahend. *Subtracting 1 from any number gives a remainder 1 less than the number.*

5. $6 - 3 = 3$ is a double. Say five more doubles. How can you use doubles to help you with subtraction facts that are near doubles? *4-2=2; 8-4=4; 2-1=1; 10-5=5; 12-6=6; etc. by going "up 1" or "down 1" from the double*

6. For help in subtracting from a teen number, think of the number as a 10 and some ones. Subtract first from the 10. Then add the ones to that remainder. Make a dot picture on the board and show how this rule works for the example $15 - 7$. *10-7=3; 3+5=8*

7. How can you use an A. fact for help with a S. fact? *For example: since 3+6=9, 9-6=3*

Related addition and subtraction facts go together to make whole stories. The related addition and subtraction facts in the whole story about 8, 9, and 17 are:

$8 + 9 = 17$; $9 + 8 = 17$; $17 - 8 = 9$; $17 - 9 = 8$.

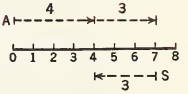
8. Say the four related addition and subtraction facts for

a. 4, 8, and 12; *4+8=12 8+4=12 12-4=8 12-8=4*

b. 6, 3, and 9; *6+3=9 3+6=9 9-6=3 9-3=6*

c. 9, 7, and 16. *9+7=16 7+9=16 16-9=7 16-7=9*

On this number line, we may add by counting to the right from 0 and subtract by counting to the left from the end of the distance for the minuend.



9. Explain how this line shows $4 + 3 = 7$ and $7 - 3 = 4$.

10. On a line at the board, show $3 + 4 = 7$ and $7 - 4 = 3$.

14

Teaching Pages 14 and 15

Pupil's Objectives. (*a*) To formulate helpers useful in remembering difficult subtraction facts; (*b*) to rediscover the relationship that exists between addition and subtraction facts (whole stories); (*c*) to review higher-decade subtraction without and with borrowing; (*d*) to write original subtraction problems.

Background. When children have difficulty with subtraction facts, they need some way to remember or rediscover the unknown facts. That is the reason for the helpers in Ex. 2–7 on page 14 in the text. Some pupils will not need to use helpers. Others will find them necessary, but should be led away from their use when possible.

If pupils are thoroughly familiar with the meanings of addition and subtraction, they will have no difficulty in seeing how addition and subtraction facts may be related. If we have two groups of

objects which can be combined to make one larger group, we can, in turn, think of separating this larger group into the two smaller groups. If the two smaller groups are 4 and 3, which combine to make 7, four facts (the whole story in addition and subtraction) result: $4 + 3 = 7$; $3 + 4 = 7$; $7 - 3 = 4$; $7 - 4 = 3$. Of course, doubles have but two facts in the whole story (i.e., $3 + 3 = 6$, $6 - 3 = 3$).

Whenever in subtraction there is a two-place minuend and a one-place subtrahend, the subtracting may be done by thinking of families of subtraction facts. The two cases that arise are shown in boxes A and B on page 15 in the text. In box A, the ten's figure remains the same. In box B, the pupil must think the family ending, 9, and then make the ten's figure in the remainder one less. This latter case is called higher-decade subtraction (the minuend is a 2-place number) with bridging (to obtain the ten's figure in the remainder, we "bridge" back into the previous decade).

Book Lesson (page 14). *Ex. 1–10:* Oral work.

Book Lesson (page 15). *Ex. 1–4:* Oral work. *Ex. 5–13:* Written work.

Differentiations and Extensions

1. Rather than spending tedious hours working on skills in which they are already proficient, *more capable pupils* should be given an opportunity to enrich their understandings and to engage in stimulating experiences. For example, they might collect information and develop problem situations related to budgets. Special types of expenses with which pupils ought to become familiar might include Federal and state taxes, tips, and mailing expenses, all with their special categories.

2. *Slower learners*

a. When facts needing study have been isolated, have these pupils record them on study cards or in a notebook. Go over each individual's list with him to see if a pattern of weakness is revealed (e.g., minuends of 13, or subtrahends of 4, etc.). Also check 0 difficulties.

b. Help them to make dot patterns (as for Ex. 6, page 14 in the text) for teen numbers.

c. For learning more about whole stories, use dominoes or domino cards. The illustration which

(*Continued on page 39*)

Subtraction Families

Higher-decade S. [O]

Each subtraction fact gives its name to a family of examples. If you know a subtraction fact, you should be able to say the remainders for all the examples in its family.

1. What subtraction family is illustrated in box A? Give four more examples in this family. 38-5=33; 58-5=53; 63-5=63; 88-5=83

2. In the 8 – 5 family, how do you know without subtracting what the ten's figure in each answer will be? Since 5 ones can be subtracted from 8 ones, a 10 need not be borrowed.

3. What subtraction family is illustrated in box B? Give four more examples in this family. 36-7=29; 66-7=59; 76-7=69; 96-7=89

A

The 8 – 5 family

18	28	48	78
−5	−5	−5	−5
13	23	43	73

B

The 16 – 7 family

26	46	56	86
−7	−7	−7	−7
19	39	49	79

4. In the 16 – 7 family, why is the ten's figure in each answer 1 less than the ten's figure in the minuend? 6 ones are not enough to allow subtracting 7 ones, so a 10 must be borrowed. [W]

On folded paper write the remainders for rows 5–12.

	a	b	c	d	e
5.	15 – 2 13	26 – 3 23	84 – 3 81	51 – 3 48	62 – 7 5
6.	39 – 6 33	62 – 5 57	35 – 6 29	64 – 8 56	74 – 6 6
7.	42 – 3 39	75 – 4 71	63 – 4 59	24 – 7 17	81 – 8 7
8.	63 – 8 55	45 – 7 38	23 – 6 17	35 – 8 27	56 – 5 5
9.	27 – 3 24	82 – 9 73	71 – 6 65	81 – 7 74	52 – 4 4
10.	34 – 2 32	64 – 9 55	37 – 5 32	59 – 2 57	46 – 4 4
11.	45 – 3 42	59 – 3 56	48 – 9 39	80 – 4 76	52 – 6 4
12.	25 – 9 16	40 – 5 35	26 – 4 22	68 – 5 63	93 – 5 8

13. Make up a set of 5 subtraction problems, using the numbers in row 5.★

★Answers Not on Reproduced Page 15

13. For example:

a. Joe caught a fish 15 inches long. Bill's fish was 2 inches shorter. How long was Bill's fish?

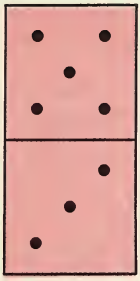
b. The Browns live 26 miles from the city. The Whites live 3 miles nearer the city. How far from the city is the Whites' house?

c. Sandra weighs 84 pounds. Kendra weighs 3 pounds less. How much does Kendra weigh?

d. Tom had 51 bottle caps in his collection. He gave Jim 3 caps. How many caps did Tom have left?

e. Gary's grandfather is 62 years old. His grandmother is 7 years younger. How old is his grandmother?

follows shows a domino and the whole story that may be derived from it.



$5 + 3 = 8$
$3 + 5 = 8$
$8 - 5 = 3$
$8 - 3 = 5$

d. Exercises which may be assigned to these pupils for further practice are given below.

Subtract 8 from each number below.

29 35 41 58 19 36

Subtract 6 from each number below.

34 23 18 50 47 76

In which of the facts below does the ten's figure in the remainder stay the same?

29	40	31	28	20	62	57	95	19	76
-5	-6	-1	-9	-6	-4	-2	-4	-8	-7

In which of the facts below is the ten's figure 1 smaller in the remainder?

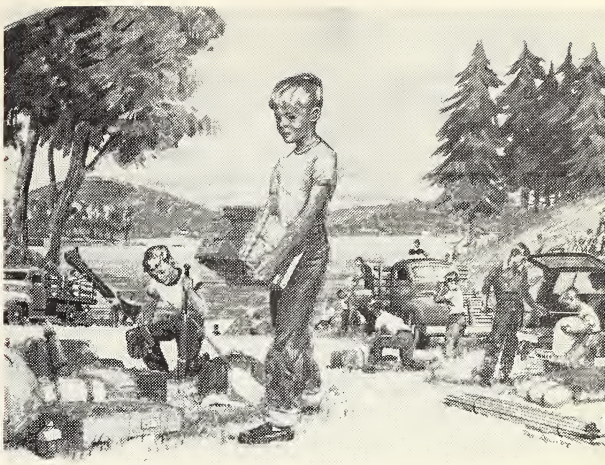
42	33	58	41	34	60	75	17	84	95
-5	-7	-4	-1	-9	-8	-4	-2	-5	-3

3. Look through Part IV of this Manual (pages 310–319) for devices and games which might be used by all pupils for increasing understanding, or for stimulating practice. For devices, consider using any or all of these: bead board, calculating blocks and strips, bead line, peg board, a pocket chart. For games, consider these: "Old Hat," "Single Search," "Zooks" (the prescribed remainder may be 7, 5, or any other number from 1–9), or "Double Draw" (for whole stories).

Teaching Pages 16 and 17

Pupil's Objectives. (*a*) To take Inventory Test 4; (*b*) to review regrouping as needed for borrowing in subtraction; (*c*) to review subtraction involving two-place numbers, without and with borrowing.

Background. The results of Inventory Test 4 on page 16 will help you plan your reteaching program in regard to subtracting larger numbers without and with borrowing.



Can You Subtract Larger Numbers?

Inventory Test 4 [W]

1. When plans were made for the June science camping trip, Mr. Lane, the head counselor, decided to take only 56 boys.

In May, he had taken 63 boys. How many fewer boys were signed up for the June trip than for the May trip? *7*

2. On the way to the Park camping ground, 75 miles away, the campers stopped for lunch. They still had 49 miles to go. How many miles had they already driven? *26*

3. When they left home, the campers had brought 176 sandwiches. At the lunch stop along the way, 149 sandwiches were eaten. How many sandwiches were left? *27*

4. One luggage truck carried about 4,500 lb. of duffel bags and equipment. A smaller truck carried tents and a canoe weighing in all about 1,785 lb. How much more weight did the big truck carry than the small one? *2,715 lb.*

16

If help is needed, it will be found on pages 17 to 19. Pupils start out on page 17 by learning how to subtract by parts, but you may, if you wish, start by using concrete materials as suggested in the Pre-book Lesson on the next page. In any case, pupils should eventually arrive at the point where they can subtract the short way as shown in boxes A and B on page 17. They should, however, be encouraged to drop the use of the crutch (the red 7 and circled 14 in box B), since insistence on its use will affect their ability to handle the subtraction step in division later on.

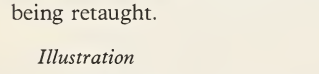
Be sure children understand that when there are not enough ones from which to subtract, a ten must be borrowed and thought of as 10 ones which when combined with the given ones makes subtraction in one's place possible. If the crutch is used, pupils should realize that this changing is indicated by circling the two figures (a ten and some ones) that are to be thought of as ones, by making the figure in the next place (ten's place) one less, and by crossing out the figures in the minuend that have been changed.

Book Lesson (page 16). *Ex. 1–4:* Oral work. Assign Inventory Test 4 to all. Study the results to find general weaknesses of the class and to help you decide what groups are needed within the class.

Pre-book Lesson (page 17). Try to give a new, interesting, and even humorous approach to review work to avoid having it become monotonous. Present situations using representative materials or sketches that show practical applications of skills being retaught.

Illustration

a. Some children were planning favors for a party. They had 48 pieces of candy. If they used 16 pieces of candy for the first favor, how many did they then have left?



40 and 8, or 48
− 10 and 6, or 16
30 and 2, or 32

b. If they then took 16 of the 32 candies, how many would they have remaining?

Allow a pupil to demonstrate what he would do when he could not take 6 candies from 2. Elicit information from the class which will lead to recording the regrouping in both the long and short ways shown below.

30 and 2	20 and 12	2 (12)
− 10 and 6	− 10 and 6	3 2
	10 and 6, or 16	− 1 6
		1 6

Book Lesson (page 17). *Ex. 1 and 2:* Oral work. *Rows 3–6:* Written work.

1. Encourage pupils to work without a crutch. There may be a few, but only a few, children who actually need to depend upon it.

2. *More capable pupils* may be assigned only rows 3 and 4, or possibly just row 4 for written work. Scan the papers of the *slower learners* in an attempt to diagnose their difficulties and to group them for remedial teaching.

Subtracting with Two-Place Numbers

Without and with borrowing [O]

Just as we can add numbers by parts, we can subtract by parts too.

1. 56 − 12 = 44 We can separate these numbers according to tens and ones and subtract, as in box A. Tell how to subtract first ones then tens, the short way.

A	Short way
50 and 6	56
− 10 and 2	− 12
40 and 4, or 44	44

2. 84 − 16 = 68 When we separate 84 into 80 and 4, why can we not subtract the ones? We can't take 6 ones from 4 ones.

B	Short way
70 and 14	7 (14)
− 10 and 6	8 4
60 and 8, or 68	− 1 6
	6 8

But we can separate 84 as in box B, letting one part be 10 less than 80, or 70, and the other part be 10 more than 4, or 14. Tell how to subtract the short way. We call this changing of groups **borrowing.**

The little figures in box B show how we borrow. You should try to do the subtracting without writing these figures.

[W]

Copy these examples and find the remainders:

	a	b	c	d	e	f	g
3.	78	56	95	69	87	$0.98	$0.46
	− 21	− 27	− 30	− 6	− 13	− 0.05	− 0.15
	57	29	65	63	74	$0.93	$0.31
4.	46	32	62	51	46	$0.82	$0.63
	− 13	− 16	− 8	− 17	− 19	− 0.36	− 0.07
	33	16	54	34	27	$0.46	$0.56
5.	76	82	52	67	94	$0.37	$0.46
	− 20	− 48	− 27	− 7	− 87	− 0.28	− 0.29
	56	34	25	60	7	$0.09	$0.17
6.	56	83	64	25	34	$0.60	$0.82
	− 32	− 25	− 56	− 7	− 28	− 0.20	− 0.23
	24	58	8	18	6	$0.40	$0.59

17

Differentiations and Extensions

1. *More capable children* may continue with some of the projects suggested in the last lesson, probably formulating problems or reports to present to the class.

2. *Slower pupils* may need continued practice with concrete materials which they can manipulate in demonstrating regrouping (borrowing). Pictured symbols (using ∅ for ten, and / for one, as illustrated at the top of the next column) also develop understanding and afford good practice.

64 − 26 = ?

Step 1.

60 and 4, or 64

Step 2. Because 6 (ones) cannot be taken from 4 (ones), regroup (borrow a ten).

50 and 14, or $\not{5}\ 1\not{4}$

Step 3. Cross out the number subtracted.

50 and 14	$\not{5}\ 1\not{4}$
− 20 and 6	− 2 6
30 and 8, or	3 8

Teaching Pages 18 and 19

Pupil's Objectives. (*a*) To review 3-place and 4-place subtraction with emphasis on regrouping; (*b*) to review subtraction examples containing 0's in the minuend.

Background. Pupils have already been introduced to the idea of changing the minuend (borrowing) in order to make subtraction possible. Now, on pages 18 and 19, the idea will be expanded to include 3- and 4-place minuends, some of which contain zeros. The basic idea is no more difficult for 3- and 4-place minuends; it is only more complicated.

Again, be sure pupils understand the rationale of borrowing and help them to do away with the necessity for using a crutch as soon as they evidence understanding of the process.

Zeros in the minuend often cause pupils great difficulty. However, if they really understand place value and the idea of changing the minuend, this should not be so. For example, notice in box A, on page 19 in the text, how convenient it is to think of 206 as 20 tens and 6 ones when it becomes apparent that a ten must be borrowed. The same procedure may be applied to examples such as the one in box D on page 19, in which 3,005 is thought of as 300 tens and 5 ones. When a ten is borrowed, 299 tens remain and may be written as shown by the little red figures.

Book Lesson (page 18 and top of page 19). *Ex. 1–9:* Oral work. *Rows 10 and 11:* Written work.

Three Places in the Minuend

5 is smaller than 8; 1 is smaller than 4 — Borrowing [0]

1. In Ex. A why did we not have to borrow?

Begin at the right. Subtract ones from ones, tens from tens, and hundreds from hundreds.

2. In this way tell how to subtract
a. 768 − 24; **b.** 385 − 24; **c.** 426 − 320.

3. In Ex. B we could not subtract the ones, so we changed the grouping of the tens and ones without changing the value of the number. Why are 7 tens and 12 ones equal to 8 tens and 2 ones?
Tell about the subtracting in Ex. B.

70+12=82
80+2=82

4. On the board show the borrowing in
a. 734 − 216; **b.** 545 − 37; **c.** 627 − 19;
d. 846 − 228; **e.** 723 − 498; **f.** 543 − 195.

5. In Ex. C we cannot subtract in ten's place, so we borrow 1 hundred (10 tens) from the 4 hundreds and add the 10 tens to the 2 tens. Explain.

6. On the board show the borrowing in
a. 548 − 263; **b.** 627 − 43; **c.** 326 − 251; **d.** 405 − 95;
e. 607 − 127; **f.** 243 − 173; **g.** 308 − 78; **h.** 216 − 85.

7. In Ex. D we cannot subtract in one's or in ten's place. So we borrow 1 ten and 1 hundred. Explain the borrowing. Why does 400 + 130 + 12, the changed minuend, equal 542?★

8. On the board show the borrowing for these examples:
a. 413 − 258; **b.** 762 − 68; **c.** 592 − 294; **d.** 827 − 98;
e. 935 − 862; **f.** 742 − 18; **g.** 243 − 89; **h.** 635 − 78.

9. In subtracting money numbers, what must we remember?
Write dollar sign and cents' point in answer.

When you cannot subtract in any place, borrow 1 of the units of next higher value and change it to make 10 of the units needed.

1. Consider the generalization at the bottom of page 18, particularly the phrase *next higher value.* Allow *more capable pupils* to explain its meaning in their own way, but let *slower learners* apply its meaning in small-group practice.

★Answers Not on Reproduced Page 18

7. 542 = 5 hundreds + 4 tens + 2 ones,
or (4 hundreds + 1 hundred)
+ (3 tens + 1 ten) + 2 ones,
or 4 hundreds + (10 tens + 3 tens)
+ (10 tens + 2 ones),
or 4 hundreds + 13 tens + 12 ones,
or 400 + 130 + 12

[W]

Subtract. Check by adding the subtrahend and the remainder.

	a	b	c	d	e	f
10.	389	264	429	605	$1.80	$7.63
	−46	−128	−284	−275	−0.91	−0.81
	343	136	145	330	$0.89	$6.82
11.	707	326	514	453	$3.62	$2.44
	−77	−248	−67	−259	−0.68	−0.75
	630	78	447	194	$2.94	$1.69

0's in the Minuend

3-place and 4-place minuends [O]

1. In Ex. A we cannot subtract in one's place, so we must borrow a ten. But what figure is in ten's place? 0

Why can't we borrow anything from 0? 0 means "nothing"

In 206 there are 20 tens, so we can borrow 1 ten, leaving 19 tens. Explain. 20 tens − 1 ten = 19 tens

2. In Ex. B we borrow 1 ten from how many tens? 7 How many tens are left? 69

3. In Ex. C we borrow twice. The 4,637 becomes 3,000 + 1,500 + 130 + 7. Explain.

4. In Ex. D we borrow 1 ten from 300 tens. Why can we use 2,990 + 15 instead of 3,005? 3,005 = 300 tens + 5 ones = 299 tens + 1 ten + 5 ones, or 2,990 + 15

Try to change minuends in your head. Study Ex. E.

Ones: *Think*, "3 from 7 = 4."

Tens: *Think*, "8 from 3, no; borrow 1 from 6 hundreds, leaving 5; 8 from 13 = 5."

Hundreds: *Think*, "6 from 5, no; borrow 1 from 4 thousands, leaving 3; 6 from 15 = 9."

Thousands: *Think*, "2 from 3 = 1."

The remainder in Ex. E is 1,954.

[W]

Extra Practice. Work Sets 9–13.

A
1 9 16
2 0 6
− 7 8
1 2 8

B
6 9 10
7 0 0
− 2 4 6
4 5 4

C
3 15 13
4,6 3 7
− 2,6 8 3
1,9 5 4

D
2 9 9 15
3,0 0 5
− 2 8 6
2,7 1 9

E
4,6 3 7
− 2,6 8 3
1,9 5 4

2. For rows 10 and 11 at the top of page 19, the following labels may be used in explaining the check.

76 (total, or sum)
− 58 (one part)
18 (other part)
76 (total, or sum)

Both parts put together again give the *total* or *sum*.

Differentiations and Extensions (page 18 and top of page 19)

Slower learners

a. Have individuals demonstrate their ability to subtract using abstract numbers only. Lead them away from practice with concrete or semi-concrete materials as soon as possible.

b. Be sure pupils understand the meaning of checking subtraction. Use classroom equipment and have these pupils put together and separate groups of paper clips, pencils, etc.

Pre-book Lesson (bottom of page 19). Review borrowing in terms of the generalization at the bottom of page 18, letting pupils apply it in exercises similar to those below.

a. In the number 436,

	100's	10's	1's
	4	3	6
change a ten to ones. →	4	2	16

b. In the number 5,972,

	1,000's	100's	10's	1's
	5	9	7	2
change a hundred to tens; →	5	8	17	2
then change a thousand to hundreds. →	4	18	17	2

Book Lesson (bottom of page 19). *Ex. 1–4:* Oral work. *Bottom of page:* Written work. If you wish to chart the regrouping for this lesson, it is best done at the board concurrently with the study of Boxes A, B, and D in the text.

Box A. The minuend is 206. We need a ten, so we think of 206 as 20 tens and 6 ones. After borrowing (regrouping), we have 19 tens (or 1 hundred and 9 tens) and 16 ones in the minuend.

	100's	10's	1's
	2	0	6
Thinking of 20 tens →		20	6
Regrouping →	1	9	16

Box B. The minuend is 700. We need a ten.

	100's	10's	1's
	7	0	0
Thinking of 70 tens →		70	0
Regrouping →	6	9	10

Box D. The minuend is 3,005. We need a ten.

	1,000's	100's	10's	1's
	3	0	0	5
Thinking of 300 tens →			300	5
Regrouping →	2	9	9	15

Differentiations and Extensions (bottom of page 19)

1. *Slower learners* may use graph paper to demonstrate regrouping to the class. The paper should be cut so that there are squares representing ones, tens (a row of 10), and hundreds (10 rows of 10).

2. **Extra Practice Sets 9–13**, reproduced below with answers, may be assigned as needed.

Set 9. Subtraction; higher-decade and two 2-place numbers; no bridging or borrowing

	a	b	c	d	e	f	g	h	i	j
1.	94	76	69	52	28	86	37	19	97	57
	− 4	− 3	− 25	− 31	− 7	− 80	− 4	− 13	− 70	− 6
	90	73	44	21	21	6	33	6	27	51
2.	74	89	97	83	36	49	81	65	72	53
	− 13	− 6	− 97	− 31	− 16	− 20	− 30	− 12	− 32	− 13
	61	83	0	52	20	29	51	53	40	40
3.	38	45	98	68	79	99	37	56	98	88
	− 24	− 14	− 42	− 55	− 51	− 52	− 2	− 54	− 86	− 10
	14	31	56	13	28	47	35	2	12	78

Set 10. Subtraction; higher-decade and two 2-place numbers; with bridging and borrowing

1.	21	13	96	22	54	30	47	65	88	32
	− 4	− 5	− 8	− 7	− 9	− 8	− 9	− 6	− 9	− 23
	17	8	88	15	45	22	38	59	79	9
2.	44	71	56	66	90	84	61	75	92	20
	− 26	− 69	− 47	− 39	− 13	− 57	− 16	− 37	− 36	− 16
	18	2	9	27	77	27	45	38	56	4
3.	94	77	81	65	92	53	81	50	33	85
	− 85	− 48	− 25	− 59	− 48	− 26	− 77	− 14	− 17	− 48
	9	29	56	6	44	27	4	36	16	37
4.	82	91	42	93	64	81	73	92	71	93
	− 19	− 78	− 34	− 59	− 48	− 32	− 28	− 65	− 53	− 24
	63	13	8	34	16	49	45	27	18	69

Set 11. Three-place minuends; no borrowing

	a	b	c	d	e	f	g	h
1.	432	641	397	258	796	854	595	978
	− 12	− 20	− 42	− 24	− 31	− 23	− 63	− 55
	420	621	355	234	765	831	532	923
2.	859	737	974	678	945	898	756	869
	− 351	− 623	− 534	− 612	− 214	− 776	− 415	− 848
	508	114	440	66	731	122	341	21
3.	812	573	398	907	674	846	631	459
	− 211	− 573	− 196	− 506	− 230	− 815	− 420	− 338
	601	0	202	401	444	31	211	121

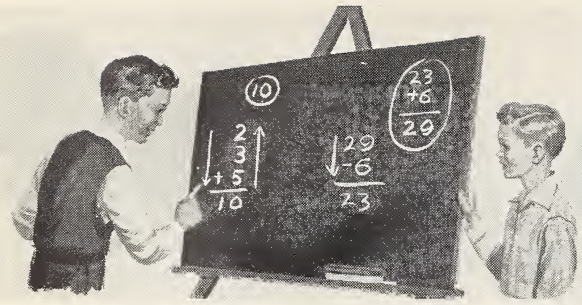
Set 12. Three-place minuends; with borrowing

1.	580	745	535	640	121	920	603	894
	− 84	− 66	− 97	− 89	− 95	− 47	− 98	− 97
	496	679	438	551	26	873	505	797
2.	212	347	274	451	764	636	222	984
	− 88	− 259	− 187	− 368	− 485	− 577	− 173	− 797
	124	88	87	83	279	59	49	187
3.	855	931	873	700	321	733	500	452
	− 689	− 643	− 494	− 308	− 123	− 669	− 486	− 295
	166	288	379	392	198	64	14	157

Set 13. Harder subtraction; without and with borrowing

	a	b	c	d	e
1.	550	2,734	8,357	98,765	12,381
	− 431	− 742	− 3,365	− 86,444	− 3,582
	119	1,992	4,992	12,321	8,799
2.	65,345	7,998	96,822	75,442	8,895
	− 7,667	− 2,722	− 8,967	− 65,578	− 4,845
	57,678	5,276	87,855	9,864	4,050
3.	976	12,455	985	32,033	6,318
	− 372	− 8,888	− 632	− 9,999	− 3,475
	604	3,567	353	22,034	2,843
4.	7,106	31,076	4,904	805	9,151
	− 499	− 5,328	− 3,956	− 689	− 298
	6,607	25,748	948	116	8,853

NOTES



Checking Sums and Remainders

[O]

Tom's young brother Ted asked, "What does checking mean?"

Tom said: "I'll show you on your blackboard. In adding, we find the sum by adding down. Then we add up and find the sum. When we compare the two sums to see if they are equal, we are checking our addition."

Tom also told Ted how to check subtraction. He said: "The remainder and the subtrahend are the two parts of the minuend. Then we can check our work by adding these two parts together and comparing their sum with the minuend."

1. Explain the checking of the addition which Tom showed Ted in the picture and the checking of the subtraction. *Add up to check A.; add remainder and subtrahend to check S.*

2. In box A, compare the sum of 47 and 35 with the minuend, 82. Are they equal? *Yes* Then why is 47 the correct other part? *47+35=82*

3. The check is shorter in box B. Add upward. Ones: 7 + 5 = 12. The 2 equals the 2 in the minuend. Tens: 1 + 4 + 3 = *8* Does this sum equal the 8 in the minuend? *Yes*

A		
	82	Check
	−35	47
	47	+35
		82
B		
	82	
	−35	Add to check
	47	

[W]

4. Solve and check: **a.** 94 − 38; *56* **b.** 73 − 56; *17* **c.** 62 − 29. *33*

Teaching Pages 20, 21, and 22

Pupil's Objectives. (*a*) To discover the relationship between the checks for addition and subtraction; (*b*) to estimate sums and remainders by rounding numbers; (*c*) to obtain practice in addition and subtraction.

Background. In practical affairs, it is very important for children to be able to check and estimate sums and remainders. They will receive help in developing these skills by working the exercises on pages 20 and 21.

There should be very little difficulty with the check for addition and subtraction, since these have both been introduced in the previous reteaching for Grade 6. However, in this lesson *both* checks are shown, and children are given a chance to explain their meanings in relation to one another.

Estimating sums and remainders may be more difficult for some children. The basic idea behind the kind of estimating shown on page 21 is that if we can produce round numbers, we can add tens, hundreds, etc. as easily as we add ones. If we were adding a group of 3-place numbers, we could neglect all but the figures in hundred's place, adding the latter like ones. A better way would be to compensate for the loss of the figures in one's and ten's places by rounding to the next higher hundred all of those numbers having a figure 5 or larger in ten's place. This is the method shown in box A on page 21.

Book Lesson (page 20). *Ex. 1–3:* Oral work. *Row 4:* Written work.

Differentiations and Extensions (page 20)

1. Have all pupils make up, record, and do the computation for two examples in addition and subtraction. From these choose several to write on the board so that the class may check the sums and remainders given.

2. *More capable children* who have learned casting out 9's as an addition check (see page 30 in the Manual) may attempt to devise a similar check for subtraction (the difference in the excesses of 9's for the minuend and subtrahend should equal the excess of 9's in the remainder).

Book Lesson (page 21). *Ex. 1–3:* Oral work. *Ex. 4–8:* Written work.

1. Emphasize the fact that for any specific example we must choose one, and only one, place to which to round, i.e., we must determine whether to estimate in terms of tens or hundreds or thousands.

2. Before assigning written work, provide help with examples that have "ragged" columns.

Illustration. For each example below, tell whether you would round to tens, hundreds, or thousands. Explain.

a.	*b.*	*c.*	*d.*
49	379	84	4,056
16	24	62	3,915
75	415	114	264
6		29	

Estimating Sums and Remainders

[O]

In box A, the addends are rounded in order to estimate the sum. In box B, the numbers are rounded to estimate the remainder. Rounding helps you to add or subtract "in your head."

A	Estimate
428	400
753	800
519	500
+94	+100
1,794	1,800

B	Estimate
824	800
−596	−600
228	200

1. In box A, tell how each addend is rounded. to nearest hundred Then explain why 1,794 is a reasonable answer. very close to estimate

2. In box B, is 228 close enough to 200 to be considered reasonable? Yes

3. Rounding to tens, estimate the sum of these numbers: 28, 56, 14, 97. 30+60+10+100=200

[W]

4. Add; then check by rounding to thousands and estimating: 7,254, 6,902, 14,073. Write your work as in box A. 7,000+7,000+14,000=28,000 (Exact sum, 28,229)

5. Subtract; check by rounding to thousands and estimating: 8,640 − 3,251. Write your work as in box B. 8,640 − 3,251 = 5,389 9,000 − 3,000 = 6,000

Work the examples below and check by estimating.

	a	b	c	d	e	f
6.	96	332	604	3,480	4,059	$82.54
	84	416	576	926	768	79.05
	27	764	482	6,748	2,574	2.68
	31	295	951	5,281	10,364	15.27
	+50	+974	+430	+7,827	+6,928	+97.62
	288	2,781	3,043	24,262	24,693	$277.16
7.	854	786	401	7,052	4,307	$70.13
	−474	−87	−273	−2,059	−1,289	−4.52
	380	699	128	4,993	3,018	$65.61
8.	359	544	800	6,205	5,049	$94.09
	−168	−168	−381	−2,698	−763	−57.18
	191	376	419	3,507	4,286	$36.91

Practice in Addition and Subtraction

[W]

Copy each example. Estimate the remainder and write it beside the example. Subtract, and use the estimate as a check.

	a	b	c	d	e
1.	628 (200)	$7.64 ($7.00)	5,460 (5,000)	3,516 (2,000)	21,958 (12,000)
	−419	−0.38	−531	−1,604	−10,439
	209	$7.26	4,929	1,912	11,519
2.	406 (200)	$40.00 ($25.00)	3,900 (3,300)	2,900 (2,500)	47,604 (30,000)
	−218	−15.36	−627	−436	−18,417
	188	$24.64	3,273	2,464	29,187
3.	700 (640)	$90.07 ($80.00)	4,210 (2,000)	4,003 (3,300)	94,058 ($89,000)
	−59	−12.52	−1,762	−685	−4,629
	641	$77.55	2,448	3,318	89,429
4.	400 (390)	$60.00 ($60.00)	7,823 (7,200)	40,079 (39,000)	30,003 (26,000)
	−8	−0.07	−587	−816	−4,208
	392	$59.93	7,236	39,263	25,795
5.	215 − 60 (160) 155	503 − 27 (470) 476	786 − 197 (600) 589	216 − 38 (180) 178	462 − 87 (400) 375
6.	600 − 46 (550) 554	214 − 98 (100) 116	906 − 818 (100) 88	340 − 76 (270) 264	500 − 60 (400) 440

Copy each example. Estimate the sum and write it beside the example. Then add, and use the estimate as a check.

7.	547 (1,200)	$9.75 ($11.00)	7,809 (12,000)	4,989 (6,000)	28,507 (45,000)
	+658	+0.69	+3,608	+763	+16,389
	1,205	$10.44	11,417	5,752	44,896
8.	297	$4.87	1,376	7,657	82,970
	78 (450)	3.67 ($9.00)	295 (2,400)	499 (12,000)	45,098 (170,000)
	+46	+0.59	+684	+3,564	+37,596
	421	$9.13	2,355	11,720	165,664
9.	387	$5.95	2,684	75	59,284
	746	7.89	367	2,097	7,691
	527 (2,200)	6.98 ($34.00)	4,019 (7,000)	644 (3,000)	28,052 (118,000)
	492	4.95	38	7	6,548
	+69	+8.48	+76	+281	6,037
	2,221	$34.25	7,184	3,104	+9,580
					117,192

Extra Practice. Work Sets 14 and 15.

3. Show how to record estimates in a form somewhat similar to that given below for Ex. 6**a** and 6**b** in the text.

Ex. 6a. Rounding to tens

96 rounds to	100
84 rounds to	80
27 rounds to	30
31 rounds to	30
50	50
288 is close to	290

Ex. 6b. Rounding to hundreds

332 rounds to	300
416 rounds to	400
764 rounds to	800
295 rounds to	300
974 rounds to	1,000
2,781 is close to	2,800

Book Lesson (page 22). *Rows 1–9:* Written work. Pupils receive practice in estimating and in computation involving addition and subtraction.

Differentiations and Extensions

1. *Slower pupils*

a. Give more detailed practice of the kind suggested below.

Rounding to tens

32 is between 30 and 40, but nearer 30.	30
50 is already an even ten.	50
7 is between 0 and 10, but nearer 10.	10
65 is halfway between 60 and 70; so we round upward.	70
154	160

Rounding to hundreds

400 is already an even hundred.	400
62 is between 0 and 100, but nearer 100.	100
350 is halfway between 300 and 400; so we round upward.	400
114 is between 100 and 200, but nearer 100.	100
926	1,000

b. Very likely these pupils will need help in rounding money numbers. Suggest that they think in terms of rounding to the nearest dime or dollar. Practice in the form given below may be helpful.

Rounding to the nearest dime

$0.93 rounds to	$0.90
0.47 rounds to	0.50
0.55 rounds to	0.60
0.84 rounds to	0.80
$2.79 is close to	$2.80

Rounding to the nearest dollar

$2.76 rounds to	$3.00
0.50 rounds to	1.00
1.04 rounds to	1.00
4.85 rounds to	5.00
$9.15 is close to	$10.00

2. Before *more capable pupils* find exact answers for the written work on page 22 in the text, let them try to tell whether their estimates will be larger or smaller than the exact answers. This kind of thinking often leads to short cuts for adding and subtracting and helps pupils understand more clearly the nature of rounding.

3. **Extra Practice Sets 14 and 15**, reproduced below, may be assigned as needed.

Set 14. Adding and subtracting money numbers; with carrying and borrowing

	a	b	c	d	e
1.	$0.26	$4.68	$0.98	$1.54	$5.84
	+ 0.76	− 0.65	+ 3.47	+ 8.57	+ 7.48
	$1.02	**$4.03**	**$4.45**	**$10.11**	**$13.32**
2.	$18.75	$54.55	$46.79	$27.89	$236.75
	− 3.75	− 47.96	− 9.92	+ 42.96	− 95.88
	$15.00	**$6.59**	**$36.87**	**$70.85**	**$140.87**
3.	$5.67	$9.86	$0.35	$5.46	$71.28
	5.56	0.86	7.78	5.78	81.97
	+ 0.87	+ 9.87	+ 3.52	+ 0.68	+ 9.36
	$12.10	**$20.59**	**$11.65**	**$11.92**	**$162.61**
4.	$2.36	$7.69	$26.25	$36.28	$857.90
	9.53	4.31	42.54	68.93	847.98
	+ 8.65	+ 0.76	+ 7.64	+ 52.95	+ 48.56
	$20.54	**$12.76**	**$76.43**	**$158.16**	**$1,754.44**

Set 15. Addition and subtraction; with carrying and borrowing

1.	872	604	9,421	6,385	78,568
	381	749	− 7,899	− 5,776	+ 78,762
	+ 758	+ 595	**1,522**	**609**	**157,330**
	2,011	**1,948**			
2.	544	629	82,325	4,738	9,759
	878	423	− 36,695	4,544	6,235
	736	665	**45,630**	8,646	3,394
	+ 867	922		+ 9,948	1,361
	3,025	+ 816		**27,876**	+ 139
		3,455			**20,888**

NOTES

Finding n

The letter **n** can stand for any number.

Missing addend [O]

A

n + 9 = 16	7
8 + n = 14	6
12 = n + 7	5
17 = 9 + n	8

1. In the addition examples in box A, tell what the addend **n** stands for. Explain what you did to find the missing addend in each example. *Subtract smaller number from larger.*

2. To find **n** in n + 9 = 16, we could write our work, "16 − 9 = n." Tell how you would write each of the other examples in box A to find **n**. *14-8=n; 12-7=n; 17-9=n*

3. Tell how to find **n** in 68 + n = 99; in n + 306 = 419. *99-68=n=31; 419-306=n=113*

When we know the sum of two addends and we know one of the addends, we can subtract to find the other addend.

Missing minuend

B

n − 7 = 8	15
6 = n − 9	15
n − 6 = 7	13
5 = n − 8	13

4. In each subtraction example in box B, tell what the minuend, **n**, stands for. Explain what you did to find **n** in each example. *Add*

5. To find **n** in n − 7 = 8, we could write, "8 + 7 = n." Tell how to write the work for the other examples in box B. *6+9=n=15; 6+7=n=13; 5+8=n=13*

6. Tell how to find **n** in n − 25 = 13; in 163 = n − 82. *13+25=n=38; 82+163=n=245*

In subtraction, the minuend is the sum of the remainder and the number subtracted. If we know the two numbers, we can add them to find the minuend.

[W]

Find **n** in Ex. 7 to 16.

7. n + 28 = 75 *47*
8. n − 46 = 23 *69*
9. 86 = n − 17 *103*
10. 123 + n = 476 *353*
11. n − 78 = 145 *223*
12. n + 264 = 398 *134*
13. 742 = n − 325 *1067*
14. n + 29 = 81 *52*
15. 39 = n − 87 *126*
16. 278 + n = 905 *627*

Teaching Page 23

Pupil's Objective. To relearn the procedure for finding *n* when it represents a missing addend or minuend.

Background. It frequently happens that we need to solve a problem of this type: "I have saved $8 toward a $17 enamelware kit for making jewelry. How much more must I save?" This may be expressed in the form 8 + ? = 17. There is nothing wrong with using the question mark for the missing number, but it is much better to prepare children for the literal numbers that they will meet in later grades by using *n* as shown in box A on page 23 in the text. In this lesson, *n* is used to stand for a missing addend or a missing minuend. From their understanding of the relation between addition and subtraction, children will quickly see why they must subtract to find a missing addend and why they must add to find a missing minuend. The letter *n* may also be used to stand for a missing sum or a missing remainder, but this use results in very simple exercises, for it is exactly like finding answers for regular addition or subtraction examples.

Pre-book Lesson. An *n* stand (page 314 in the Manual) may be used in showing pupils how the letter *n* may be used to represent a missing number.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7–16:* Written work. After Ex. 3, have pupils tell in their own words how they would find *n* when it represents a missing addend. After Ex. 6, let them explain the finding of a missing minuend.

Differentiations and Extensions. For *slower learners* use a number line in conjunction with an oak-tag card on which is printed the letter *n*, as illustrated below.

5 + n = 8 n = ?

5 + n = 8, so n = 8 − 5, or 3

NOTES

The Museum of Science

Differentiating A. and S.; estimating answers [O]

To combine groups, we add.
To take a group apart or compare groups, we subtract.

Tell for each problem whether you would add or subtract. Then estimate the answer.

1. A group of 19 boys and 15 girls in Miss Pike's class visited the Museum of Science. How many were in the group? A.; (35); 34

2. Jack found that the catalogue listed these exhibits: astronomy, 3; communications, 9; history of man, 4; natural history, 17; physical science, 16; public health, 3; transportation, 16. How many exhibits were on display? A.; (75); 68

3. Sam found an exhibit showing the first telephone, with only 46 parts, and today's telephone, with 433 parts. How many more parts than the first telephone does today's telephone have? S.; (390); 387

4. The temperature of the aquarium water, 73°, was how much warmer than the room temperature of the Museum, 68°? S.; (5°); 5°

5. Ann found a model of pyramids in Egypt, with 2,500 tiny figures of workmen. Ann began to count the little workmen but gave up after counting 27. How many more were there? S.; (2,470); 2,473

6. In the Museum shop, Joe spent 65¢ for a mineral collection and 25¢ for a book on shells. How much did he spend? A.; (90¢); 90¢

[W]

Now go back and solve the problems. Check each answer.

Hidden Questions

2-step problems; A. and S. [O]

1. Amy bought a skirt for $3.98 and a blouse for $2.98. a. How much did she pay for both? $6.96 b. How much change did she receive from a $10 bill? $3.04

This problem has two questions. You must answer the first question before you can answer the second. Below is the same problem with one question asked and the other question "hidden":

Amy bought a skirt for $3.98 and a blouse for $2.98. How much change did she receive from a $10 bill?

There are still 2 steps in the solution of the problem. The first step is to answer hidden question a, "How much did she pay for both the skirt and the blouse?"

In these problems tell why you must answer question a before you can answer question b:

2. Ted's mother gave him $2.00 to pay for some groceries. Ted put this money in his wallet with his own money. He paid $1.36 for the groceries, and bought nothing else. When he got back from the store he had $1.19 in his wallet. a. How much change did he owe his mother? $0.64 b. How much of his own money was in the wallet? $0.55

3. Mrs. Evans wants to get to the station 15 minutes before her train leaves at 5:15 P.M. She needs 30 minutes for the trip from her house to the station. a. At what time must she reach the station? 5 P.M. b. When should she leave home? 4:30 P.M.

4. Mrs. Howe's regular telephone service for local calls costs $3.75 a month. Last month she paid 56¢ more for extra local calls. Her total bill, including long-distance calls, was $7.00. a. What was the total charge for local service? $4.31 b. What did her long-distance calls cost her last month? $2.69

[W]

Now write each problem with only question b asked. Then solve the problem.

Teaching Pages 24, 25, and 26

Pupil's Objectives. (*a*) To select the correct process for solving addition and subtraction problems; (*b*) to practice estimating answers for addition and subtraction problems; (*c*) to study and solve problems containing hidden questions.

Background. Ability to solve problems cannot be confined to special problem-solving lessons. Pages 24, 25, and 26, then, are intended to help pupils extend whatever ability in problem-solving has already been developed. On page 24, children learn to differentiate between the need for addition or subtraction in problems. At the same time, they obtain practice in estimating answers. As pupils decide orally what process is needed to solve a problem, they learn to distinguish the need for a certain process in a problem, and they learn how addition and subtraction may be called for in problem-solving situations.

Two-step problems are introduced on pages 25 and 26 for review. Since the biggest difficulty in

Finding Hidden Questions

2-step problems: A. and S. [O]

Some of the following problems have a hidden question which must be answered before you can answer the problem question. Read each problem carefully and tell whether or not it has a hidden question. If it has, tell what the hidden question is.

1. To patch some holes in the doghouse roof, Max needed four pieces of board with the following lengths: 16 in., 21 in., 13 in., and 9 in. He sawed these pieces from a board 64 in. long. How long was the piece that was left? [How many inches in the four pieces? (59)] 5 in.

2. When Tony went to visit his grandparents, his father took him 8 miles to the railroad station. Then he rode 135 miles on the train and 26 miles on a bus to get to his grandparents' farm. How far did Tony travel in all? 169 mi.

3. A candy store bought 25 lb. of sour balls. After 12 lb. had been sold, the rest was mixed with other hard candy to make a mixture that weighed 30 lb. in all. How many pounds of other candy had to be added to the sour balls? [How many pounds of sour balls in the mixture? (13 lb.)] 17 lb.

4. At the end of the year, the Gold Thimble Sewing Club had $4 left in the treasury, so the girls decided to give a party for their mothers. They could get 2 quarts of ice cream for $1.25 and cupcakes for $1.12. That would leave how much money for favors and prizes? [How much did the ice cream and cupcakes cost? ($2.37)] $1.63

5. The girls in Grade 6 sold 131 calendars to earn money for the Science Club, and the boys sold 96. The girls sold how many more calendars than the boys did? 35

6. On Monday, Mrs. Bishop put 37 dozen pansy plants on sale at her roadside stand. That day she sold 19 dozen. Tuesday morning she put 13 dozen more plants with the unsold ones. How many dozen plants in all were on sale Tuesday morning? [How many were not sold on Monday? (18 doz.)] 31 doz. [W]

Now go back and solve the problems. Write all your work.

solving two-step problems is discovering the hidden question, pupils are first presented with problems (page 25) which contain two parts, the first of which must be answered before the second part can be answered. Then, on page 26, children are presented with problems some of which contain a hidden question which they first locate orally.

Book Lesson (page 24). *Ex. 1–6:* Oral work. *Bottom of the page:* Written work. Ask pupils to justify their choices of addition or subtraction so that you may be sure their decisions are based on understanding.

Book Lesson (page 25). *Ex. 1–4:* Oral work. *Bottom of the page:* Written work. You may want to isolate certain important items of information to record on the board during discussion, as illustrated below for Ex. 1 in the text.

Information Given		*To find*	*Need to know*
skirt	$3.98	change from	total cost of skirt
blouse	$2.98	$10.00 bill	and blouse

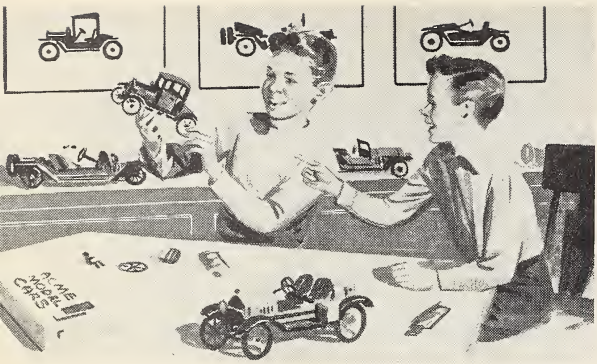
Book Lesson (page 26). *Ex. 1–6:* Oral work. *Bottom of the page:* Written work.

Differentiations and Extensions

1. Have *all children* state experiences of their own which required the solving of two-step problems (addition and subtraction only). Categories might include allowances, travel expenses, scores in games, and the like.

2. *More capable pupils* may write two-step problems containing hidden questions using information written in abbreviated form on the board.

NOTES



When We Multiply

Inventory Test 5 [W]

Each week Bill and Joe buy kits of parts for building model antique automobiles.

1. One week Joe bought 2 kits for making models. Each kit cost 63¢. What did Joe pay for the 2 kits? $1.26

2. Bill bought eight 5¢ tubes of cement so they would be sure to have enough. What did he spend for the cement? 40¢

3. The next week Bill bought 2 kits for making electric coupés. At 89¢ each, what did Bill pay for the 2 kits? $1.78

4. So they could each have two model roadsters, Joe bought 4 kits of those parts one week. At 59¢ each, what did Joe pay for these kits? $2.36

5. Joe painted the wheels of all his models green. Bill made his wheels red. Bill bought 6 tubes of paint at 25c a tube. How much did he pay for the paint? $1.50

6. What did 2 racers cost at $1.35 each? $2.70

7. At the end of twelve weeks, each boy had spent $14.25. What did both boys spend in all? $28.50

The Meaning of Multiplication (M.)

Order of factors [O]

1. If 8 children can sit at each of the lunchroom tables in the picture, how many children can sit at five of the tables? 40

First find the total by adding five 8's, as in box A. The sum is 40.

Since the groups of children are equal, a quicker way to find the total is to use the fact, "five 8's are 40." Explain the two ways of writing this multiplication fact that are shown in box B.

A

8
8
8
8
+8
40

To see five 8's, draw on the board a 45-inch line and number the inches. Start at the 0 point and mark off five groups of 8 inches. At what point on the line does the last group end? 40 Do five 8's equal 40? Yes

B

8 *multiplicand*
×5 *multiplier*
40 *product*

5 × 8 = 40

2. Look at box B again. We say that the numbers, 8 and 5, are **factors** of 40. Which factor tells the size of each of the equal groups? 8 Which factor tells the number of equal groups there are? 5 The factor that tells the size of the groups is called the multiplicand, and the factor that tells the number of groups is called the multiplier. The answer in a multiplication example is called the product.

Teaching Pages 27, 28, and 29

Pupil's Objectives. (*a*) To take Inventory Test 5; (*b*) to review the meaning of multiplication; (*c*) to review the terms *multiplicand*, *multiplier*, *product*, and *factor*; (*d*) to devise rules that may be used as helpers in learning difficult multiplication facts.

Background. Inventory Test 5 on page 27 will help you discover which pupils need reteaching of the understanding, skills, and abilities related to the meaning of multiplication facts and to multiplying tens, hundreds, and thousands, both when carrying does and does not occur.

The essential concept to be understood as far as the meaning of multiplication is concerned is that it is a combining process involving equal groups. It is, therefore, a short way of adding equal groups. Also reintroduced on pages 28 and 29 are such other matters related to multiplication as the reading of a multiplication fact; the terms *multiplicand*, *multiplier*, *product*, and *factor*; the idea that the multiplier tells the number of groups, whereas the multiplicand tells the size of the

3. Another lunchroom has 8 tables. Only 5 children can sit at each of these tables. How many children can be seated in all? 40
Eight 5's = 40

This time, 5 tells the size of the equal groups, so 5 is now the multiplicand; 8 tells the number of equal groups, so 8 is now the multiplier. The numbers multiplied (the factors) have changed places. Is the product changed? No Does 8×5 equal 5×8? Yes Explain. We may use the factors in any order.

4. $3 \times \$52 = \$156$, so $52 \times \$3 = \$156$. Tell why. any order.

5. In working the example 982×6, John wrote:

$$\begin{array}{r} 982 \\ \times 6 \end{array}$$

Was he wrong? No Explain why we can multiply by the smaller number in examples of this kind. Changing order of factors does not change the products.

Multiplication is a short way of adding equal groups.

The multiplicand tells the size of the groups, and the multiplier tells the number of groups. These two numbers are factors of the answer, or product.

We may use the factors in any order.

Remembering the Multiplication (M.) Facts

Helpers [O]

1. Tell a fact in which 3 and 5 are factors; in which the product is 72. 3X5=15, or 5X3=15 8X9=72, or 9X8=72

2. Now turn to page 293, cover the answers, and quickly say the 81 multiplication facts. 7X4=28, or 4X7=28

3. Now say the 19 multiplication facts in which 0 is one of the factors. Then make a rule for finding the product of 0 and any other number. The product of 0 and any other number is 0. (first row and first column of facts on page 293)

4. Say all the facts that have 1 as a factor. Then make a rule for finding the product of 1 and any other number. The product of 1 and any other number is the other number.

5. If you know the product of 6×9, do you also know the product of its reverse, 9×6? Yes Explain. We may use the factors in any order.

groups; and the idea that factors may be used in any order.

Since factors may be taken in any order and since it is usually easier to multiply by the smaller number in an example like $52 \times \$3$ (Ex. 4, top of page 29 in the text), we usually reverse the factors and multiply. However, it must be remembered that the multiplier always shows the *number of equal groups*, making it impossible to think of \$3 as a multiplier, since the number 3 with a dollar sign refers to the size of a group. Therefore, we disregard the dollar sign and multiply 3 times 52, interpreting only the answer as a money number by inserting a dollar sign.

By now pupils should be used to the idea of using helpers. Some that are suitable for use in multiplication are given at the bottom of page 29 in the text.

Book Lesson (page 27). *Ex. 1–7:* Written work. Inventory Test 5.

Book Lesson (page 28 and top of page 29). *Ex. 1–5:* Oral work.

1. As you develop the meaning of multiplication, relate it to addition, bringing out clearly the similarities and differences (i.e., that both are used for putting groups together, addition for unequal groups and multiplication for equal groups).

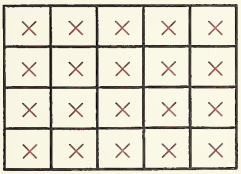
2. For Ex. 2, have a pupil put a diagram on the board to show eight 5's as presented in this problem.

Book Lesson (bottom of page 29). *Ex. 1–5:* Oral work. You may wish to have partners ask each other the facts referred to in Ex. 2 so that each pupil's difficult facts may be recorded. If you prefer to give a written test, be sure to include some of the 0-facts and some of the facts for 1 and for 2.

Differentiations and Extensions

1. *All pupils*

a. Use the following type of practice to develop the meaning of multiplication, to review reverse facts (like 4×5 and 5×4), and to review the terms that apply to a multiplication example.



By rows

Size of groups 5 (the multiplicand)
Number of groups $\times$ 4 (the multiplier)
20 (the product)

By columns

Size of groups 4 (the multiplicand)
Number of groups $\times$ 5 (the multiplier)
20 (the product)

b. Give practice also with facts written in horizontal form, as below.

Name the multiplicand that tells the size of each group.

$4 \times 7 = ?$ $9 \times 6 = ?$ $5 \times 8 = ?$ $6 \times 4 = ?$

Name the multiplier that tells the number of groups.

$3 \times 6 = ?$ $4 \times 1 = ?$ $2 \times 5 = ?$ $8 \times 3 = ?$

2. *Slower learners*

a. It is important that these pupils isolate facts needing study. Have them record these facts on study cards or in notebooks. It may be necessary in a few cases to have the facts organized by "families" (all facts with multiplicand 4, 5, etc.), or in small units for study and mastery.

b. Devices and equipment (see pages 310–315 in this Manual) for use in helping these pupils might include bead lines, bead boards, calculating blocks and strips, graph paper, and a multiplication pocket chart. Games (see pages 316–319) applicable to studying multiplication facts are: "Old Hat," "Single Search," "Clover," "Zooks," "Yay," and "Mads." Also check the list of commercial aids (pages 320–322) for films, filmstrips, and other such equipment.

Teaching Pages 30 and 31

Pupil's Objectives. (*a*) To review multiplying tens, hundreds, and thousands; (*b*) to review checking products by repeated addition; (*c*) to obtain practice in multiplying money numbers.

Background. The essential thing for pupils to remember in multiplying tens, hundreds, and thousands is that they may be multiplied just like ones (i.e., 3×2 hundreds = 6 hundreds, just as 3×2 ones = 6 ones) and that we can multiply by parts, as shown in box C on page 31 in the text. Box E also shows multiplying by parts and should help pupils see why it is that when we carry from one place to another we add the carried figure *after* we have multiplied in the next place. This sometimes causes difficulty, since in addition the carried figure may be added first.

With a firm background of understanding, pupils should progress quickly to multiplying the short way shown in boxes D and F on page 31 in the text.

Helps for Multiplying Tens, Hundreds, Thousands

Without carrying [O]

1. Sometimes matches come in books of 20. How many matches are there in 3 books? 60

3×20, or 3×2 tens = 6 tens, or 60.

Explain the check in box A. Three 20's = 20+20+20=60

2. On the board find 4×50. 200 Check by adding. 50+50+50+50=200

A	
20	Check
×3	20
60	20
	+20
	60

3. What is the cost of 4 boxes of strawberries at 40¢ a box?

You can multiply this way: $4 \times 40¢ = 160¢$, or $1.60.

Or you can write "$0.40" instead of "40¢" and multiply this way: $4 \times \$0.40 = \$1.60$.

4. Toothpicks come in boxes of 300. How many toothpicks are there in 4 boxes?

For 4×300, *think*, "4×3 hundreds." 4×3 hundreds = 12 hundreds, or 1,200. Explain the check in box B.

B	
300	Check
×4	300
1,200	300
	300
	+300
	1,200

5. On the board find 2×600. 1,200 Check.

6. $3 \times 2{,}000 = ?$ 3×2 thousands = 6 thousands, or 6,000

7. On the board find $3 \times 4{,}000$. 12,000 Check by adding. 4,000+4,000+4,000=12,000

We multiply tens, hundreds, and thousands as we multiply ones.

[W]

Find the products in rows 8 and 9.

	a	b	c	d	e	f
8.	30	50	20¢	40¢	3,000	$6.00
	×2	×7	×4	×5	×3	×3
	60	350	80¢	$2.00	9,000	$18.00
9.	400	700	100	5,000	8,000	$40.00
	×6	×5	×8	×6	×9	×2
	2,400	3,500	800	30,000	72,000	$80.00

When money numbers are involved, there is nothing new to be learned except that the product must show two places for cents and a dollar sign to indicate dollars.

Book Lesson. *Ex. 1–7:* Oral work. *Rows 8 and 9:* Written work. *Ex. 10–12:* Oral work. *Ex. 13–18:* Written work.

1. Emphasize multiplying first by ones, then by tens, etc., beginning with Ex. 10 on page 31.

2. Remind pupils to use a dollar sign and decimal point for money-number products.

3. Ask pupils to check some of the examples in the written work on both pages 30 and 31 by repeated addition.

With carrying [O]

10. Mrs. Long bought 3 cases of canned corn. If a case holds 24 cans, how many cans did she buy? 72

In box C, how is 24 separated into parts? into tens and ones Why can we add 60 and 12 to get our final answer? The product equals the sum of the tens and ones. Explain the short way in box D.

C	D
20 + 4 ×3 60 + 12, or 72	Short way 24 ×3 72

11. $4 \times 879 =$ 3,516

In box E we multiply ones, then tens, then hundreds, and show each product separately. Explain.

The carried figures are added mentally in box F. Explain the multiplying and the carrying.

E	F
879 ×4 36 28 32 3,516	Short way 879 ×4 3,516

12. Mrs. Peck bought 5 chairs at $13.54 each. How much did she pay? $67.70

Box G. Explain the work. We write the dollar sign and the decimal point in the product, allowing 2 places at the right for cents. [W]

G
$13.54 ×5 $67.70

Multiply the short way in Ex. 13–18.

	a	b	c	d	e	f
13.	62	90	18	79	$2.03	2,408
	×4	×6	×6	×5	×7	×3
	248	540	108	395	$14.21	7,224
14.	842	706	420	567	$5.80	3,056
	×5	×8	×9	×7	×5	×8
	4,210	5,648	3,780	3,969	$29.00	24,448
15.	260	508	657	700	$2.89	4,687
	×9	×6	×8	×7	×6	×9
	2,340	3,048	5,256	4,900	$17.34	42,183

16. $5 \times 4{,}006$ 20,030 17. $7 \times \$5.00$ $35.00 18. $8 \times 15{,}730$ 125,840

Extra Practice. Work Sets 16 and 17.

3. **Extra Practice Sets 16 and 17** may be assigned as needed.

Set 16. 1-place multipliers; 2-place multiplicands; without and with carrying

	a	b	c	d	e	f	g	h	i
1.	34	16	35	48	72	81	53	97	72
	×2	×3	×4	×6	×4	×9	×8	×6	×7
	68	48	140	288	288	729	424	582	504
2.	59	68	36	71	65	52	74	89	62
	×3	×2	×6	×5	×7	×2	×9	×4	×9
	177	136	216	355	455	104	666	356	558
3.	31	86	73	35	68	46	71	18	35
	×2	×5	×3	×9	×8	×4	×8	×7	×5
	62	430	219	315	544	184	568	126	175

Set 17. 1-place multipliers; 3- and 4-place multiplicands; carrying; some money numbers

	a	b	c	d	e	f	g
1.	623	$0.75	$1.98	6,314	$4.75	9,425	$0.48
	×4	×6	×3	×2	×3	×5	×6
	2,492	$4.50	$5.94	12,628	$14.25	47,125	$2.88
2.	$0.87	$3.45	8,475	$52.69	2,574	$0.96	$27.89
	×8	×7	×4	×8	×9	×6	×7
	$6.96	$24.15	33,900	$421.52	23,166	$5.76	$195.23
3.	$7.98	6,387	$0.34	$3.98	3,285	$8.16	546
	×2	×5	×8	×9	×6	×7	×9
	$15.96	31,935	$2.72	$35.82	19,710	$57.12	4,914

NOTES

Differentiations and Extensions

1. *Slower learners* may use a multiplication pocket chart to demonstrate the finding of products for simple examples such as Ex. 8**a** and 13**c** in the text.

2. Some pupils have difficulty retaining the number to be carried. Discourage the practice of writing carried figures above the multiplicand, as difficulties arise when there are two or more figures in any one place-value column in the multiplicand. If a "crutch" is necessary, let pupils write the figures to the right of the example, as below.

496 4 3
×5
2,480

Can You Multiply by Larger Numbers?

Inventory Test 6 [W]

Linda works after school at the Argus Candy Company. She helps count boxes and check prices of candy bought and sold by the company. See if you can solve these problems of Linda's:

1. What would 45 pounds of Mint Sticks cost at 83¢ a pound? $37.35

2. How much would 125 pounds of chocolate-coated nuts cost at $1.28 a pound? $160.00

3. How much would the Oceanside Candy Shop be billed for 165 pounds of the De Luxe Assortment at $1.15 a pound? $189.75

4. The Great Plains Candy Company shipped 95 pounds of Foamy Fudge at 85¢ a pound. What should the bill show for total cost? $80.75

5. The Argus Company bought 115 pounds of chocolates at $1.08 a pound. What was the total cost? $124.20

Helps with 2-Place Multipliers

[O]

1. Jean is buying ten 3¢ stamps. How much do they cost? $10 \times 3¢ = 30¢$

$1 \times 3 = 3$ i ten $\times$ 3 = 3 tens, or 30. The stamps cost 30¢.

2. On the board show, by adding ten 3's, that $10 \times 3¢ = 30¢$. $10+10+10=30$

3. How much will twenty 3¢ stamps cost? $20 \times 3¢ = 60¢$

$2 \times 3 = 6$ 2 tens $\times$ 3 = 6 tens, or 60. The stamps will cost 60¢.

Multiply by tens as you multiply by ones. Make the product mean tens by writing 0 in one's place.

4. In the box, think of 26 as 6 ones and 2 tens and multiply by these two parts: $6 \times 14 = 84$; 2 tens $\times$ 14 = 28 tens. Where do we write the 28 to show that it means 28 tens? with the tens column Why are the 84 and the 28 called **partial products**? Explain the rest of the work. Each is a part of the whole product.

14	Check
×26	26
84 } partial products	×14
28 }	104
364	26
	364

[W]

Multiply. Be sure to write the 0 in one's place.

	a	b	c	d	e	f	g	h
5.	3	9	21	62	32	41	86	12
	×40	×30	×20	×10	×50	×30	×20	×30
	120	270	420	620	1,600	1,230	1,720	360

Multiply as in the box and check.

6.	41	84	36	74	28	23	53	65
	×26	×16	×42	×13	×24	×84	×39	×47
	1,066	1,344	1,512	962	672	1,932	2,067	3,055

Extra Practice. Work Set 18.

Teaching Pages 32, 33, and 34

Pupil's Objectives. (*a*) To take Inventory Test 6; (*b*) to receive help in multiplying by 2-place numbers; (*c*) to review checking by reversing the multiplier and multiplicand; (*d*) to overcome difficulties related to the occurrence of zeros in multiplication; (*e*) to obtain practice in multiplication involving 2-place multipliers and money numbers.

Background. You may find, after pupils have taken Inventory Test 6, that help in multiplying by 2-place multipliers is required. This help is given on pages 33 and 34.

Pupils first learn that we can multiply by tens just as we can multiply by ones (2 tens $\times$ 3 = 6 tens, just as $2 \times 3 = 6$). Then they learn that we can multiply first the ones, then the tens, adding together these parts to obtain the complete product. The written form for this is shown in the box on page 33 and the term *partial products* is introduced. Some pupils may need to write a 0 in one's place of the second partial product, but this crutch should be eliminated as quickly as possible.

The development is extended to include larger multiplicands on page 34. Pupils learn in box B to write a zero in one's place in the product when the multiplier is an even ten's number.

Writing Partial Products Correctly

Multiplicands of 2, 3, and 4 places [O]

1. A bushel of corn weighs 56 lb. How much do 24 bu. weigh?

Box A: 4 ones × 56 = 224 ones. 2 tens × 56 = 112 tens. Where do we start to write the second partial product? Explain.

A	B
Thousands Hundreds Tens Ones 5 6 × 2 4 2 2 4 1 1 2 1, 3 4 4	32 ×70 2,240 C 50 ×30 1,500

2. 70 × 32 = 2,240 Study box B. 0 ones × 32 = 0 7 tens × 32 = 224 tens. Why can we write both partial products on the same line? 0 is in ones place of the multiplier.

3. 30 × 50 = 1,500 Explain the multiplying in box C above.

4. A grocer paid $0.67 a dozen for 36 dozen eggs. How much did he pay in all? $24.12

Explain the multiplying in box D.

5. Don reads 213 words a minute. How many will he read in 15 min.?

Multiplying a 3-place number is as easy as multiplying a 2-place number. Explain Ex. E.

D	E
$0.67 ×36 4 02 20 1 $24.12	213 ×15 1 065 2 13 3,195

[W]

In rows 6–8 find the products.

	a	b	c	d	e	f
6.	62	45	58	978	$0.47	2,481
	×34	×26	×30	×42	×38	×23
	2,108	1,170	1,740	41,076	$17.86	57,063
7.	27	70	65	196	$3.16	7,458
	×60	×81	×18	×79	×65	×34
	1,620	5,670	1,170	15,484	$205.40	253,572
8.	39	70	68	432	$6.54	6,512
	×45	×90	×50	×36	×73	×56
	1,755	6,300	3,400	15,552	$447.42	364,672

Extra Practice. Work Set 19.

34

Book Lesson (page 32). *Ex. 1–5:* Written work. Give Inventory Test 6 to determine strengths and weaknesses of pupils in working with 2-place multipliers.

Book Lesson (page 33). *Ex. 1–4:* Oral work. *Rows 5 and 6:* Written work. You may put the following record on the board after the discussion related to the work in the box. Pupils should understand the meaning of the terms used in this record.

42
× 68
336 ← One's partial product
2 520 ← Ten's partial product
2,856 ← Product (sum of the partial products)

Pre-book Lesson (page 34). Give some quick practice to be sure that pupils understand that 0 times a number equals 0. 0 × 2 = 0, 0 × 7 = _?_ 0 × 25 = _?_ 0 × 47 = _?_

Book Lesson (page 34). *Ex. 1–5:* Oral work. *Rows 6–8:* Written work.

1. Notice that 0 in the multiplier is not merely a place-holder, but signifies that there are *no ones*. Therefore, the 0 is not merely brought down as a place-holder, but is used as a multiplier (i.e., in box B we multiply 0 × 32 and write the product, 0, in one's place below the line).

2. Before assigning the written work, label the parts for an example with a 2-place multiplier containing a zero, as below.

32
× 70
2,240

Ten's partial product — One's partial product

3. Observe pupils as they do the written work, so you may group those who need help. Provide small-group guidance as needed, but allow individuals to work independently when their progress warrants it.

Differentiations and Extensions

1. *More capable pupils* may list specific situations in which a newsboy, club treasurer, or baby sitter might need to use 2-place multipliers. Be sure situations that do not involve money numbers are also mentioned. Then ask these pupils to write original problems to present to the rest of the class.

2. Do not allow *slower learners* to adopt wrong habits of procedure. It is far better to have them do fewer examples and to do them correctly than to proceed without help. Let these pupils use crutches and the records written on the board during discussions, but continue to build strength so that the need for these aids is gradually lessened.

3. Some teachers have found that in each step of learning or review concerned with the fundamental operations in arithmetic, children enjoy and learn from a demonstration of the process on a calculating machine. If possible, obtain a desk calculator and demonstrate multiplication with two-place multipliers to your class. Most of the manufacturers of these machines will be happy to send you information which will make your demonstration more interesting.

4. **Extra Practice Sets 18 and 19** may be assigned as needed.

Set 18. 2-place multipliers and multiplicands; with carrying

	a	b	c	d	e	f	g	h	i
1.	45	38	52	79	46	38	93	72	36
	× 27	× 14	× 38	× 59	× 86	× 24	× 87	× 28	× 36
	1,215	532	1,976	4,661	3,956	912	8,091	2,016	1,296
2.	74	95	79	62	54	48	63	75	58
	× 24	× 12	× 36	× 75	× 43	× 37	× 29	× 47	× 95
	1,776	1,140	2,844	4,650	2,322	1,776	1,827	3,525	5,510
3.	29	58	27	76	95	79	37	68	42
	× 24	× 56	× 16	× 79	× 12	× 16	× 35	× 25	× 59
	696	3,248	432	6,004	1,140	1,264	1,295	1,700	2,478

Set 19. 2-place multipliers; 3- and 4-place multiplicands; with carrying

	a	b	c	d	e	f	g
1.	$4.75	368	$12.98	756	2,149	984	638
	× 35	× 24	× 16	× 67	× 34	× 78	× 53
	$166.25	8,832	$207.68	50,652	73,066	76,752	33,814
2.	1,752	$79.25	1,468	$2.19	357	635	4,832
	× 48	× 29	× 69	× 75	× 47	× 98	× 16
	84,096	$2,298.25	101,292	$164.25	16,779	62,230	77,312
3.	468	2,579	$3.75	$46.98	$8.63	1,368	$7.35
	× 75	× 83	× 65	× 24	× 18	× 39	× 27
	35,100	214,057	$243.75	$1,127.52	$155.34	53,352	$198.45

Teaching Pages 35, 36, and 37

Pupil's Objectives. (*a*) To review multiplying by 3-place numbers; (*b*) to learn to reverse factors when the true multiplicand has fewer figures than its multiplier; (*c*) to receive help in multiplication involving zero.

Background. There is nothing new on page 35, for pupils have already learned about partial products and reversing factors as in box B. However, since the procedure with 3-place multipliers involves one more partial product, *slower learners* may need the help and practice offered on this page.

Zeros in multiplication frequently cause great difficulty. This should not happen if pupils understand what they are doing. They should be able to show two partial products together as in the first example in box A on page 36 in the text, and they should be thoroughly familiar with multiplication and addition facts involving zero. For example, in box C on page 37, pupils should realize that starting from the right and reading left, the first two zeros in the product stand for the partial products in one's and ten's places, and that the third zero results from multiplying 5 (hundreds) times 8 and obtaining 40 (hundreds), the 4 being carried to thousand's place ($5 \times 0 + 4 = 4$).

Helps with Three-Place Multipliers

Multiplying by smaller number [O]

1. A farmer sold 327 hogs whose average weight was 468 lb. Find the total weight.

Box A. We multiply the average, 468 lb., as if it were the real weight of each hog.

How do we get the one's partial product? the ten's and the hundred's partial products? 7×468=3,276 (ones); 2(tens)×468=9,36(tens); 3(hundreds)×468=1,404(hundreds)

The partial product, 936, means 936 tens. What does the partial product, 1404, mean? 1404 hundreds

A
468
×327
3 276
9 36
140 4
153,036

2. How many partial products will there be in the example 39×451? 2 in $176 \times 4{,}338$? 3 How can you tell? One for each figure in the multiplier.

3. How many pencils are in 235 boxes with 12 in each box? 2,820

Box B. Because 12 is the size of each group, we write, "235 × 12." Can we get the same product by finding 12×235? Yes Why is this better? Only 2 partial products

B	
Not this	But this
12	235
×235	×12

4. Tell how to change the following before multiplying:

a. 462×8 8 × 462 b. 246×23 23 × 246 c. $1{,}954 \times 75$ 75 × 1,954

[W]

Copy, and find the products. You may reverse factors.

	a	b	c	d	e	f
5.	237	569	27	$4.38	3,129	7,615
	×128	×256	×419	×127	×645	×462
	30,336	145,664	11,313	$556.26	2,018,205	3,518,130
6.	857	236	41	$6.24	7,632	8,546
	×794	×587	×392	×361	×958	×398
	680,458	138,532	16,072	$2,252.64	7,311,456	3,401,308
7.	764	581	64	$5.89	351	9,837
	×218	×736	×319	×215	×5,872	×264
	116,552	427,616	20,416	$1,266.35	2,061,072	2,596,968

Extra Practice. Work Set 20.

Book Lesson (page 35). *Ex. 1–4:* Oral work. *Rows 5–7:* Written work. Record the following chart on the board following the discussion of Ex. 1.

```
  468
× 327
 3 276 ←— One's partial product
 9 36  ←— Ten's partial product
140 4  ←— Hundred's partial product
153,036 ←— Product (sum of the partial products)
```

Book Lesson (pages 36 and 37). *Ex. 1–12:* Oral work. *Rows 13–15:* Written work.

Helps in Multiplying with 0

In multiplier [O]

A

One zero or more in the multiplier

46	427	843	3,572
×80	×340	×206	×400
3,680	17 080	5 058	1,428,800
	128 1	168 60	
	145,180	173,658	

Box A. When there is a 0 in any place in the multiplier, we multiply the whole multiplicand by the 0 (as $0 \times 46 = 0$ or $0 \times 843 = 0$). We then write the product 0 in its correct place.

1. Explain how the red product 0's in box A are placed.

Copy on the board and finish these examples:

2. $20 \times 7{,}134$ *142,680*
3. $360 \times 2{,}781$ *1,001,160*
4. $602 \times 4{,}826$ *2,905,252*

In multiplicand

B

One zero or more in the multiplicand

280	3,070	5,009	4,800
×7	×6	×15	×428
1,960	18,420	25 045	38 400
		50 09	96 00
		75,135	1 920 0
			2,054,400

Box B. We multiply the zeros just as we multiply the other figures in the multiplicand. If there is a carried figure in any place, we add it to the product 0.

5. Explain each red product figure in box B.

Copy on the board and finish these examples:

6. $15 \times 6{,}047$ *90,705*
7. $62 \times 6{,}050$ *375,100*
8. $293 \times 7{,}000$ *2,051,000*

0 in both multiplier and multiplicand

C	D	E
308	4,600	6,003
×500	×340	×702
154,000	184 000	12 006
	1 380 0	4 202 10
	1,564,000	4,214,106

9. Study box C above. Why are there three 0's in the product? Explain how we get each product 0. How do we get the 4 in thousand's place? *2 zeros for the 2 in the multiplier; 1 zero from 5 (hundreds) × 8 = 40 (hundreds)*

10. Copy these examples on the board and work them:

a. 40×208 *8,320*
b. $60 \times 5{,}007$ *300,420*
c. 30×406 *12,180*
d. 300×702 *210,600*
e. 600×405 *243,000*
f. 400×308 *123,200*

11. Box D. What is the first partial product? *0* Where is it written? *ones column* What is the second partial product? *13400 (tens)* the third? *13800 (hundreds)* Why do we start to write the third partial product in hundred's place? *To show that 13800 means 13800 hundreds*

12. Box E. Why do there seem to be only two partial products? Explain all the work in box E. *0 is the second partial product; 42021 the third*

[W]

Copy rows 13–15 and write your work.

	a	b	c	d	e	f
13.	406	600	250	631	4,700	603
	×27	×29	×16	×500	×29	×381
	10,962	*17,400*	*4,000*	*315,500*	*136,300*	*229,743*
14.	6,800	7,006	4,800	6,003	300	2,806
	×560	×904	×509	×470	×200	×405
	3,808,000	*6,333,424*	*2,443,200*	*2,821,410*	*60,000*	*1,136,430*
15.	2,806	1,945	1,080	3,007	8,920	9,040
	×659	×704	×906	×600	×300	×25
	1,849,154	*1,369,280*	*978,480*	*1,804,200*	*2,676,000*	*226,000*

Extra Practice. Work Sets 21 and 22.

1. Discuss Ex. 1, asking pupils to explain why the partial products for tens and hundreds are so placed.

2. While some children write the examples on the board, as directed for Ex. 2–4, 6–8, and 10, allow others to do them at their desks.

Differentiations and Extensions

1. *Slower learners* may need to use crutches, such as those mentioned below, in order to place partial products correctly.

a. Suggest that pupils cover with a finger the figures to the left of the one they are working with in the multiplier in order to avoid being confused by too many figures. The movement of the finger to the left for the next figure in the multiplier also helps pupils remember to start a new partial product.

b. Let pupils who have difficulty placing partial products correctly use guide lines, as shown below, particularly when 0's occur in the multiplier.

	5	2	3
×	2	4	0
2 0	9	2	0
1 0 4	6		
1 2 5 ,	5	2	0

2. *More capable pupils*, with a little instruction, may apply the procedure they learned in casting

out 9's for checking addition (page 30 in the Manual) and subtraction (page 44) to the method for checking multiplication.

Illustrations

a. 237 × 128 = 1 896, 4 74, 23 7, 30,336

$2 + 3 + 7 = 12$ $1 + 2 + 8 = 11$ $12 \div 9 = 1$ and 3 R
$11 \div 9 = 1$ and 2 R
$6(2 \times 3)$
$6 \div 9 = 0$ and 6 R
$3 + 0 + 3 + 3 + 6 = 15$
$15 \div 9 = 1$ and 6 R
6 is the remainder for both processes

b. 425 × 180 = 34 000, 42 5, 76,500

$4 + 2 + 5 = 11$ $1 + 8 + 0 = 9$ $9 \div 9 = 1$ and 0 R
$0(0 \times 2)$
0 R
$7 + 6 + 5 + 0 + 0 = 18$
$18 \div 9 = 2$ and 0 R
0 is the remainder for both processes.

Stress the fact that with this type of check the only parts used are the multiplicand, multiplier, and product (*not* the partial products).

3. **Extra Practice Sets 20, 21, and 22** are reproduced below with answers.

Set 20. 3-place multipliers; with carrying

	a	b	c	d	e	f
1.	475 × 123 = 58,425	64 × 527 = 33,728	42 × 876 = 36,792	386 × 648 = 250,128	3,589 × 957 = 3,434,673	64 × 918 = 58,752
2.	18 × 491 = 8,838	438 × 365 = 159,870	64 × 914 = 58,496	45 × 934 = 42,030	97 × 849 = 82,353	647 × 579 = 374,613
3.	6,389 × 238 = 1,520,582	325 × 327 = 106,275	2,167 × 579 = 1,254,693	2,196 × 236 = 518,256	752 × 386 = 290,272	92 × 523 = 48,116

Set 21. 3-place multipliers; with carrying; some money numbers

1.	36 × 468 = 16,848	579 × 312 = 180,648	$32.50 × 645 = $20,962.50	1,023 × 273 = 279,279	$0.59 × 527 = $310.93	$35.97 × 170 = $6,114.90
2.	8,641 × 185 = 1,598,585	127 × 608 = 77,216	$8.64 × 397 = $3,430.08	$0.94 × 179 = $168.26	468 × 204 = 95,472	3,690 × 698 = 2,575,620
3.	5,280 × 124 = 654,720	1,728 × 605 = 1,045,440	90 × 345 = 31,050	369 × 285 = 105,165	1,056 × 637 = 672,672	2,479 × 837 = 2,074,923

Set 22. Zeros in multiplication; 3-place multipliers; some money numbers

1.	375 × 142 = 53,250	8,064 × 253 = 2,040,192	$0.68 × 957 = $650.76	6,802 × 407 = 2,768,414	914 × 174 = 159,036	3,824 × 806 = 3,082,144
2.	$75.39 × 305 = $22,993.95	$0.97 × 793 = $769.21	8,542 × 139 = 1,187,338	106 × 316 = 33,496	$73.90 × 812 = $60,006.80	$0.52 × 921 = $478.92
3.	1,695 × 490 = 830,550	4,071 × 507 = 2,063,997	$0.79 × 289 = $228.31	356 × 257 = 91,492	$0.89 × 601 = $534.89	3,407 × 649 = 2,211,143

NOTES

Teaching Pages 38, 39, and 40

Pupil's Objectives. (*a*) To review multiplication involving 4-place multipliers and 4-place multiplicands; (*b*) to use a set of examples as the basis for writing multiplication problems; (*c*) to learn a quick way to multiply when both factors end in zero; (*d*) to estimate products for multiplication examples by rounding numbers; (*e*) to solve problems involving addition, subtraction, and multiplication.

Background. No new difficulties occur on page 38 except greater length (multipliers and multiplicands are 4-place numbers). Notice that more help is given when zeros occur and that the check for multiplication may now be used freely, since, with the reintroduction of 4-place multipliers, all children may reverse factors and multiply again.

Pupils should be led to see why it is that there is usually a much greater discrepancy between estimated answers and exact answers in multiplication than would be the case in addition and subtraction. When the multiplicand 144 is rounded to 100, the amount neglected (44) will accumulate according to the size of the multiplier. The same reasoning is applied in rounding the multiplier.

Pupils have already differentiated addition and subtraction (page 24, in the text) and now, on page 40 in the text, they will differentiate addition, subtraction, and multiplication. This helps them to see the many different ways in which these processes may be called for in problems and helps clarify the uses of these processes.

Pre-book Lesson (page 38). Pupils should realize that ability to work with the large numbers presented on page 38 is dependent upon previous understandings and upon accuracy.

Book Lesson (page 38). *Ex. 1–5:* Oral work. *Ex. 6–16:* Written work. As box A is being discussed, show on the board the example and its check with the partial products labeled as below.

		Check	
3,604		1,297	
1,297		3,604	
25 228	(ones)	5 188	(ones)
324 36	(tens)	778 20	(tens and hundreds)
720 8	(hundreds)	3 891	(thousands)
3 604	(thousands)	4,674,388	
4,674,388			

(*Continued on page 60*)

Four-Place Multipliers and Multiplicands

[O]

In box A, a 4-place number is multiplied by a 4-place number.

A	
3,604	Check
×1,297	1,297
25 228	×3,604
324 36	5 188
720 8	778 20
3 604	3 891
4,674,388	4,674,388

1. How many partial products are there in the solution in box A? 4 Why? 4 figures in multiplier

2. Why is the 4 of the fourth partial product written under the 1 of the multiplier? To show that 3604 means thousands

3. Study the check in box A. Explain why there seem to be only three partial products. There is a zero in the multiplier.

B	
	Check
	3,125
	×6,030
	93 750
	18 750 0
	18,843,750

4. For the example ×3,125, when you start to multiply by 3 (thousands) where will you write the 0 of the partial product? Explain. in thousands place

5. The check for Ex. 4 is shown in box B. Where are the partial products written when you multiply by the two 0's in 6,030? one's place and hundred's place

[W]

Multiply each number in Ex. 6 to 10 by 3,125. Check by reversing the factors.

6. 6,715 (20,984,375) 7. 9,006 (28,143,750) 8. $81.56 ($254,875.00) 9. 7,080 (22,125,000) 10. 4,670 (14,593,750)

(51,658,495) (69,283,158) ($627,441.08) (54,466,440) (35,926,310)

11–15. Multiply each number in Ex. 6 to 10 by 7,693 and check.

16. In a recent year there were 4,693 pupils enrolled in the schools of Callwood. The cost per pupil was $154.60. What was the total cost for schools in Callwood that year? $725,537.80

Turn back to page 33 and write multiplication problems for the examples in row 6. ★

Find the answer for each of your problems.

★Answers Not on Reproduced Page 38

Bottom of page. Illustrative problems for the first three examples in row 6 on page 33 in the text are given below.

a. To save the tomatoes from frost, Bill picked them and laid them in neat rows to ripen. He put 41 tomatoes in a row and made 26 rows. How many tomatoes were there? (1,066)

b. Mary's mother makes 7 dozen cupcakes each day for the Food Shop. How many cupcakes does she make in 16 days? (1,344)

c. There are 36 inches in a yard. How many inches are there in 42 yards? (1,512)

Estimating Products

[O]

1. In Ex. a, compare the number of 0's in the product with the number in both multiplier and multiplicand. *the same*

a. $40 \times 700 = 28{,}000$
b. $200 \times 600 = 120{,}000$
c. $20 \times 8{,}000 = 160{,}000$

2. Make the same comparison in Ex. b and Ex. c.
3. Make a rule about the number of 0's in these products. *Number of 0's in product = sum of 0's in factors*
4. How do we get 28 in Ex. a? *4 × 7* 12 in Ex. b? *2 × 6* 16 in Ex. c? *2 × 8*

Without paper and pencil, tell the products in Ex. 5–8.

5. 60×300 *18,000* 6. 400×200 *80,000* 7. 80×50 *4,000* 8. $30 \times 5{,}000$ *150,000*

If both factors of a multiplication example end in 0, you can quickly get the product by multiplying the other figures and writing beside that product as many 0's as there are at the ends of both factors.

To estimate any product, you can round each factor and multiply as above. Your answer will be a round number.

[W]

For each example three estimated answers are given. Copy the one that is nearest right. Check by working the example.

9. $4 \times 789 =$ *3,156* 2,800 32,000 (3,200)
10. $63 \times 38 =$ *2,394* (2,400) 1,800 2,800
11. $72 \times 58 =$ *4,176* 5,600 (4,200) 3,500
12. $89 \times 196 =$ *17,444* 1,800 (18,000) 16,000

Write an estimated answer for each example. Then work the example and compare your estimate with the product.

	a *(490)*	b *(2,000)*	c *(5,600)*	d *(2,700)*
13.	7×69 *483* (*$58*)	38×51 *1,938* (*24,000*)	82×68 *5,576* (*$36*)	94×27 *2,538* (*$4*)
14.	$93 \times \$0.58$ *$53.94* (*280,000*)	62×389 *24,118* (*320,000*)	$6 \times \$5.97$ *$35.82* (*35,000*)	$8 \times \$0.52$ *$4.16* (*$280*)
15.	742×365 *270,830*	821×394 *323,474*	52×684 *35,568*	$42 \times \$6.95$ *$291.90*

The Lunchroom

Differentiating A., S., M.; estimating answer [O]

For each problem tell whether you should add, subtract, or multiply. Then give an (estimate) of the answer.

1. Mary paid 23¢ for lunch at school, and her father paid 99¢ at a restaurant. How much less did Mary's lunch cost than her father's? *S.; (75¢); 76¢*

2. One day at a large table there were seated 16 children, then 14 children, then 11, then 15. How many children in all used the large table? *A.; (55); 56*

3. The lunchroom floor is 74 feet long and 42 feet wide. How many feet greater is the length than the width? *S.; (35); 32*

4. The lunchroom manager has ordered 18 dozen glasses at $0.64 a dozen. How much will the glasses cost in all? *M.; ($12); $11.52*

5. Lunch tickets cost 23¢. How much will 15 tickets cost? *M.; ($3); $3.45*

6. About 145 glasses of milk are sold a day. At that rate, how many glasses of milk will be sold in 22 days? *M.; (3,000); 3,190*

[W]

Now go back and write your work for problems 1–6.

Pre-book Lesson (page 39)

1. Present a problem related to the experiences of your class which can be answered approximately.

Illustration. A church has 81 pews, each of which seats about 10 people. About how many people could be seated in the church?

81 rounds to 80
× 10 × 10
about 800 people

2. Let pupils try to tell how they would round the numbers in the examples at the right. Point out that it is not necessary to round both factors to the same place-value column. Contrast this situation to that in addition in which all numbers in a given example must be rounded to one place-value column only. At this time do not work out the examples or estimates, but leave them on the board for use during the book lesson.

34 is close to 30	832 (800)	739 (700)
× 3	× 3	× 25 (30)	× 450 (500)
2,146 (2,000)	6,200 (6,000)	4,712 (5,000)
× 70 (70)	× 368 (400)	× 7,435 (7,000)

Book Lesson (page 39). *Ex. 1–8:* Oral work. *Ex. 9–15:* Written work. The examples and rounded numbers left on the board after the pre-book lesson may be completed after the generalization in the middle of the page has been discussed.

Pre-book Lesson (page 40). You may want to summarize with your class the principal uses of addition, subtraction, and multiplication, as below.

Addition: putting together unequal groups
Subtraction: separating parts, or finding the difference
Multiplication: putting together equal groups

Book Lesson (page 40). *Ex. 1–6:* Oral work. *Bottom of the page:* Written work.

1. Study with your pupils each problem on the page with respect to the use of each process.

2. After the written assignment has been completed and corrected, go over the problems again in order to discuss the reasons for selecting each process and any other questions that arise.

Differentiations and Extensions

1. *More capable pupils* may check some of the products by casting out 9's.

2. *Slower learners* may temporarily need to write the rounded numbers as side work, whereas the *more capable pupils* may be able to compute with them mentally.

Teaching Pages 41, 42, and 43

Pupil's Objectives. (*a*) To evaluate understanding of certain facts, meanings, and relationships; (*b*) to analyze computation in order to identify and remove sources of difficulty; (*c*) to evaluate problem-solving ability; (*d*) to test computational ability in the skills learned or relearned.

Background. These three pages constitute a simple but well-rounded testing program. A rereading of the learning outcomes given at the beginning of the teaching lessons for this chapter would be helpful at this time. It will be seen that most of the goals listed there can be evaluated through use of the tests for meaning, computation, and problem-solving. The proper appraisal of the child's total arithmetic ability must be based on the results of all three of these tests. In addition, observe what desirable arithmetic attitudes, appreciations, and values the child possesses.

Do You Understand?

Test of Information and Meaning 1

Write your answers.

1. In two different ways, write the meaning of 742. *e.g., 7 hundreds + 4 tens + 2 ones; 74 tens + 2 ones; etc.*

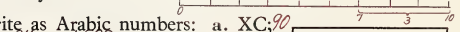
2. $(5 \times 1{,}000) + (6 \times 100) + (2 \times 10) + (7 \times 1) =$ *5,627*

3. Round to the nearest thousand: a. 7,829; *8,000* b. 2,824,726. *2,825,000*

4. Write in figures: a. three tens; *30* b. seven hundreds; *700* c. forty-five tens; *450* d. two hundred seven hundreds. *20,700*

5. Round to the nearest million: a. 8,194,608; *8,000,000* b. 3,408,852,016; *3,409,000,000* c. 19,822,694. *20,000,000*

6. Show by a number line that $10 - 3 = 7$.



7. Write as Arabic numbers: a. XC; *90* b. XXXIV; *34* c. MD; *1,500* d. LXXXV. *85*

5	9	4
+2	−3	×8
7	6	32

8. From the box copy a. an addend; *2 or 5* b. a minuend; *9* c. a sum; *7* d. a product; *32* e. a remainder; *6* f. a multiplier. *8*

9. Write two examples in the $3 + 8$ addition family. *13+8=21; 3+18=21; 23+8=31; etc.*

10. To find the difference between two numbers, do you add, subtract, or multiply?

11. For $28 + n = 46$, what would you do to find n? *subtract*

12. To check subtraction, what should you do? *add remainder and subtrahend*

13. Write in Roman numerals: a. 75; *LXXV* b. 107; *CVII* c. 1,050. *ML*

14. Write two examples in the $14 - 8$ subtraction family. *24-8=16; 34-8=26; etc.*

15. Does the multiplier or the multiplicand tell the size of the equal groups?

16. Do you put $ and decimal point in partial products? *No*

17. In working the example $4{,}285 \times 86$, by which number will you multiply? *86*

18. Estimate the product for 9×418. *10×400=4,000*

41

Pre-book Lesson

1. Let pupils quickly scan the four pages of tests, reading directions and noticing content; then have all books closed.

2. Set the stage for the evaluation program by discussing with the children what they learned and what they think they should retain from Chapter 1. Make a note of what they emphasize and omit in order to evaluate your teaching program. Promote a healthy attitude toward tests, indicating the value to the individual of the diagnostic test at the top of page 42.

Book Lesson (page 41). *Ex. 1–18:* Written work. Test of Information and Meaning 1.

Do You Make Mistakes?

Diagnostic Test 1

Copy and work the examples in rows 1–5. If you make mistakes in any row, review the Study Pages. For more practice with examples of the same kind, use the Practice Sets.

	a	b	c	d	Study Pages	Practice Sets
1.	24 +32 56	472 +15 487	215 +385 600	6,494 +8,363 14,857	8, 11	1-8
2.	38 −25 13	795 −84 711	491 −79 412	9,423 −895 8,528	14, 17, 18	9-11
3.	405 −28 377	700 −275 425	3,002 −589 2,413	$50.00 −25.98 $24.02	19	12-13
4.	43 ×3 129	97 ×8 776	$4.05 ×7 $28.35	4,852 ×6 29,112	30, 31	16-17
5.	54 ×23 1,242	$0.68 ×46 $31.28	$6.75 ×108 $729.00	2,050 ×3,704 7,593,000	33-38	18-22

Can You Work Problems?

Problem Test 1

1. Jack is saving money for a bicycle. He has $18 and needs $14 more. How much does the bicycle cost? $32

2. The sea route between New York and Vancouver by way of the Panama Canal is 6,992 miles. The route around South America is 16,333 miles. How much shorter is the route by way of the Panama Canal than around South America? 9,341 miles

3. Mr. Ambrose has an orchard with 12 rows of apple trees set 14 trees to a row. How many apple trees does Mr. Ambrose have in all? 168

4. For her television set, Mrs. Parks paid $42 down and $12 a month for 12 months. Find the total cost of the set. $258

5. Mr. Hill receives $75 a week. Mr. Fox receives $3,500 a year (52 weeks). Which man receives more per year? How much more? Mr. Hill, $400 more

6. A farm of 121 acres has 35 acres in corn, 28 acres in alfalfa, 16 acres in tobacco, and 3 acres in yards and buildings. The rest is in pasture. How many acres are there in pasture? 39

7. A tank holds 175 gallons of fuel oil. When there were still 50 gallons left in it, the tank was filled again. How much did the filling cost at 13¢ a gallon? $16.25

8. Mr. Jackson washes windows in an office building at an average rate of 65 windows a day. At 15¢ a window, how much will Mr. Jackson earn in a week of 5 days? $48.75

9. Don spent $9.95 of the $14 that he earned selling magazines. He put the rest in the bank with the $65 that he already had there. How much did he then have in the bank? $69.05

How Well Can You Figure?

Computation Test 1

	a	b	c	d	e		
1.	47 −34 13	32 +26 58	$5.78 ×4 $23.12	82 −23 59	756 −214 542	5.	8 7 4 6 3 +5 33
2.	175 +286 461	604 −128 476	520 ×7 3640	46 ×57 2,622	4,257 −1,874 2,383		
3.	35 −29 6	628 −165 463	472 −48 424	843 +179 1,022	537 −248 289	6.	$41.11 63.02 50.34 +21.41 $175.88
4.	700 −285 415	654 ×37 24,198	$5.05 ×46 $232.30	3,470 ×609 2,113,230	$53.83 +26.65 $80.48		

Book Lesson (top of page 42). *Rows 1–5:* Written work. Diagnostic Test 1. Explain the function of a diagnostic test and the use of study pages and practice sets. Children should be advised that they can do much to remedy their errors at this time by working practice sets whenever they can.

Book Lesson (bottom of page 42 and top of page 43). *Ex. 1–9:* Written work. Problem Test 1.

Book Lesson (bottom of page 43). *Ex. 1–6:* Written work. Computation Test 1. This test may be used for practice as there is an alternate test on page 302 in this Manual.

Differentiations and Extensions

1. All four tests may be used for teaching as well as evaluation purposes. Tabulate errors item by item and discuss with pupils their errors and reasons for them. Some areas may need reteaching.

2. Allow time for Extra Practice Sets for those who need them. (See under *Practice* in the Manual Index for the location of the sets, with answers, referred to in the diagnostic test.) *More capable* children may help *slower learners* with these and the study pages listed in the diagnostic test at the top of page 42 in the text.

3. Make a check list of desirable arithmetic attitudes, appreciations, and values. Discover how

many of your pupils seem to possess them in their daily contact with the use of numbers both in and out of school.

4. Discuss your arithmetic program with the class. They may have valuable suggestions for changes or improvements which will be helpful to you as you adapt the program to their needs.

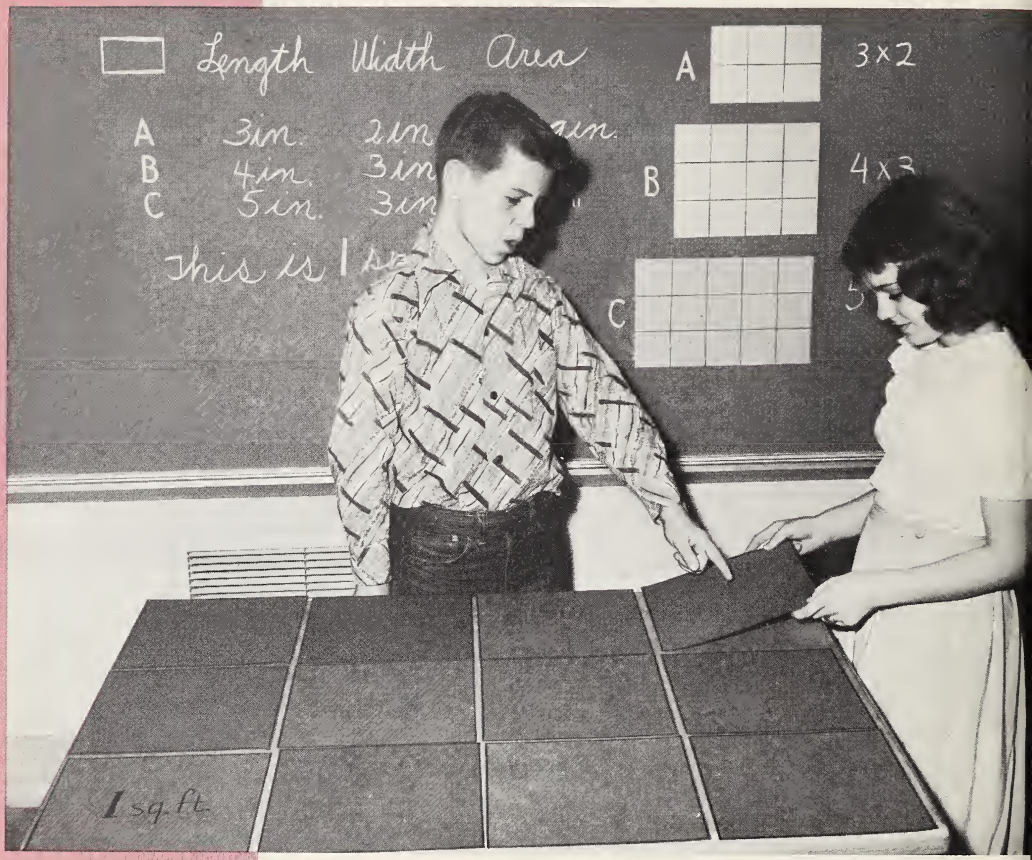
5. Keep problem-solving and computation test scores or marks for each child on a $3'' \times 5''$ card similar to the one shown below. Per cents to be entered for this chapter may be obtained from the table of per cents at the foot of this column. For example, a score of four correct on Problem Test 1 would give, in per cent, a score of 44 to be entered on the individual test record card. If you use the alternate computation test for this chapter, the same table may be used for obtaining per-cent scores. This holds true also for the per-cent tables and alternate computation tests in the remaining chapters.

Name		Grade	Date
Chapter	Per Cent Right Problem-Solving Test		Per Cent Right Computation Test
1			
2			
3			
4			
5			
6			
7			

TABLE OF PER CENTS FOR CHAPTER 1 SCORES

Score	Per Cents for Problem Test 1
1	11
2	22
3	33
4	44
5	56
6	67
7	78
8	89
9	100

Score	Per Cents for Computation Test 1	Score	Per Cents for Computation Test 1
1	5	12	55
2	9	13	59
3	14	14	64
4	18	15	68
5	23	16	73
6	27	17	77
7	32	18	82
8	36	19	86
9	41	20	91
10	45	21	95
11	50	22	100



Detroit Public Schools

Using foot-squares to find the number of square units that will cover a surface helps pupils discover the meaning of area measurement.

Teaching Chapter 2 of Grade 6

Introduction

The work of building readiness for new learnings in Grade 6 is continued in Chapter 2. The principal purpose is to present briefly a review of division and denominate numbers.

I. Learning Outcomes in Chapter 2

1. Understanding of division and knowledge of the even and uneven division facts
2. Ability to work harder division examples with up to three places in the divisor and five places in the quotient
3. Understanding of measures and ability to use equivalents in these tables of measures: linear, liquid, dry, time, weight
4. Ability to add, subtract, multiply, and divide denominate numbers
5. Understanding and skill in finding perimeters and areas
6. Ability to solve problems
7. Understanding of various specific concepts, generalizations, relationships, and skills
8. Desirable emotionalized responses (attitudes, appreciations, values)

II. The Nature of the Major Outcomes

Understanding of Division and Knowledge of the Even and Uneven Division Facts (pages 45–50)

Division, the last of the four basic processes, is presented in Chapter 2 for reteaching. It is a separating process, like subtraction, but division involves separating into several equal groups, rather than into two unequal groups. For this reason, division provides a short way of finding how many times a smaller group can be repeatedly subtracted from a larger group. Once this idea is made clear, pupils must learn the difference between the two kinds of division. In measurement division we find the number of equal groups in a larger group, and in fractional-part division we find how many are in each equal part or share of a group (the quotient is a fractional part of the dividend). This idea pupils review on pages 46 and 47, and on page 48 they learn to distinguish the two kinds of division in problems.

When pupils have shown that they are proficient in using the division facts, with the aid of helpers (page 49) if necessary, the uneven division facts are presented. Success with the uneven division facts is dependent upon a thorough mastery of the even division facts and upon an understanding of the meaning and use of table numbers. Each dividend for an even division fact is a table number for use in making a group of related uneven division facts. Since $35 \div 5 = 7$ and $35 \div 7 = 5$, 35 is a table number for dividing 36, 37, 38, and 39 by 5, and for dividing 36, 37, 38, 39, 40, and 41 by 7. As soon as uneven division has been introduced, pupils are shown how to write remainders with R, and how to add the remainder in checking division computation.

Ability to Work Harder Division Examples (pages 51–53, 57–62, 64–69)

After a review of the meaning and use of division, and after work to gain mastery of the basic even and uneven division facts, pupils are given a systematic redevelopment of the following kinds of examples:

a. 1-place divisors with
- 2-place dividends and quotients
- 3-, 4-, and 5-place dividends

b. 2-place divisors with
- 1-place quotients when
  - the trial quotient is the true quotient
  - the first trial quotient is not the true quotient
  - several trial quotients are required (teen-number divisors)
- 2-place quotients, apparent and non-apparent quotient figures
- 3-place quotients, apparent and non-apparent quotient figures

c. 3-place divisors with 1-, 2-, and 3-place quotients

Some of the more important generalizations that pupils are helped to discover are given below.

a. We divide tens, hundreds, and thousands just as we divide ones.

b. We divide the dividend by parts. In a 4-place dividend we first try to divide the thousands. If there aren't enough thousands to be divided, we change thousands to hundreds and divide first hundreds, then tens, and then ones, and so on.

c. Each time a part of the dividend is divided we must write a figure in the quotient. Sometimes this figure is zero.

d. The product of the quotient and the divisor must not be greater than the part of the dividend used.

e. Try no quotient figure larger than 9. If the quotient figure is too large, try the next smaller number until the product is no longer greater than the part of the dividend used.

f. The quotient is correct if the divisor times the quotient plus the remainder equals the dividend.

Understanding and Ability in Working with Measures (pages 71–77, 88–91)

In Inventory Test 9, pupils review some of the equivalents for linear measures as well as for measures of capacity (liquid and dry), weight, and time. Pupils then learn the difference between counting and measuring numbers and are led to the important understanding that measuring numbers are never entirely accurate. As part of the presentation, they learn that units of measure have not always been standardized. Then, adopting a procedure similar to the historical development, pupils are taught certain reference measures that can be used in estimating length, weight, time, and the like. For example, a part of the pupil's finger may be used as a reference measure in estimating things measured in inches; a man's shoe may be thought of as about a foot long; a door as about seven feet high; a bag of sugar weighing five pounds or ten pounds may be used as a reference measure for weight; etc.

Equivalents in the tables of measures have developed through the ages on the basis of use and are not, therefore, part of a unified system with a common base, as is the metric system. This is the reason pupils must learn the equivalents, or must learn to refer to tables of measures for the equivalents, if they are to develop skill in changing from one unit to another. Once information concerning the relationship of one unit to another has been found, the ratio may be used in changing from larger to smaller or from smaller to larger units, multiplication being used for the first kind of change and division for the second.

Ability to Add, Subtract, Multiply, and Divide Denominate Numbers (pages 78–82, 84)

The four basic processes with denominate numbers are used very much as they are used with whole numbers, except that we do not always group by tens or by powers of ten, since the base for grouping depends on the measures being used. If pounds and ounces are being added, 16 ounces must be gathered and grouped in ounce's column before carrying to the pound's column. In this case, we group by 16 rather than 10. If 4 yd. 2 ft. is to be divided by 7, the 4 yards must be thought of as 12 ft., which, with the 2 feet, makes 14 feet. In this case, neither the base 10 nor the base 16 is used in changing the dividend, but a base of 3, since 3 ft. = 1 yd. If pupils really understand the nature and use of the decimal system of notation, they can carry over some of the processes used with whole numbers to work with measures, providing that they realize the base for changing or grouping depends upon the measures being used.

Understanding and Skill in Finding Perimeters and Areas (pages 84–87)

Finding a perimeter is simply a matter of finding the total distance around a figure or object. In finding areas, one usually works with linear measures, but a new kind of measurement unit, the square inch, square yard, etc., is introduced for use in finding surface area. While the material under this heading might have been included under the heading that discusses denominate numbers, some of the concepts behind measuring surface area involve such an extension of previously developed ideas that special attention seems desirable.

Ability to Solve Problems (pages 48, 54–56, 63, 69, 70, 92)

It should be stressed again that the problem-solving program cannot be confined to a few specific pages. One of the most important developments in Chapter 2, as far as the problem-solving program is concerned, relates to the meaning of the two kinds of division. Just as soon as these have been introduced, pupils receive practice in telling the kind of division that is involved in several problems.

Once pupils have reviewed the meanings of division, average problems are introduced. These are usually two-step problems involving first addition of several groups, then division to find how large the groups would be if all were equal. This quotient is called an average and is very useful in describing situations that involve a sequence of data. In some average problems the total is given, and it is only necessary to divide this number into

several equal groups. In other problems, the average is given to be used in finding the total or in finding the number of groups to which the average applies. Of course, these are not two-step problems.

More practice is given in Chapter 2 in finding hidden questions in two-step problems, but now division is added to the other three processes. Pupils also obtain practice in distinguishing the process to be used, and in making up division problems using examples given to them. In Chapter 1 pupils differentiated the process to use in addition, subtraction, and multiplication problems. Now that division has been reintroduced, they receive practice in differentiating the process to use in sets of problems involving all four basic processes. Finally, on page 92, pupils are given a set of problems involving measurement and some of the concepts that they have learned in Chapter 2.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

acre (A.)
A.D. (anno Domini)
area
average
B.C. (before Christ)
bushel (bu.)
calendar
century
common year
counting number
day (da.)
decade
dimension
divide, division (D.)
dividend
division tables
divisor
foot, feet (ft.)
fractional-part division
gallon (gal.)
hour (hr.)
inch (in.)
length
linear
measurement division
measuring numbers
mile (mi.)
minute (min.)
ounce (oz.)
partial dividend
peck (pk.)
perimeter
pint (pt.)
pound (lb.)
quart (qt.)
quotient
rectangle
reference measure
rod (rd.)
second (sec.)
side work
square foot (sq. ft.)
square inch (sq. in.)
square mile (sq. mi.)
square yard (sq. yd.)
standard unit (of measure)
table numbers
time line
ton (T.)
trial quotient figure
true quotient figure
width
yard (yd.)
year (yr.)

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 2 will be found on the following pages in the text, and on the corresponding reproduced pages in this Manual; 46, 47, 51, 55, 60, 61, 70, 73, 76, 77, 83, 85, 91.

3. *Skills beyond those already listed:* Ability to find *n* when it represents a missing factor (page 83)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

In Chapter 2 it is hoped that pupils will:

a. Gain increased appreciation of the logic of the number system and of the arithmetical processes as they increase their ability in division computation and in work with measurement numbers.

b. Realize that individuals within a class may differ widely in achievement, but that steady growth in understandings and skills is possible for all.

c. Recognize the importance of analyzing their own strengths and weaknesses as well as taking the initiative in seeking the kind of assistance needed.

d. Gain increased appreciation of the value of arithmetic as part of their daily living.

When We Divide

Inventory Test 7 [W]

Bob, Jack, and Tom are working on their stamp collections to get them ready for the Hobby Show.

1. Bob stopped at the post office and bought some new commemorative stamps. He got 32 stamps in blocks of 4. How many blocks did he get? *8*

2. Tom bought a packet of 25 mixed stamps. He offered to trade $\frac{1}{3}$ of them for some of Bob's extra commemorative stamps. How many stamps would Tom give Bob? *8* Is this exactly $\frac{1}{3}$ of the 25 stamps? *No; 25 ÷ 3 = 8, R1*

3. Jack's stamp album has 250 pages. He has divided it into 5 equal sections. How many pages are there in each of the sections? *50*

4. Bob's album once belonged to his father, who had begun 6 different kinds of collections in it. There are now 1,824 stamps in the album. Divided equally among the collections, how many stamps would there be in each collection? *304*

5. Tom ordered some approval sheets from which he kept 139 stamps. He put them on the table in rows of 8. How many complete rows did he have? *17* How many other stamps? *3*

6. The 3 boys together bought 500 transparent hinges to use in fixing stamps on the pages. What was each boy's share of the hinges? *166 (and 2 hinges left over)*

7. They also bought 3 sets of tongs to use in handling the stamps and a magnifying glass for help in reading the letters and figures on the stamps. These tools cost $12 in all. The boys shared the cost equally. How much did each boy pay? *$4*

8. The boys spent about 36 hours during February working on their stamp collections. If the time was divided equally among the 4 weeks, how many hours a week did they spend? *9*

Teaching Pages 44 and 45

Pupil's Objective. To discover, by taking Inventory Test 7, how much relearning in division will be required.

Background. The inventory test on page 45 in the text should indicate to you how well children understand the two meanings of division; how proficient they are in using even and uneven division facts; and how well they can divide by one-place numbers. The material on pages 46 through 53 provides a thorough program of reteaching in these areas, including a carefully planned redevelopment of meanings.

Book Lesson. *Ex. 1–8:* Written work. Inventory Test 7.

1. Remind pupils that the purpose of an inventory test is to help them discover their strengths and weaknesses so that they may improve their work in the most efficient way possible.

2. Whether you correct the test or have the pupils correct their own papers, go over the examples one by one with the class. Such a review will give you added information about your pupils and may help them remember certain learnings forgotten temporarily.

Differentiations and Extensions

1. On the basis of the results in the inventory test, separate your class into smaller groups. *Slower learners* may need to proceed through all of the work up to and including page 53. *More capable children* may omit some of these pages.

2. Many devices useful in reteaching division are suggested on the pages of the Manual which follow. Let *more capable children* help in constructing these devices, or in preparing demonstrations which employ them. These children may also read for enrichment from some of the sources listed in the bibliography (pages 323–324 in the Manual).

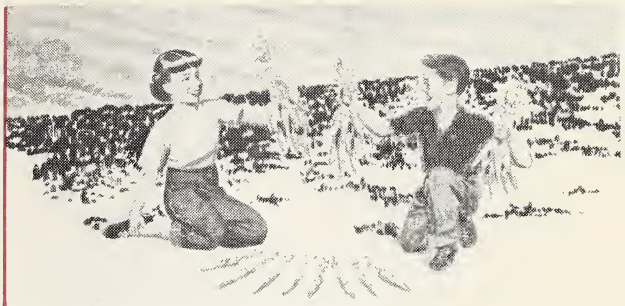
Teaching Pages 46 and 47

Pupil's Objective. To obtain help that will lead to more thorough understanding of the two meanings in division (measurement division and fractional-part division).

Background. Just as addition and multiplication are related, so are subtraction and division related. Subtraction involves separating a group into *unequal* parts while division involves separating a group into several *equal* groups. There are, however, two meanings for division, and it is important that children understand these meanings if they are to be successful in problem-solving.

The chart on page 72 summarizes the differences between measurement division and fractional-part division. The work on pages 46 and 47 in the text will help pupils develop an understanding of these meanings.

Teacher's Preparation. If possible, have on hand some of the following materials: bead line (page 312 in this Manual); calculating strips (stored in numbered boxes as suggested on page 319) and number line; peg board and pegs (page 314); measuring cup; two large containers; cardboard discs. You may want to ask pupils to save as many candy and cigar boxes as will be needed for storing these materials.



What Division Does

Measurement D.; meaning [O]

1. Sue and Al had 32 carrots to tie in bunches of 8. After Sue had tied 1 bunch and Al had tied 2 bunches, how many carrots were left? 8 How many bunches could they make in all? 4

Sue and Al were really measuring. They measured the large group, 32 carrots, by the small group of 8 and found that there are four 8's in 32. This is **measurement division.**

2. The answer for Ex. 1 can be found by subtracting 8's, as in box A. How many 8's were subtracted in box A? 4

3. When we know the division facts, it is quicker to divide, as in box B. How many 8's are there in 32? 4

Box B shows two ways to write the division fact. Each way can be read, "8's in 32 = 4" or "32 divided by 8 is 4" or "8 is contained in 32 four times."

A		B
32		
−8	(1)	
24		4
−8	(2)	8)32
16		or
−8	(3)	
8		32 ÷ 8 = 4
−8	(4)	
0		

Measuring a group by a smaller group to find how many of the small groups it contains is called measurement division.

46

Pre-book Lesson

Measurement Division (page 46)

1. Use many illustrations to show the measurement idea in division. For example, present a problem like the following: "To make lemonade

you may dilute 1 cup of lemon juice with 4 cups of water. How many cups of lemon juice will be needed to dilute 12 cups of water?" From a vessel containing 12 cups of water, measure 4 cups of water, then 4 more, then 4 more. Write the record on the board as shown below.

12 cups of water	4's in 12 = 3
− 4 (1)	
8 cups	12 ÷ 4 = 3
− 4 (2)	
4	3
− 4 (3)	4)12
0	

Call attention to the repeated subtraction and to the fact that the quotient in division corresponds with the number of times a smaller group (here, 4) may be subtracted from a larger group.

2. Use representative objects and let pupils engage in some of the activities suggested below.

a. Bead line. Separate a group of 24 beads from the rest of the beads. Find 6's in 24. Write these two forms:

$$6\overline{)24}^{\,4} \qquad 24 \div 6 = 4$$

b. Calculating strips and number line. How many 4-inch strips correspond to 20 inches on the number line?

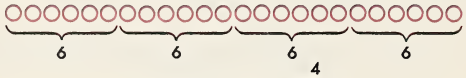
$$4\overline{)20}^{\,5} \qquad 20 \div 4 = 5$$

c. Show a row of 10 pegs on a peg board. How many 5's in 10?

$$5\overline{)10}^{\,2} \qquad 10 \div 5 = 2$$

3. As below, let pupils illustrate measurement division using pictures made up of lines or circles.

a.



6's in 24 = 4 $\quad 6\overline{)24}^{\,4}$

b.

4's in 20 = 5 $\quad 4\overline{)20}^{\,5}$

c.

5's in 10 = 2 $\quad 5\overline{)10}^{\,2}$

4. Have pupils record on the board some examples with "name" words.

a. 6 beads)24 beads (4 groups	*c.* 4 inches)20 inches (5 strips
b. 5 pegs)10 pegs (2 groups	*d.* 4 cups)12 cups (3 times

5. All pupils should state generalizations about the type of examples done. Record them on the board, being sure that they include the following understandings:

a. A large group was divided into smaller parts.
b. The smaller parts were equal in size.
c. In each case, we knew the total and the number in each equal part.
d. We found the number of small parts that were contained in the large group.

Fractional-part Division (page 47)

1. Use a problem such as the following: "18 cookies were to be divided among 6 children. How many cookies did each child receive?"

Use 18 cardboard discs to represent the cookies. Separate the discs into 6 equal groups by the process of placing 1 disc at a time in each of 6 equal piles until all 18 discs are used. Show the semi-concrete form and the number record on the board as below.

one sixth of 18 = 3
18 divided by 6 = 3
$\frac{1}{6}$ of 18 = 3

18 ÷ 6 = 3 $\quad 6\overline{)18}^{\,3}$

2. Use representative objects and let pupils engage in some of the activities suggested below.

a. Bead line. Separate a group of 14 beads from the rest of the beads. What is $\frac{1}{2}$ of 14 beads? Move one from the right end, then one from the left end until all 14 beads have been used in making two equal groups.

$\frac{1}{2}$ of 14 = 7 $\qquad 2\overline{)14}^{\,7}$

b. Peg board and pegs. What is $\frac{1}{3}$ of 15? Using 15 pegs and three rows on the peg board, place a peg in each row, repeating the process until all the pegs have been used. The finished demonstration should show 3 rows of pegs with 5 pegs in each row, illustrating that $\frac{1}{3}$ of 15 is 5.

$\frac{1}{3}$ of 15 = 5 $\qquad 3\overline{)15}^{\,5}$ $\qquad 15 \div 3 = 5$

(*Continued on page 71, second column*)

Fractional-part D; meaning [O]

When we divide, we do not always use measurement division. Ex. 4 and 5 tell about a different kind of division.

4. Fred and Kay also had 32 carrots, but they wanted to divide theirs into 8 equal bunches. How many carrots would there be in each bunch? 4

The picture above shows how Fred and Kay started to divide the 32 carrots into 8 equal groups. Explain their method. Make a diagram on the board to show how they finished the dividing. How many carrots are there in each of the equal groups? In each $\frac{1}{8}$ of 32 carrots there are 4 carrots.

When we divide in order to find the size of one of the equal parts of a group or a number, we are using **fractional-part division.**

5. A quicker way to find the answer to Ex. 4 is to think that each bunch will be 1 of the 8 equal parts of 32 carrots, or $\frac{1}{8}$ of 32 carrots. To find $\frac{1}{8}$ of 32, we divide 32 by 8. $32 \div 8 = 4$

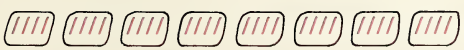
$\frac{1}{8}$ of 32 carrots = 4 carrots.

The kind of division in which we find how many in one of the equal parts of a group or number is called fractional-part division.

We write our work for both fractional-part division and measurement division the same way.

★Answers Not on Reproduced Page 47

4. To divide 32 carrots into 8 equal groups, Fred and Kay used 8 carrots to start 8 groups. Then they kept placing carrots one at a time in each group until all the carrots were used.



3. As suggested in the diagram below, let pupils illustrate fractional-part division using dot pictures and "name" words.

a.

$\frac{1}{2}$ of 14 beads = 7 beads

7 beads in each group

2 (groups))14 beads

b.

$\frac{1}{3}$ of 15 pegs = 5 pegs

5 pegs in each row

3 (rows))15 pegs

4. As pupils state generalizations about each demonstration, look for evidence that would indicate that the following understandings have been developed:

a. A large part was divided into smaller parts.
b. The smaller parts were equal in size.
c. In each case we knew the total and the number of small parts.
d. We found how many objects would be in each part.

Book Lesson. *Ex. 1–5:* Oral work.

1. As you conclude the discussion for problems 1 and 4, show the appropriate written form using labels, as below.

Ex. 1. 4 bunches
8 carrots)32 carrots

$32 \div 8 = 4$

Ex. 4. 4 carrots
8 (bunches))32 carrots

$\frac{1}{8}$ of 32 = 4

2. Compare the generalizations on these pages with those formulated by pupils.

Differentiations and Extensions

1. *Slower learners.* If possible, provide each child in this group with a number box for use in independent study. These pupils may then manipulate objects while a demonstration related to measurement division or fractional-part division is being directed by the teacher or a student.

2. *More capable pupils* may be asked to write measurement-division problems based on information such as the following:

a. 2 shoes to a pair
b. 3 feet to a yard
c. 4 wheels to a cart
d. 5 pennies equal a nickel

Measurement Division	Fractional-part Division
Clyde has 8¢. How many 2¢ pencils can he buy?	Clyde has 8¢. How much of it did he give away if he gave $\frac{1}{2}$ of it to his brother?
Differences in object manipulation	
To find the number of 2's in 8, count the number of times groups of 2 objects may be separated from the 8 objects.	To find $\frac{1}{2}$ of 8, separate the 8 objects into 2 equal groups by the continual process of placing 1 unit at a time in each of 2 piles until all 8 objects are used.
Differences in semi-concrete picturization	
(XX) (XX) (XX) (XX) (4 groups)	(XXXX) (XXXX) (4 objects in each group)
Differences in definition	
In measurement division you find the number of equal groups in a large group of objects when the size of the smaller groups is known.	In fractional-part division, you find the size of the equal groups when a large group is separated into a specified number of equal groups.
Differences in writing and reading the algorisms	
Writing: 2's in 8 = 4 $8 \div 2 = 4$ $2\overline{)8}\ 4$ Reading: two's in 8 = 4 8 divided by 2 is 4	Writing: $\frac{1}{2}$ of 8 = 4 $8 \div 2 = 4$ $2\overline{)8}\ 4$ Reading: one half of 8 is 4 8 divided by 2 is 4
Differences in relating the "labels"	
$2¢\overline{)8¢}\ 4$	$2\overline{)8¢}\ 4¢$ ←

They may also write fractional-part division problems based on information similar to the following:

a. $\frac{1}{2}$ of 60 minutes
b. $\frac{1}{3}$ of 60 minutes
c. $\frac{1}{4}$ of 60 minutes
d. 4 people sharing 100 envelopes
e. planning work periods of 3 equal parts
f. separating apples into 6 baskets equally

3. *All pupils* may enter in their notebooks some of the diagrams, examples written with "name" words, and the various forms of writing division examples presented in the pre-book lesson.

Teaching Page 48

Pupil's Objectives. (*a*) To relearn the terms *quotient*, *dividend*, and *divisor*; (*b*) to differentiate between measurement division and fractional-part division in problems with 1-figure divisors.

Background. Note that the terms for identifying the parts of a division example are introduced on page 48 in the text and that pupils go on to tell the kind of division that is used for six specific problems, thereby calling on the understandings reviewed on pages 46 and 47.

Pre-book Lesson. A discussion of the forms given below will provide the basis for a brief review of the generalizations arrived at in previous lessons and will serve as a point of reference for problems presented in this lesson.

Measurement Division

number of equal groups (to find)
size of each part (known)) total (known)

Fractional-part Division

size of each part (to find)
number of equal groups (known)) total (known)

Book Lesson. *Ex. 1–4:* Oral work. *Ex. 5–10:* Written work.

Which Kind of Division (D.) Problem?

[O]

1. From the box, tell what we call the number that is divided; the number by which we divide; the answer. dividend, divisor, quotient

$$\begin{array}{r} 4 \leftarrow \text{quotient} \\ 8\overline{)32} \leftarrow \text{dividend} \\ \uparrow \\ \text{divisor} \end{array}$$

2. In $54 \div 6 = 9$, which number is the divisor? 6 Which is the dividend? 54 Which is the quotient? 9

In measurement division, the quotient tells the number of equal groups that are contained in a larger group (the dividend). In fractional-part division, the quotient tells how many in each of the equal parts of the dividend.

3. How many rows of 8 pins each can Ann make from 48 pins? 6 We are measuring 48 with 8 to find the number of groups. Why is this measurement division? Does the quotient tell how many 8's are contained in 48? Yes $48 \div 8 = 6$.

4. Ann bought 6 pencils for 30¢. Each pencil cost 5 ¢. We are finding the number in each part of the whole. Why is this fractional-part division? Does it take $\frac{1}{6}$ of 30¢ to pay for each pencil? Yes $\frac{1}{6}$ of 30¢ = 5 ¢.

[W]

For each of Ex. 5–10, write "measurement" if the quotient tells the number of equal groups and "fractional-part" if the quotient tells the size of each of the equal parts.

5. Our schoolroom has 40 seats in 5 equal rows. How many seats are there in each row? fractional part

6. Ann's step is 2 feet long. How many steps will she take in walking 100 feet? measurement

7. At $5 a pair, how many pairs of shoes will $10 buy? measurement

8. Mr. Ashley drove 237 miles in 6 hours. This was at the rate of how many miles an hour? fractional part

9. How many pecks are there in 40 qt.? (8 qt. = 1 pk.) measurement

10. Forty-two children were divided equally into 6 rows. How many children were in each row? fractional part

1. As Ex. 3 and 4 are discussed, relate the numbers to the forms on the board.

Ex. 3.

$$8\text{ pins (size of each part)}\overline{)48\text{ pins (total)}}\quad 6\text{ rows (number of equal groups)}$$

Ex. 4.

$$6\text{ (number of equal groups)}\overline{)30¢\text{ (total)}}\quad 5¢\text{ (size of each part)}$$

Continue the discussion, emphasizing the meaning of each quotient.

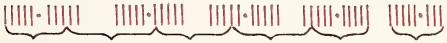
2. Attention may be called to the generalizations at the top of the next column if divisors, dividend, and quotients are labeled as they are written on the board during the discussion:

a. Measurement Division. The divisor is an actual part of the dividend. (The labels are the same.)

b. Fractional-part Division. The divisor may be written as a fraction if it is not already so written. The quotient is an actual part of the dividend.

Differentiations and Extensions

1. Do not confuse *slower learners* with too many words nor expect them to formulate generalizations beyond the limits of their abilities. Encourage them to "picture" situations with diagrams, as illustrated below.



$$8\text{ pencils}\overline{)48\text{ pencils}}\quad 6\text{ groups}$$

2. *More capable pupils* may complete the following chart in analyzing the problems in Inventory Test 7 on page 45:

Problem	Meaning of Quotient	Kind of Division
1	number of equal groups	measurement D.
2	size of each part	fractional-part D.
3	?	?

NOTES

Helps for Learning Division Facts

[O]

Read each of Ex. 1–4 in two ways. Tell the dividend, the divisor, and the quotient in each exercise. (i.e., for Ex. 1, 16 is the dividend, 8 the divisor, and 2 the quotient; 16 divided by 8 = 2, 8 2s in 16 = 2)

1. $16 \div 8 = 2$ 2. $5\overline{)40} = 8$ 3. $7\overline{)21}$ 4. $27 \div 9 = 3$

5. In each of Ex. 1–4, multiply the divisor by the quotient. Compare your answer with the dividend. What is true in each case? Why can we say that the quotient and the divisor are factors of the dividend? (quotient × divisor equals dividend; 3 × 7 = 21; Their product equals the dividend.)

6. In $36 \div 9 = ?$ which number can be thought of as a product? (36) as a factor? (9) How do we find the missing factor? (Divide)

7. Whenever we know a product and one of its two factors, how can we find the missing factor? (Divide product by known factor.)

8. The 81 division facts are given on page 294. Turn to that page, cover the answers, and quickly say all the facts.

9. $0 \div 1 = ?$ Say all the facts that have a zero quotient. Then make a rule for dividing 0 by any number. (0 divided by any number is 0.)

10. Say the facts in which 1 is the divisor. Then make a rule for dividing any number by 1. (The quotient of any number divided by 1 is that number.)

11. Say the facts that have 1 as the quotient. Then make a rule for dividing any number by an equal number. (Any number divided by an equal number equals 1.)

12. If you know the division fact $42 \div 6 = 7$, how does it help you in finding the answer for $42 \div 7$? (From 42 ÷ 6 = 7, you know that 42 ÷ 7 = 6)

13. If you know the multiplication fact $8 \times 9 = 72$, how does it help you in finding the answer for $72 \div 9$? (From 8 × 9 = 72, you know that there are eight 9s in 72)

Multiplication facts go with related division facts to make whole stories. The whole story about 4, 8, and 32 is:

$4 \times 8 = 32$; $8 \times 4 = 32$; $32 \div 8 = 4$; $32 \div 4 = 8$.

Tell the whole story in multiplication and division about

14. 3, 7, and 21. 15. 4, 9, and 36. 16. 7, 8, and 56.

Helps for Learning Uneven Division Facts

Table numbers [O]

1. How many stamps at 6¢ each can I buy with 20¢?

I can buy as many stamps as there are 6's in 20. In box A, how many times is 6¢ taken from 20¢? (3) How much is left over? (2¢)

In box B, how many 6's do we take from 20 all at one time? (3)

Box C shows two short ways to write this fact with its remainder.

A	B	C
20¢ −6¢ 14¢ −6¢ 8¢ −6¢ 2¢	3, R2 6)20 18 (3 × 6) 2	3, R2 6)20 or 20 ÷ 6 = 3, R2

For box C *think*, "20 ÷ 6 is not in any division table, but 18 ÷ 6 is. 18 is the **table number** to use because it is next smaller than 20. 18 ÷ 6 = 3; so 20 ÷ 6 = 3, R2."

2. Tell the table number for: 7)24; (2) 5)32; (3) 3)16; (5) 8)35. (2)

3. In Ex. 1, the quotient tells how many stamps. Does the remainder mean stamps or cents?

4. As he plants corn, Don puts 4 kernels of corn in each hill. How many hills will he make from 25 kernels? (See box D.) (6 hills, 1 kernel left)

What does the quotient tell? (no. of hills) the remainder? (kernels left) Explain the check in E. (Quotient × divisor + remainder = dividend)

D	E
6, R1 4)25 24 1	Check 4 ×6 24 +1 25

[W]

Divide and check as in boxes D and E above.

	a	b	c	d	e	f	g
5.	2)7 (3, R1)	6)17 (2, R5)	3)16 (5, R1)	8)27 (3, R3)	5)33 (6, R3)	7)30 (4, R2)	8)51 (6, R3)
6.	4)37 (9, R1)	6)53 (8, R5)	7)46 (6, R4)	8)45 (5, R5)	9)32 (3, R5)	9)58 (6, R4)	7)36 (5, R1)
7.	4)13 (3, R1)	9)25 (2, R7)	6)43 (7, R1)	4)31 (7, R3)	5)37 (7, R2)	3)26 (8, R2)	9)65 (7, R2)
8.	5)42 (8, R2)	9)50 (5, R5)	7)53 (7, R4)	8)61 (7, R5)	4)35 (8, R3)	9)80 (8, R8)	6)35 (5, R5)

Teaching Pages 49 and 50

Pupil's Objectives. (*a*) To identify and study division facts needing mastery; (*b*) to review the idea of letting a quotient represent a missing factor; (*c*) to review "whole stories" in multiplication and division; (*d*) to use "table numbers" in relearning uneven division facts; (*e*) to review ways of writing uneven division facts; (*f*) to tell the meaning of remainders in uneven division problems; (*g*) to relearn the method for checking quotients having remainders.

Background. The fact that the quotient and the divisor may be considered factors of the dividend (Ex. 5, page 49) is an important concept, since it is the basis for the multiplication step in the long form of division, as shown in box B on page 50 in the text, and for the division check shown in box E on page 50. Moreover, this idea is also useful in working expressions of the form $n \div 9 = 2$.

Ex. 9–11 on page 49 present helpers for division facts involving zero and one. Ex. 12 reintroduces the idea that a related or reverse fact may be used as a helper (i.e., $42 \div 6 = 7$ may be used when a pupil has difficulty with the fact $42 \div 7$). Then in Ex. 13–16 pupils are reminded of the fact that there are whole stories in multiplication and

division just as there are for addition and subtraction and that these whole stories may also be used for helpers.

Ability to work with uneven division facts is exceedingly important in harder division. The most important help introduced on page 50 in the text is the use of "table numbers." Table numbers themselves are simply the dividends taken from the tables of division facts. They are introduced to your pupils because their use conforms to the best thinking process employed in division. Through this approach the pupil is expected to look at $15 \div 4$ and think, "15 can't be divided evenly by 4, what is the next *smaller* number that *can* be divided by 4 evenly?" If the division facts are well known it will be seen immediately that the table number that helps is 12.

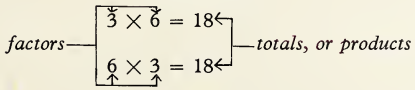
In Ex. 3 and 4 on page 50, pupils are given help in interpreting the meaning of remainders. This is an important understanding in problem-solving, especially when pupils wonder whether remainders should be expressed in a fraction or with R.

The check in box E should cause no difficulty except that *slower learners* may need a variety of experiences to understand why the remainder is incorporated into the check.

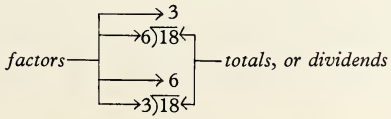
Pre-book Lesson

1. See whether pupils recall that the term *factors* refers to the two numbers which multiplied together form a product. Relate this understanding to division. Then, as illustrated below, place an "X" pattern on the board and let pupils label the terms of the four facts in multiplication and division which are thus shown.

Multiplication



Division



2. Lead pupils to state generalizations which include some of the following points:

a. The dividend is the total.
b. The divisor is one factor.
c. The quotient is the other factor.
d. If we know the total (dividend), and one of the factors, we can find the other factor by dividing.

Book Lesson (page 49). *Ex. 1–16:* Oral work.

1. "Easy" division facts. Have the pupils name three more division facts which belong with each fact below.

$1\overline{)7}$ (divisor 1) $\quad 1\overline{)6}$ (quotient 1)

$2\overline{)16}$ (divisor 2) $\quad 2\overline{)8}$ (quotient 2)

$5\overline{)25}$ (doubles–divisor and quotient are the same)

2. "Harder" facts. Have pupils say the facts to partners or, possibly, let them write them as a test from dictation. Help pupils build co-operative attitudes toward partners. Try to organize your class so that partners are of about equal ability in respect to knowing facts. Cite instances where partners are, or may be, really helpful to each other.

Book Lesson (page 50). *Ex. 1–4:* Oral work. *Rows 5–8:* Written work. Have pupils use representative materials or let them make diagrams concurrently with Ex. 1 and 4 on this page in the text. For Ex. 4, pupils may put materials together again to demonstrate the check.

Differentiations and Extensions

1. *All pupils* should isolate facts needing study and record them on study cards or in notebooks. Help pupils who have many facts to study to organize them into "family" groups (4's, 5's, etc.). Hold each pupil responsible for learning the facts within his own individual time limit.

2. *Slower learners*

a. More than likely, only a few pupils will need to continue to use the "longer" form for uneven division examples, as shown in box B on page 50. Some may need to use it only when bridging a ten occurs in subtracting to find the remainder (i.e., not for $8\overline{)59}$ $^{7,\ R3}$ but for $8\overline{)62}$ $^{7,\ R6}$). When possible, encourage these pupils to try shorter forms as shown in box C on page 50.

b. Select some of the division facts that seem to cause the most trouble for *slower learners*. After they have been identified as table numbers, let pupils write on the board some of the uneven division facts that are derived from them.

Illustration

The division fact $8\overline{)56}$ (table number 56) may be used with these uneven division facts: $8\overline{)57}$, $8\overline{)58}$, $8\overline{)59}$, $8\overline{)60}$, $8\overline{)61}$, $8\overline{)62}$, $8\overline{)63}$.

When several such sets of facts are completed, erase all but one uneven division fact in each set. Have pupils complete each using the short way

$$8\overline{)60} \quad 7, \text{R}4$$

(i.e., or $60 \div 8 = 7, \text{R}4$).

c. For practicing even and uneven division facts consider the following materials: bead lines, calculating blocks, calculating strips with number line, graph paper.

d. Some of the games mentioned in Part IV of the Manual that may be used by *slower learners* in studying division facts are: "Double Draw," "Clover," (the criterion being particularly the "family" groups needing study), "Yay," "Mads."

3. Each of the *more capable pupils* may make a number line extending from 1 to 100. Then they may cross out any numbers that are evenly divisible (except by 1 or the number itself). The numbers that are left ("prime" numbers, or those which are not divisible) may be circled, as shown below for the numbers 1 to 10.

① ② ③ ~~4~~ ⑤ ~~6~~ ⑦ ~~8~~ ~~9~~ ~~10~~

Jane's Corn Roast

2-place dividend and quotient [O]

1. For the corn roast, Jane put 60 ears of corn in 3 equal piles. How many ears did she put in each pile? 20 $60 \div 3 = 20$

60 is 6 tens. 6 tens ÷ 3 = 2 tens, or 20. There were 20 ears of corn in each pile.

2. Tell the quotients: **a.** $80 \div 2$ 40 **b.** $120 \div 4$ 30 **c.** $400 \div 8$ 50

We divide tens just as we divide ones.

3. How many 4¢ ice-cream cups can Jane buy with 96¢? 24

Tens: Why is 8 the table number for $4\overline{)9}$? $8 \div 4 = 2$ We write "2" in ten's place in the quotient. next smallest number evenly divisible by 4

When we subtract 8 tens, or 80, we are subtracting twenty 4's. Explain. 2 tens (20) × 4 = 80

When we bring down the 6 ones to go with the 1 ten left over, the 16 to be divided next is 16 ones. Why? 1 ten = 10 ones; 10 ones + 6 ones = 16 ones

Tens	Ones	
2	4	
4)9	6	
8		(20 × 4)
1	6	
1	6	(4 × 4)

ones place

Ones: $16 \div 4 = 4$

In what place in the quotient do we write "4"? Explain. it is the quotient for 16 (ones) ÷ 4 [W]

Write your work for these as in the box:

4. $3\overline{)69}$ 23 (60 (20×3), 9 (3×3))
5. $2\overline{)76}$ 38 (60 (30×2), 16 (8×2))
6. $4\overline{)92}$ 23 (80 (20×4), 12 (3×4))
7. $3\overline{)96}$ 32 (90 (30×3), 6 (2×3))
8. $5\overline{)75}$ 15 (50 (10×5), 25 (5×5))

Teaching Pages 51, 52, and 53

Pupil's Objectives. (*a*) To review dividing by 1-place numbers when quotients of two or more places result; (*b*) to use the terms *first partial dividend* and *second partial dividend;* (*c*) to learn how to estimate quotients.

Background. When divisors are 1-place numbers and quotients contain two or more figures, the long division form is the most appropriate written record of the computational process. An essential understanding for this kind of division is that we can divide by parts (i.e., we can use partial dividends). Therefore, pupils are reminded on page 51 in the text that they can divide tens just as they divide ones. For example, 6 tens ÷ 3 = 2 tens, just as 6 ones ÷ 3 = 2 ones.

If pupils place a piece of paper along the dotted line in the box on page 51 in the text so that only the ten's column and the divisor remain uncovered, they will see how dividing by parts (in this case tens) greatly simplifies division when quotients are 2-place numbers. Notice in the discussion at the left of the box that the idea of a table number is immediately put to work and that the relationship between subtraction and division is maintained.

Helps for Dividing Larger Numbers

2-place quotient [O]

1. $\frac{1}{8}$ of 743 = 743 ÷ 8 (box A).

Can 7 hundreds be divided into 8 parts giving 1 hundred to each part? No

Then 74 tens becomes the **first partial dividend.** It is only part of the whole dividend. Why is this called a partial dividend? Why is a small x placed over the 4 of 74? To show that first quotient figure is in tens place.

A

Hundreds	Tens	Ones
	9 x	2, R7
8)7	4	3
	7 2	(9 tens × 8)
	2	3
	1	6 (2 × 8)
		7

First Partial Dividend (getting tens)

Divide tens: 74 ÷ 8 = ? *Think,* "The table number is 72. 72 ÷ 8 = 9." Write "9" in ten's place in the quotient.

Multiply and compare: 9 × 8 = 72. 72 tens are less than 74 tens, so 9 is not too large for a quotient figure.

Subtract and compare: 74 − 72 = 2. Can 2 be divided by 8? No If it could be, you would know that the quotient figure was too small.

Bring down the 3 ones. Think of the 2 tens and 3 ones as 23 ones, the **second partial dividend.**

Second Partial Dividend (getting ones)

Explain what was done in dividing the 23 ones in box A.

B

Check
92
×8
736
+7
743

Remember that the divisor and quotient are factors of the dividend. Explain the check in box B. ∨ divisor × quotient + remainder = dividend

For each exercise below tell the first partial dividend. Then tell where you would put x to show the place of the first quotient figure. (x in ten's place in each) (underlined figures below to be read as tens)

	a	b	c	d	e	f
2.	35 5)175	62 3)186	63, R2 4)254	53, R2 8)426	72, R1 9)649	82, R3 6)495
3.	55, R1 2)111	96, R1 3)289	54, R1 6)325	76, R3 5)383	55, R2 3)167	73, R3 8)587
4.	43 4)172	96 2)192	78, R1 3)235	82, R1 7)575	68, R1 2)137	93, R1 7)652

[W]

Now copy rows 2–4, divide, and check, as in boxes A and B.

Deciding what the first partial dividend will be usually causes difficulty in finding 2-place quotients when there are 3-place dividends. Notice the help that is given in Ex. 1 on page 52 in the text for this phase of the work. As a further aid, pupils are shown how to use a small x (box A) to help them remember where to write the first figure in the quotient. Notice, also, on page 52 in the text, that the steps for division are given and that in box B the division check is shown again to remind pupils that they can multiply the divisor times the quotient when the divisor is the smaller number.

Quotients of 3, 4, and 5 places, some of which contain a zero, appear on page 53 in the text. The kind of help given in exercises 7 and 8 and boxes D and E should help pupils overcome difficulties caused by the fact that a zero must be written in the quotient as a place-holder.

Estimating for an example like 11**a** on page 53 in the text is straightforward. Pupils simply round to hundreds and divide hundreds like ones, obtaining the estimate 500. For an example like 11**b**, they round to thousand's place and divide thousands like ones, obtaining 7 (thousands) ÷ 4 = 1 (thousand), R3 (thousands), or almost 2 thousand.

Book Lesson (page 51). *Ex. 1–3:* Oral work. *Ex. 4–8:* Written work.

1. For Ex. 2, provide additional help by writing examples on the board with words and figures.

Illustration

80 ÷ 2 = ?

4 tens	40
2)8 tens	2)80

2. For Ex. 3, direct attention to the boxed example, where the first product 8 (tens) is shown as 20 × 4 (the quotient, 2 tens, × divisor, 4). Thus it can be determined that twenty 4's are being subtracted all at once from 96.

Book Lesson (pages 52 and 53). *Ex. 1–4:* Oral work. *Bottom of page 52:* Written work. *Ex. 5–10:* Oral work. *Rows 11–14:* Written work.

1. Have a pupil work Ex. 1 at the board so that the class will encounter step-by-step the particular subtopics as they are met on the page.

2. For Ex. 7, ask pupils why only one product shows in box D when there are two figures in this quotient. Also, for Ex. 8, inquire why only two products show in box E when the quotient has three figures.

Differentiations and Extensions

1. *Slower learners* may profit from a record such as the one below, but they should work toward eliminating its use as quickly as possible.

Steps in Dividing 743 by 8

First Partial Dividend

a. Divide. 9 / 8)743

3-, 4-, or 5-place quotient; 0 quotient figure [O]

5. In box C, why is the first partial dividend 13 hundreds? ÷4 not possible in thousands place

6. In box C, how many 4's were subtracted in all? 339

7. In box D, why is 0 in one's place in the quotient? 2÷9 not possible in ones place

8. In box E, why is 0 in ten's place in the quotient? 4÷8 not possible in tens place

9. For 42,653 ÷ 5 = ? tell the first partial dividend and the number of quotient figures. 4 (42 thousands)

C	D	E
339, R1 4)1,357 12 15 12 37 36 1	50, R2 9)452 45 2	305 8)2,440 24 40 40

Study the examples in row 10 and tell the mistakes.

	a	b	c	d	e
	150, R6	590	800, R6	806	8750
10.	7)1,056	5)2,950	8)6,406	6)4,836	4)35,000
	7	25	64	48	32
	35	45	06	36	20 30 28 20 20
	35	45		36	0
	6	0			

Estimate the quotients for rows 11–14 below. For Ex. 11a, *think*, "15 hundreds ÷ 3 = 5 hundreds. The quotient will be about 500." For Ex. 11b, *think*, "7 thousands ÷ 4 = 1 thousand, remainder 3. The quotient will be almost 2,000."

[W]

Copy and divide the following as in boxes C to E above:

11.	3)1,527 (509)	4)7,228 (1,807)	2)9,411 (4,705, R1)	3)27,017 (9,005, R2)	4)68,037 (17,009, R1)
12.	5)3,600 (720)	9)6,350 (705, R5)	4)8,360 (2,090)	7)25,619 (3,659, R6)	8)36,015 (4,501, R7)
13.	8)8,185 (1,023, R1)	6)2,401 (400, R1)	8)9,075 (1,134, R3)	9)4,826 (536, R2)	5)74,283 (14,856, R3)
14.	2)7,017 (3,508, R1)	7)7,058 (1,008, R2)	6)2,412 (402)	5)8,605 (1,721)	9)75,785 (8,420, R5)

Extra Practice. For more practice, work Sets 23 and 24.

b. Multiply and compare. Is the product smaller than the first partial dividend?

```
    9
8)743
→ 72
```

c. Subtract and compare. Is the remainder smaller than the divisor?

```
    9
8)743
↑ 72
→ 2
```

d. Bring down. Now we have our next partial dividend.

```
    9
8)743
 72↓
→ 23
```

Second Partial Dividend. (Repeat steps *a*–*d*.)

2. Extra Practice Sets 23 and 24 may be assigned as needed.

Set 23. 1-place divisors; 2- and 3-place quotients; without and with remainders

Show any remainder with R.

	a	b	c	d	e
1.	34 2)68	244 976 ÷ 4	242, R2 3)728	$1.02 $8.16 ÷ 8	14, R2 6)86
2.	$0.35 5)$1.75	116, R3 7)815	70 630 ÷ 9	57 6)342	43, R6 307 ÷ 7
3.	14, R2 58 ÷ 4	188 3)564	49, R3 5)248	116, R6 934 ÷ 8	13, R3 9)120
4.	204, R1 2)409	53, R2 214 ÷ 4	53, R2 479 ÷ 9	106, R1 2)213	71, R4 572 ÷ 8
5.	106, R1 319 ÷ 3	150, R1 6)901	138, R3 5)693	35, R1 246 ÷ 7	$0.78 5)$3.90

Set 24. 1-place divisors; some zeros in 3-, 4-, and 5-place quotients; without and with remainders

Show any remainder with R.

1.	2,243 3)6,729	547, R1 2)1,095	26,992 3)80,976	20,345 2)40,690	1,344 4)5,376
2.	23,554, R2 4)94,218	1,350 7)9,450	1,019, R3 5)5,098	5,252, R1 8)42,017	5,809, R3 6)34,857
3.	2,671, R3 6)16,029	17,807 3)53,421	1,092, R1 7)7,645	10,936, R1 2)21,873	6,070, R3 8)48,563
4.	2,680 9)24,120	1,450 5)7,250	11,929, R7 8)95,439	1,391, R5 6)8,351	1,677 4)6,708
5.	1,768 7)12,376	893 9)8,037	4,269, R1 5)21,346	10,408, R3 9)93,675	12,574, R1 7)88,019

NOTES

Finding and Using an Average

Meaning [O]

1. Would you always use the same number to tell how many ounces there are in a pound? Yes (No) to tell how many players there are on a regular baseball team? Yes (No)

2. Would you always use the same number to tell how many ounces a baby weighs? No to tell how many runs were scored in a baseball game? No

Some numbers stand for things that always stay the same, such as the number of inches in a foot or the number of dimes in a dollar. Other numbers stand for things that do not stay the same, such as the number of inches in the height of a growing plant or the number of miles covered each hour of a trip.

There are many times when it is not convenient to use a long list of these changing numbers. It is better to use just one number that represents all the numbers in the group that we are talking about. We call this number an **average.**

3. Suppose someone asked you how many minutes you spend eating lunch. If you timed yourself every day for a week, would you be likely to find that you spent *exactly* the same number of minutes every day?

You might find that you spent 18 minutes one day, 24 the next, 23 the next, 19 the next, and so on. Instead of saying all these different numbers, could you just estimate and say, "About 20 minutes, on the average" to give someone else a good idea of the time you spend eating lunch? Yes

In Ex. 3, instead of estimating the number 20, you could have found an average by using the numbers in this way:

First find the total of 18, 24, 23, and 19,—which is 84. Then find $\frac{1}{4}$ of 84, or 21.

4. Why does 21 tell you the number of minutes you would have spent if you had spent the same number of minutes each day? Why is this a good number to use as an average? It's typical. It is one of the four equal parts of 84.

5. At the cook-out, these boys cooked the follow of hamburgers: Don, 14; Jim, 18; Bob, 15; Dick was the average number of hamburgers cooked by a b

If each boy had cooked the same number of ha boys, what part of the total number would he have made? $\frac{1}{4}$ How do you find one of the equal parts of a total? Divide total by number of equal parts.

The box below shows how to find the average number of hamburgers cooked by a boy. Explain the work.

14	(Don)		17 (average)
18	(Jim)	$\frac{1}{4}$ of 68 =	$4\overline{)68}$ ← Total
15	(Bob)	Number of parts	4
+21	(Dick)		28
68	← Total of 4 parts		28

How many boys cooked more than the average number of hamburgers? 2 (Jim, Dick) How many cooked fewer than the average? 2 (Don, Bob) Do you think that 17 is a good average to use? Explain.

Is this a two-step problem? No See if you can make up a find-the-average problem that is not a two-step problem.

To find the average of several numbers, you divide the sum of the numbers by the number of addends.

6. The lengths of 5 sticks of wood used for a trellis were 10 in., 12 in., 11 in., 9 in., and 8 in. What was the average length of stick used? 10 in.

On the board, side by side, draw lines to stand for these sticks of wood. Explain how you might take inches off the long sticks and add them to the short ones to make each stick the same length—that is, the average. (See pictures)

Work the problem on the board, using figures.

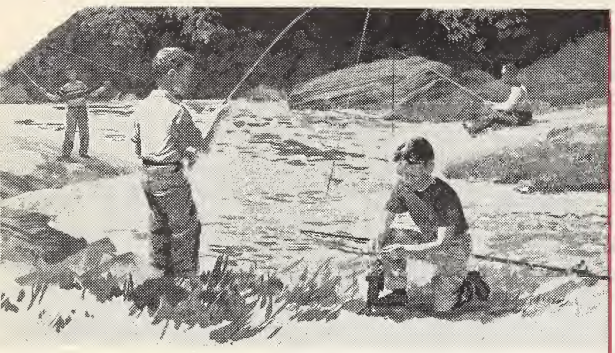
Teaching Pages 54, 55, and 56

Pupil's Objectives. (*a*) To increase understanding of the meaning of average; (*b*) to obtain practice in finding averages both when a total is and is not given; (*c*) to use averages in problems.

Background. Averages are used to such a great extent these days in communicating information that it is important for pupils to understand their meaning as well as how to find them. After studying the work on page 54 in the text, pupils should realize that an average is a number used to describe a range of numbers. More specifically, they should realize that an average is a number which indicates the size all the numbers would be if they were the same size.

On page 55, pupils learn how to find an average. They are led to realize that most problems involving averages require two steps, and that the hidden question usually is "What is the sum of all the smaller groups?" Ex. 6 on page 55 asks for an illustration of the meaning of the procedure for finding averages.

Finally, on page 56, pupils solve several problems involving averages. They also learn (Ex. 3) that some average problems are 1-step problems since the total is given, and that in other problems (Ex. 6) they may find the total by multiplying the average by the number of groups.



Problems about Averages

[W]

Write your work for Ex. 1–6.

1. On a fishing trip, Joe caught 12 fish, Don caught 14, Pete caught 9, and Bill caught 17. What was the average number caught? 13

2. The attendance at the last 4 weekly meetings of the Science Club was as follows: 12, 10, 13, and 15. What was the average attendance? 12

3. Joe rode his bicycle 21 miles in 3 hours. What was his average rate in miles per hour? 7 (Do you need to find the total before you divide in this kind of problem? no)

4. In 6 days, Mr. Fox drove 1,488 miles. How many miles did he average a day? 248 mi.

5. At an average of 8 miles an hour, how many hours will it take Sam to ride 48 miles? 6 hr. (Do you have to find the average in this example or will you use the average just as though it were a number which does not change? no, yes)

6. In our room, each person borrows an average of 3 library books a week. How many books a week will all 35 of us borrow? 105

Teacher's Preparation. See page 312 in Part IV of the Manual for an explanation of how calculating blocks may be used to explain average.

Pre-book Lesson

1. Use calculating blocks, books, and the like to demonstrate the meaning of averages.

2. Present a group of denominate numbers such as 27 in., 36 in., 14 in., and 6 in. Ask pupils to inspect each addend and tell those that are above or below the average. In this way they may be able to estimate the average.

3. Use the two problems below in discovering whether or not your pupils can handle problems involving averages both when a total must be found, and when the total is given.

a. One day Mr. Webster traveled 27 miles, 33 miles, and 15 miles on different business trips. What was the average distance that he traveled for a trip? (Must find total.)

b. In 75 miles of travel he used 5 gallons of gasoline. What was the average number of miles per gallon? (Total given.)

4. Discuss with pupils the practical advantages of knowing averages in the problems above.

Book Lesson (pages 54 and 55). *Ex. 1–6:* Oral work.

Book Lesson (page 56). *Ex. 1–6:* Written work.

1. Before pupils start this written work, let them study the problems to be sure there are no language difficulties.

2. When the written work has been finished and corrected, discuss each example with emphasis on the meaning of the quotient as well as on any specific needs of your class.

Differentiations and Extensions

1. Have *all pupils* compile a list of ways they might use averages in their own activities, as in averaging scores for a series of games, reporting the average height and weight for the class, recording the average amount of money earned or spent, listing the average hours (or minutes) of time spent in baby-sitting, lawn-mowing, and the like.

2. *More capable pupils* may use a detailed weather report obtained from the Weather Bureau or from a newspaper. They should try to explain the term *mean temperature* and the method for finding this kind of average. They may learn to interpret reports of extremes and deviations from normal temperatures.

Teaching Page 57

Pupil's Objective. To take Inventory Test 8 in order to discover how much help is needed in dividing by 2-place numbers.

Background. Inventory Test 8 prefaces a whole series of reteaching pages that run to page 69 in the text.

When We Divide by a 2-Place Number

Inventory Test 8 [W]

The sixth-grade pupils of Stockwell School have been planning a Thanksgiving party. These are some of the problems they have had to solve. Write your work for these problems:

1. The committee has planned a party lasting 120 minutes. How many hours long will the party be? 2

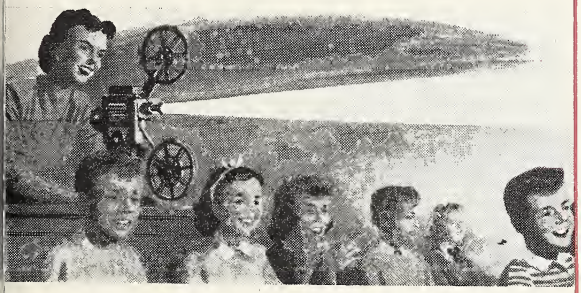
2. To pay for the use of a moving-picture projector, $3.85 had to be collected from the class. Miss Pike knew that $3.85 could not be divided evenly among the 36 pupils in the class, so she agreed to pay the remainder. What would each pupil's share be? How much would Miss Pike pay? 25¢ 10¢

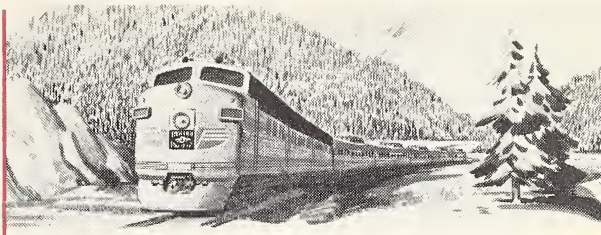
3. For use in games to be played, the committee divided a package of 288 slips of paper into 36 smaller equal packages. How many slips of paper would be in each small package? 8

4. Ice cream cups for the group of 37 (including Miss Pike) would cost in all $3.33. What was the cost per person? 9¢

5. If other classes joined the sixth grade for refreshments, ice cream and cookies could be bought at a better price. At a total cost of $10.44, what would ice cream and cookies cost apiece for 58 people? 18¢

6. A large group in another building spent $122.50 to cover the cost of costumes and equipment for entertainment as well as refreshments for 98 people. What was the cost per person? $1.25





Helps for Dividing by a 2-Place Divisor

1-place quotient [O]

1. At an average of 60 miles an hour, how many hours does it take the Rocky Mountain Limited to run 180 miles?

At the board find the answer in these ways:

Find how many 60's you can subtract from 180. 3

Make division tables with divisor 6 and with divisor 60, as in box A. Divide as far as $18 \div 6$ and $180 \div 60$ to find how many 60's in 180.

Divide as in box B. How many 60's are subtracted all at one time? 3

The side work in box B makes it easier to tell what the quotient figure will be. *Think*, "$18 \div 6 = 3$, so $180 \div 60 = 3$." Write "3" in one's place in the quotient.

Explain the check in box C: 2 quotient times divisor equals dividend [W]

A

$6\overline{)6}$	$60\overline{)60}$
1	1
$6\overline{)12}$	$60\overline{)120}$
2	2
$6\overline{)18}$	$60\overline{)180}$
3	3

B

	Side work
$60\overline{)180}$	$6\overline{)18}$
3	3
180 (3×60)	

C

Check

$$\begin{array}{r} 60 \\ \times 3 \\ \hline 180 \end{array}$$

Copy, divide, and check as in boxes B and C.

	a	b	c	d
2.	$20\overline{)80}$ 4	$30\overline{)90}$ 3	$20\overline{)160}$ 8	$70\overline{)210}$ 3
3.	$40\overline{)360}$ 9	$60\overline{)240}$ 4	$80\overline{)400}$ 5	$90\overline{)720}$ 8

Division is known to be a difficult process and you will be most fortunate if you find that there are only a few pupils who need reteaching in this area. Very likely you will find that all pupils need reteaching of one kind or another. The results of this inventory test will help you discover the best way to plan the lessons which follow in the text.

Book Lesson. *Ex. 1–6:* Written work. Inventory Test 8.

Teaching Pages 58 and 59

Pupil's Objectives. (*a*) To obtain help in dividing by 2-place numbers (1-place quotients only); (*b*) to practice dividing by tens and by tens and ones when the trial quotient is the true quotient.

Background. On page 58 in the text, pupils divide by even ten's numbers, always obtaining

a 1-place quotient with no remainder. If pupils realize that in box B they are finding how many 60's in 180, they will know that the quotient will be a one's number. The side work in box B points out that dividing by the ten's figure of the divisor is a good way to find the trial quotient.

On page 59, pupils divide by both tens and ones, obtaining remainders in some instances. The procedure is little different from the one presented on page 58 in conjunction with dividing by even tens. If difficulty occurs, it will probably be in dividing the first one or two figures in the dividend by the ten's figure in the divisor to estimate the trial quotient figure, for this step often involves uneven division.

On page 59, notice in box A and the discussion beside it that the steps in dividing are highlighted and that the division check is given again. Notice, also, that in boxes B and C pupils receive help in dividing money numbers.

Book Lesson (page 58). *Ex. 1:* Oral work. *Rows 2 and 3:* Written work. When the class is ready to do the written work, show how you want them to record the examples, side work, and checks. Encourage systematic work habits and organization.

Book Lesson (page 59). *Ex. 1 and 2:* Oral work. *Ex. 3–9:* Written work.

1. Most of your pupils probably ought to do the oral work for the purposes of review and to obtain a secure foundation for future learning. Pupils who showed ability in the inventory test may do enough rows of written practice to maintain their competence and to increase speed. Select the rows carefully so that practice with the various kinds of examples will be had.

2. At first *all pupils* may be required to show on their papers the side work for each example so that their thinking process can be followed. Undoubtedly, *more capable pupils* will reveal quickly their ability to do the operations mentally.

Differentiations and Extensions

1. *More capable pupils* may occasionally help *slower learners* by acting as partners or as group leaders in activities involving certain specific learning processes, as in this lesson. Do not overdo this procedure, as you will also want to provide *slower learners* with opportunities to help themselves. However, experiences demanding cooperation with other pupils is realistic and can be rewarding. Seek in other areas, too, opportunities in which pupils who are *slower learners* in arithmetic, and possibly in other academic areas, may share their interests and abilities with others.

2. *More capable pupils* may undertake a unit of work on consumer education. Many points related to installment buying should be within their understanding. Using whatever general information their parents provide, as well as information from other sources, let them seek instances of advertising concerning such purchases as homes, cars, refrigerators, furniture, clothing, television sets, and the like.

Dividing by Tens and Ones

Trial quotient the true quotient [O]

1. Mr. Dow had to plant 169 onion sets. He put 21 sets in each row. How many full rows did he make?

Why is this measurement division? We find number of equal groups.

We measure 169 by what smaller group? 21

Divide. To divide 169 by 21, *think*, "16 tens ÷ 2 tens = 8." Where is the quotient figure written? Why? ones place (ones) × 2 tens = 16 tens

Multiply and compare. 8 × 21 = 168. Is 168 less than 169? Yes

Subtract and compare. 169 − 168 = 1 (the remainder).

A

$$\begin{array}{r} 8,\ \text{R}1 \\ 21\overline{)169} \\ 168 \quad (8 \times 21) \\ \hline 1 \end{array}$$

Side work

$$\begin{array}{r} 8 \\ 20\overline{)160} \end{array}$$

Check

$$\begin{array}{r} 21 \\ \times 8 \\ \hline 168 \\ +1 \\ \hline 169 \end{array}$$

2. Mrs. Higgins bought a bag of 32 oranges for $1.60. How much did each orange cost?

$1.60 is 160¢, so 1 orange costs $\frac{1}{32}$ of 160¢. Why is this fractional-part division? We find 1/32 of the 32 equal parts of 160.

160¢ ÷ 32 = ? What table number do we use for 16 tens ÷ 3 tens? 15 (tens) Explain box B.

In box C, $1.60 is not changed to 160¢. Explain the division and the 0's in the quotient. [W]

B

$$\begin{array}{r} 5¢ \\ 32\overline{)160¢} \\ 160 \end{array}$$

C

$$\begin{array}{r} \$0.05 \\ 32\overline{)\$1.60} \\ 1\ 60 \end{array}$$

Divide, as in box A.

	a	b	c	d	e	f
3.	40)320 (8)	90)540 (6)	60)427 (7, R7)	82)341 (4, R13)	42)97 (2, R13)	33)$1.32 ($0.04)
4.	52)129 (2, R25)	23)139 (6, R1)	41)248 (6, R2)	64)269 (4, R13)	83)581 (7)	53)$2.12 ($0.04)
5.	71)516 (7, R19)	94)752 (8)	22)157 (7, R3)	72)579 (8, R3)	51)479 (9, R20)	44)$1.76 ($0.04)
6.	$\frac{1}{53}$ of 286 (5, R21)	7. $\frac{1}{72}$ of 448 (6, R16)	8. $\frac{1}{42}$ of 220 (5, R10)	9. $\frac{1}{32}$ of $1.92 ($0.06)		

Extra Practice. Work Set 25.

59

3. **Extra Practice Set 25** may be assigned as needed.

Set 25. 2-place divisors; 1-place quotients without and with remainders; trial quotient the true quotient

Show any remainder with R.

	a	b	c	d	e
1.	5, R4 21)109	8, R1 33)265	$0.09 60)$5.40	$0.07 83)$5.81	7, R4 22)158
2.	3, R1 35)106	5, R30 63)345	9, R30 84)786	6, R1 23)139	9, R38 40)398
3.	$0.08 67)$5.36	8, R24 30)264	$0.06 42)$2.52	8, R23 87)719	6 68)408
4.	9, R68 92)896	$0.09 31)$2.79	6, R5 45)275	9, R42 73)699	9 98)882
5.	$0.06 32)$1.92	4, R32 79)348	8 52)416	5, R5 57)290	4 94)376
6.	9, R70 75)745	$0.09 20)$1.80	2, R21 49)119	4, R55 85)395	7, R4 77)543

Teaching Pages 60, 61, and 62

Pupil's Objectives. (*a*) To rediscover the fact that trial quotients are not always true quotients; (*b*) to work division examples in which the trial quotient must be changed one or more times (1-place quotients); (*c*) to practice estimating the product of a quotient figure times the divisor as a way of identifying true quotients quickly; (*d*) to learn to use the table of 15's when divisors are the numbers 15–19.

Background. So far pupils have reviewed division involving 2-place divisors and 1-place quotients in which dividing by the number in ten's place in the divisor gave the correct trial quotient figure (see box A, page 60 in the text). Now, however, they will meet problems in which this procedure produces a trial quotient figure that is too large (see box B on page 60). The occurrence of this situation is a good time for pupils to discover and state in their own words the fact that the product of a quotient figure and the divisor must not be greater than the part of the dividend used (see the generalization in dark print at the middle of page 60 in the text). If pupils understand this idea, they should realize that the next step would involve using a trial quotient figure one smaller than the one used for their first trial quotient figure.

Dividing by Tens and Ones

Trial quotient not the true quotient [O]

For work like that in box A, you have been thinking, "8 tens ÷ 4 tens," and that gave you the correct quotient figure. But this way of thinking will not always give the correct quotient figure.

1. Mr. Hand packed 294 peaches in boxes of 36 peaches each. How many boxes did he fill?

Box B. When you *think* the division, "29 tens ÷ 3 tens" you get 9 as quotient figure, but $9 \times 36 = 324$. Since 324 is more than 294, there are not as many as nine 36's in 294.

Now try 8, the next smaller number (box C). 8 is the correct quotient because $8 \times 36 = 288$, a number smaller than 294.

A

2, R3
42)87
84
3

B

Trial 1
9
36)294
324 (9 × 36)
[Wrong]

C

Trial 2
8, R6
36)294
288 (8 × 36)
6

The product of the quotient figure and the divisor must not be greater than the part of the dividend used.

[W]

Divide and check. Try to multiply in your head.

	a	b	c	d	e	f
2.	3, R3 27)84	1, R23 26)49	8 23)184	5, R5 25)130	8 36)288	$0.06 24)$1.44
3.	3, R20 24)92	3, R12 23)81	8 26)208	8 68)544	6, R10 45)280	$0.05 36)$1.80
4.	1, R18 28)46	3, R16 22)82	4, R3 27)111	8, R3 27)219	6, R4 58)352	$0.02, R24¢ 48)$1.20
5.	3, R12 25)87	1, R20 22)42	4 38)152	8, R7 14)119	6 47)282	$0.01, R88¢ 96)$1.84
6.	7, R7 49)350	9 76)684	5, R13 39)208	8, R6 47)382	5, R6 36)186	$0.07 46)$3.22

Extra Practice. Work Set 26.

On page 61, pupils are led to realize that sometimes several trials may be necessary before the correct quotient figure is derived; that no quotient figure larger than 9 should be used even though a larger trial quotient figure may appear to be called for; and that estimating mentally will facilitate the writing of a division example.

The procedure for estimating that is shown above the written work on page 61 amounts to multiplying the number of tens in the divisor by the first trial quotient figure and adding to it the product of the same trial quotient figure times the number of ones in the divisor. If this amount is more than the dividend, a smaller trial quotient

True Quotient after Several Trials

1-place quotient [O]

1. 198 ÷ 28 = ? Study the work in the boxes.

Trial 1	Trial 2	Trial 3
9	8	7
28)198	28)198	28)198
252	224	196
		2

19÷2=9,R1 — 9×28=252, too large — next smaller numbers

Why do we begin with 9?
Why isn't 9 correct?
Why do we try 8, then 7?
Tell why 7 is correct. 7×28= 196 which can be subtracted from 198 with R. less than 28.

2. For 208 ÷ 28, why can't we *think*, "20 tens ÷ 2 tens = 10"? can't write 10 in ones place
What is the largest trial quotient figure we should ever use? 9

Work Ex. 3 and 4 on the board.

3. 175 ÷ 28 = 6,R7 4. 272 ÷ 38 = 7,R6

Try no quotient larger than 9.
If the quotient figure is too large, try the next smaller numbers until the product is no longer greater than the part of the dividend used.

You can estimate and avoid erasing. For Ex. 1 *think:*
"9 × 20 = 180, and 9 × 8 = 72; 180 + 70 is more than 198.
8 × 20 = 160, and 8 × 8 = 64; 160 + 60 is more than 198.
7 × 20 = 140, and 7 × 8 = 56; 140 + 50 is less than 198."

[W]

Divide. Estimate to find your quotient figures.

	a	b	c	d	e
5.	5 / 36)180	6,R5 / 35)215	8,R3 / 24)195	7,R23 / 67)492	$0.08,R3¢ / 57)$4.59
6.	6 / 27)162	6 / 28)168	6,R18 / 37)240	8 / 26)208	$0.08,R20¢ / 68)$5.64
7.	9,R10 / 73)667	9,R9 / 27)252	4 / 78)312	7,R19 / 29)222	$0.08 / 49)$3.92
8.	9,R11 / 23)218	8 / 47)376	8 / 38)304	6 / 47)282	$0.07 / 39)$2.73
9.	8,R48 / 58)512	8 / 25)200	8 / 57)456	7,R10 / 25)185	$0.06,R24¢ / 28)$1.92
10.	6,R24 / 26)180	9,R18 / 21)207	8,R9 / 36)297	9 / 34)306	$0.06,R13¢ / 39)$2.47

Teen-Number Divisors

[O]

It is harder to estimate with teen-number divisors because you may have to try many times before you find the correct quotient figure.

1. 92 ÷ 19 = ? Study the work in boxes A–F.

If you first tried 9 as the quotient figure and then kept on trying the next smaller number each time, how many trials would you have to make before you finally got the correct quotient figure? 6

Even if you tried 9 first, why would it be sensible to try 6 next instead of 8 and 7? 9×19 is almost twice the dividend.

If you knew the table of 15's, given below, would you be more likely to use 6 or even 5 as your first trial quotient instead of 9? Why? 15 is smaller than 19 and products for 6 or 5 x 15 are smaller than 92.

15	15	15	15	15	15	15	15	15
×1	×2	×3	×4	×5	×6	×7	×8	×9
15	30	45	60	75	90	105	120	135

Work Ex. 2 and 3 on the board. Tell how it would help you if you knew the table of 15's.

2. 94 ÷ 16 5,R14 3. 120 ÷ 18 6,R12

[W]

Divide. If you do not remember the facts you need from the table of 15's, use the table.

	a	b	c	d
4.	6 / 17)102	7,R12 / 15)117	8,R3 / 13)107	7,R4 / 15)109
5.	8,R6 / 16)134	4,R7 / 14)63	6,R9 / 18)117	6,R3 / 17)105
6.	7,R7 / 19)140	6,R12 / 18)120	7 / 16)112	8 / 14)112
7.	8 / 13)104	6,R14 / 16)110	7,R3 / 18)129	8,R5 / 15)125

Extra Practice. Work Set 27.

A
Trial 1
9
19)92
171

B
Trial 2
8
19)92
152

C
Trial 3
7
19)92
133

D
Trial 4
6
19)92
114

E
Trial 5
5
19)92
95

F
Trial 6
4
19)92
76

figure must be used. The tens are multiplied first because this product often shows immediately that the trial quotient figure is too large.

Since teen-number divisors cause great difficulty, all of page 62 is devoted to a review of this subject. To show the nature of the difficulty, pupils are first confronted with an example the working of which results in six trial quotient figures. Then the table of 15's is introduced to help pupils cut down the amount of work that would be required in this kind of example. If pupils know the products for the table of 15's, they can substitute the number 15 for the divisor whenever a teen-number divisor appears in an example. This will help them select a trial quotient figure which will result in a product somewhere near the dividend being used in the division example. Since the number 15 is about in the middle of the range of teen numbers, using this procedure will sometimes give a trial quotient figure that is too large and sometimes (when the divisors are less than 15) a trial quotient figure that is too small. In the latter case, some pupils, especially *slower learners*, may need help in developing techniques which will help them decide when the trial quotient figure must be *increased*. In addition to using the table of 15's, *more capable children* may discover the very useful technique of rounding the numbers 18 and 19

upward to 20 and the numbers 11 and 12 downward to 10 in order to estimate the first trial quotient figure more precisely.

Book Lesson (page 60). *Ex. 1:* Oral work. *Rows 2–6:* Written work.

1. For rows 2–6, expect *more capable pupils* to multiply mentally. *Slower learners* may need to multiply in a side-work step.

2. Encourage *more capable pupils* who still use side work to abandon this intermediate step.

Book Lesson (page 61). *Ex. 1–4:* Oral work. *Rows 5–10:* Written work.

1. After Ex. 1 and the work in the three boxes has been discussed, ask your pupils to try to work out at the board an example in which a number larger than 9 appears to be the first trial divisor.

Illustration

In dividing 445 by 47, we see that 44 (tens) does not contain 47 ten or more times, so the quotient will be a one's number. We put a small x in one's place to show the location of the quotient figure.

In determining the trial quotient figure, we divide 44 (tens) by 4 (tens), obtaining 11.

Since it is impossible to write 10 or 11 as a 1-place quotient, we realize that 9 must be used as the first trial quotient figure.

2. Before pupils start the written work, give oral practice in estimating, using row 5. They may then work out and leave on the board a guide like the one below to help them estimate quotients.

Trial 1

$$\begin{array}{r} 9 \\ 28\overline{)198} \end{array} \quad \begin{array}{l} 9 \times 2 \text{ tens} = 18 \text{ tens} \\ 9 \times 8 \text{ is about } \underline{\ 7} \text{ tens (ignoring 2 ones)} \\ \underline{25} \text{ tens, or } 250 \end{array}$$

250 is larger than 198.

Trial 2

$$\begin{array}{r} 8 \\ 28\overline{)198} \end{array} \quad \begin{array}{l} 8 \times 2 \text{ tens} = 16 \text{ tens} \\ 8 \times 8 \text{ is about } \underline{\ 6} \text{ tens (ignoring 4 ones)} \\ \underline{22} \text{ tens, or } 220 \end{array}$$

220 is larger than 198.

Trial 3

$$\begin{array}{r} 7 \\ 28\overline{)198} \end{array} \quad \begin{array}{l} 7 \times 2 \text{ tens} = 14 \text{ tens} \\ 7 \times 8 \text{ is about } \underline{\ 5} \text{ tens (ignoring 6 ones)} \\ \underline{19} \text{ tens, or } 190 \end{array}$$

190 is smaller than 198. Therefore, 7 *may* be the true quotient. Multiply and compare to check.

Book Lesson (page 62). *Ex. 1–3:* Oral work. *Rows 4–7:* Written work. Your pupils may work out on the board a plan such as the following to determine whether or not the table of 15's shortens the time required for finding the true quotient for the example in Ex. 1 in the text.

$19\overline{)92}$ In the table of 15's, the nearest product to 92 is 90 (6×15). Use 6 as the first trial quotient figure.

Trial 1

$$\begin{array}{r} 6 \\ 19\overline{)92} \end{array} \quad \begin{array}{l} 6 \times 1 \text{ ten} = \ 6 \text{ tens} \\ 6 \times 9 = 54, \text{ or about } \underline{\ 5} \text{ tens} \\ \underline{11} \text{ tens, or } 110 \end{array}$$

110 is larger than 92.

Trial 2

$$\begin{array}{r} 5 \\ 19\overline{)92} \end{array} \quad \begin{array}{l} 5 \times 1 \text{ ten} = 5 \text{ tens} \\ 5 \times 9 = 45, \text{ or about } \underline{4} \text{ tens} \\ \underline{9} \text{ tens, or } 90 \end{array}$$

90 is smaller than 92. 5 *may* be the true quotient. Multiply. $5 \times 19 = 95$, which is too large.

Trial 3

$$\begin{array}{r} 4 \\ 19\overline{)92} \\ \underline{76} \\ 16 \end{array}$$ 16 is smaller than 19, so the true quotient is 4.

A comparison with boxes A–F on page 62 reveals that with the usual method, 6 trials were necessary. With the help of the table of 15's, only 3 trials were necessary. Therefore, the conclusion may be drawn that the table of 15 shortens the process for some teen-number divisors. Do other examples so that the conclusion may be more thoroughly justified.

Differentiations and Extensions

1. Ask *all pupils* to identify from the following examples those which could probably be done more quickly by using the table of 15's.

a. $18\overline{)159}$	*b.* $12\overline{)110}$	*c.* $24\overline{)192}$
d. $15\overline{)105}$	*e.* $10\overline{)70}$	*f.* $42\overline{)157}$
g. $19\overline{)186}$	*h.* $16\overline{)111}$	*i.* $17\overline{)109}$

2. *More capable children* may strengthen their understanding of division with non-apparent quotient figures and their grasp of the table of 15's by writing problems similar to Ex. 1 on page 60 in the text.

3. **Extra Practice Sets 26 and 27**, reproduced below with answers, may be assigned as needed.

Set 26. 2-place divisors; 1-place quotients on the second trial; some remainders and some money numbers

Show any remainder with R.

	a	b	c	d	e
1.	2, R30 33)96	5, R40 42)250	6, R68 83)566	3, R16 24)88	4, R74 82)402
2.	7 38)266	3, R10 25)85	$0.08 68)$5.44	2, R24 26)76	8, R42 52)458
3.	6, R72 84)576	4, R83 93)455	7, R66 74)584	8, R90 92)826	4, R56 86)400
4.	2, R13 39)91	9, R12 23)219	4, R11 37)159	$0.07, R44¢ 46)$3.66	9, R16 54)502
5.	$0.07 35)$2.45	6, R15 45)285	7, R26 32)250	2, R76 78)232	$0.09 58)$5.22
6.	6, R45 75)495	$0.09 56)$5.04	2, R30 34)98	4, R44 64)300	8, R86 96)854

Set 27. 2-place divisors; 1-place quotients after several trials; remainders and some money numbers

Show any remainder with R.

1.	7, R10 13)101	9, R4 24)220	7, R55 58)461	9, R25 87)808	3, R90 94)372
2.	7, R17 26)199	8, R40 45)400	6, R7 19)121	6, R32 37)254	4, R60 76)364
3.	5, R6 49)251	5, R22 38)212	7, R48 62)482	7, R60 85)655	6, R27 29)201
4.	$0.07 17)$1.19	9, R52 53)529	$0.08 15)$1.20	8, R82 93)826	7, R62 72)566
5.	5, R10 34)180	8, R68 71)636	3, R80 82)326	7, R6 16)118	$0.07 69)$4.83
6.	7, R60 96)732	7, R39 59)452	6, R25 67)427	$0.05 48)$2.40	5, R90 91)545

Using What You Have Learned

Practice: A., S., M., D. [W]

Copy, and do what the signs tell you to do.

	a	b	c	d	e
1.	125 37 +28 190	762 509 +73 1,344	84 132 +3,658 3,874	2,106 795 +5,514 8,415	43,560 8,216 +907 52,683
2.	353 598 +65 1,016	76 463 +677 1,276	2,987 45 +761 3,793	648 83 +7,642 8,373	52,275 774 +9,438 62,487
3.	685 −342 343	1,206 −758 448	3,510 −493 3,017	5,280 −2,655 2,625	23,016 −8,942 14,074
4.	132 ×25 3,300	205 ×86 17,630	1,541 ×73 112,493	864 ×123 106,272	2,048 ×309 632,832

Show any remainder with R.

5.	5, R13 23)128	8, R6 37)302	4, R1 46)185	950, R2 8)7,602	8,402 5)42,010
6.	6, R2 19)116	7, R54 93)705	8, R43 74)635	557, R4 9)5,017	3,410 3)10,230

Distinguishing process

7. A weekly magazine which costs 20¢ on the newsstands can be bought by subscription for $6.00 a year. How much money can be saved in a year by subscribing to the magazine? $4.40

8. A laundry charges 12¢ per sheet and 5¢ per pillowcase. Find the charge for laundering 6 sheets and 12 pillowcases. $1.32

9. Mrs. Tucker pays 60¢ a week for a newspaper delivered to her house each day. The newsstand price of the paper is 5¢ for the daily (Monday-Saturday) paper and 20¢ for Sunday's. How much extra does Mrs. Tucker pay each week for delivery service? 10¢

Make up division problems for the 5 examples in row 5.

63

Teaching Page 63

Pupil's Objectives. (*a*) To maintain skills and abilities in computation involving the four basic processes; (*b*) to obtain practice in solving 2-step problems; (*c*) to make up division problems using a specific set of examples.

Background. This page in the text has been designed to help pupils maintain at a high level the skills and abilities that have already been retaught. Since the abstract practice contains examples with all of the various kinds of difficulties that have been studied, you will want to be careful in limiting the assignment for *slower learners*, making sure that you have representative examples for all the four processes.

Book Lesson. *Ex. 1–9:* Written work.

1. Note that pupils must still show remainders with R. On page 105 in the text, pupils will review showing a remainder in a fraction.

2. Let *slower learners*, as an oral lesson, tell the hidden question in problems 7–9.

Differentiations and Extensions

1. *More capable children* may discuss some of the original division problems they have written (see bottom of the page in the text) to decide

whether the problems are realistic or not. Then they may solve some of these problems.

2. Very likely many of the errors made by *slower learners* will be errors in multiplication and subtraction. Isolated practice in these processes may be of value, but also provide practice set in the framework of the division algorism. Give examples with quotients supplied, as below, so that the multiplication and division steps will receive undivided attention.

$$37\overline{)3{,}478}\quad 94 \qquad 56\overline{)46{,}440}\quad 860 \qquad 42\overline{)39{,}312}\quad 936$$

3. This lesson may be treated as a diagnostic test. Analyze the work done by pupils in order to locate areas that need reteaching. Then plan work that will help children overcome their difficulties. Right at this time, this procedure will seem time-consuming; however, in the long run, it will be the most efficient way of developing arithmetic competence for *all pupils*.

Teaching Pages 64, 65, and 66

Pupil's Objective. To obtain practice in division involving 2-place divisors and 2- or 3-place quotients, first with apparent and then with non-apparent quotient figures.

Background. There are no important new steps in this lesson. Pupils are already familiar with dividing by parts and with the terms *first partial dividend* and *second partial dividend;* they have already used an x as a crutch (see page 52 in the text) to help them decide where the first figure should be written in the quotient; they are familiar with the check for division when there is a remainder; they have just relearned the procedure involved when there are one or more trials in obtaining a quotient figure; and they know how to show dollars and cents in the quotient with a decimal point and dollar sign. Except for many specific details in the procedure for dividing, your main efforts will probably be in helping pupils place the first quotient figure correctly and in guiding them in the use of trial quotient figures for both the first and second partial dividends.

In the work with 3-place quotients on page 66, the ideas and procedures are a logical extension of those developed for dividing by 2-place divisors when 2-place quotients result.

Two-Place Divisors and Quotients

Quotient figures apparent [O]

1. Mr. Ames, the superintendent, had 788 new books to distribute equally among 32 classrooms. How many books did he send to each room?

$\frac{1}{32}$ of 788 = 788 ÷ 32 (box A).

A

Hundreds	Tens	Ones		Check
	2	4,	R20	32
	×			×24
32)7	8	8		128
6	4		(2 tens × 32)	64
1	4	8		768
1	2	8	(4 × 32)	+20
	2	0		788

First Partial Dividend

In 788, are there enough hundreds to give 1 hundred to a room? Yes Are there enough tens to give at least 1 ten to each room? (788 = 78 tens and 8 ones.) Then 78 tens is the first partial dividend, and we can put a small × over the 8 of 78 to show where the first quotient figure is to be placed.

Explain the multiplying and the subtracting.

What figure is brought down to go with the 14 tens? 8 (ones)

Second Partial Dividend

The second partial dividend is 148 ones. Explain.

Explain the dividing, multiplying, and subtracting for the second partial dividend. What is the remainder? 20

Explain the check in box A. Quotient (24) × divisor (32) + remainder (20) = dividend (788)

Explain the divisions below. We usually write our work as in Ex. 3 and 4, without showing the multiplications at the right of the example as we have been doing for 3 tens × 25 in Ex. 2.

2. $25\overline{)765}$ — 30, R15; 75 (3 tens × 25); 15

3. $21\overline{)1{,}596}$ — 76; 147; 126; 126

4. $54\overline{)\$43.70}$ — \$0.80, R50¢; 432; 50

In addition to the skills already developed for dividing when 2-place quotients result, be sure pupils can handle examples such as the one in box B on page 66 in the text in which a zero must be written in the quotient when the second partial dividend cannot be divided. They should also realize in this case that a figure may be brought down from one's place and written beside the second partial dividend to form the third partial dividend and the division continued in the usual way.

Book Lesson (pages 64 and 65). *Ex. 1–8:* Oral work. *Rows 9–13:* Written work.

1. Allow pupils to use a small x in indicating the place where the first figure in the quotient will

Quotient figures non-apparent [O]

5. A truck averaged 28 mi. an hour over a distance of 980 mi. How many hours did it take to cover the distance? 980 ÷ 28 = ? (box B).

What is the first partial dividend? 98 How can you tell? 98 can be divided by 28 in ten's place.

How many figures will the quotient have? 2 (ten's and one's)

Then how many divisions must you make? 2

In the first division, what would be your first trial quotient figure? 4 Why? because 7 ÷ 2 = 4, R1

What is the first true quotient figure? 3 In what column do you write it? tens Why? 1st partial dividend is 98 tens

What is the second partial dividend? 140

In the second division, what would be your first trial quotient figure? 14 ÷ 2 = 7 How many trials do you make? 3 (7, 6, and 5)

6–8. Answer the questions in Ex. 5 for box C; for box D; for box E.

B

35
28)980
84
140
140

C

43
18)774
72
54
54

D

$0.54
26)$14.04
13 0
1 04
1 04

E

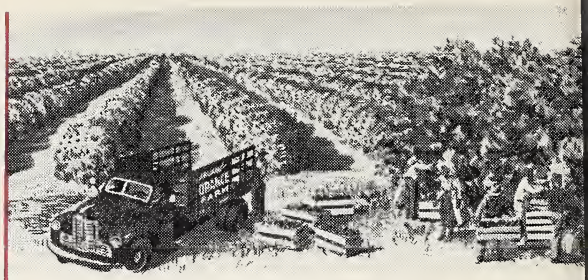
40, R23
39)1,583
1 56
23

[W]

Divide and check. Show any final remainder with R.

	a	b	c	d	e
9.	46, R38 53)2,476	30, R6 24)726	70 27)1,890	98 58)5,684	$0.56 27)$15.12
10.	82 78)6,396	60, R20 37)2,240	84, R40 44)3,736	69, R6 32)2,214	$0.62 48)$29.76
11.	87, R27 58)5,073	94, R21 36)3,405	73 78)5,694	30, R7 28)847	$0.45 16)$7.20
12.	54, R16 28)1,528	40, R16 48)1,936	68, R20 59)4,032	58, R44 51)3,002	$0.64 27)$17.28
13.	60 36)2,160	76 19)1,444	50 68)3,400	37 16)592	$0.46 19)$8.74

Extra Practice. Work Sets 28 and 29.



Three-Place Quotients

Apparent and non-apparent quotient figures [O]

1. An orange grove has 11,622 trees, planted in 78 equal rows. Each row has how many trees?

11,622 ÷ 78 = ? Look at box A.

Why is 116 hundreds the first partial dividend? 116 can be divided by 78 in hundred's place.

The dividend has two more figures. Then how many figures will the quotient have in all? 3 Why? 1 for each partial dividend

How many times do we divide? 3 Explain all the division in box A.

2. Box B. Why do we have to write a 0 in ten's place in the quotient? Explain all the division. 19 ÷ 48 not possible in ten's place.

A

149
78)11,622
7 8
3 82
3 12
702
702

B

204
48)9,792
9 6
192
192

[W]

Divide in rows 3–5. Show any remainder with R.

	a	b	c	d	e
3.	221, R22 26)5,768	190 52)9,880	217 16)3,472	503 23)11,569	$2.10 13)$27.30
4.	400, R18 46)18,418	300 27)8,100	200, R5 48)9,605	876 32)28,032	$4.00 16)$64.00
5.	509, R6 18)9,168	526, R4 17)8,946	687 19)13,053	824, R7 14)11,543	$2.07 28)$57.96

Extra Practice. Work Set 30.

be written only until they can locate the place with assurance, at which time you should encourage them to abandon it.

2. Call particular attention to the following details in the examples at the bottom of page 64:

a. The finding of the first and second partial dividends in all examples

b. The 0 in one's place in the quotient for Ex. 2

c. The placement of the first quotient figure in Ex. 3

d. The occurrence of dollars and cents and the 0 in dollar's place in the quotient for Ex. 4

3. In the oral work at the top of page 65, be sure pupils recall the recently taught method of using trial quotients to find true quotients.

4. Have pupils scan the rows of written work to see if they can find any examples in which the table of 15's might be helpful. Encourage mental calculation as far as possible.

Book Lesson (page 66). *Ex. 1 and 2:* Oral work. *Rows 3–5:* Written work.

Differentiations and Extensions. Assign **Extra Practice Sets 28, 29, and 30** as needed.

Show any remainder with R.

	a	b	c	d	e
1.	11 R 14)154	82, R20 32)2,644	23 26)598	47, R5 51)2,402	83 84)6,972
2.	31, R10 38)1,188	42 23)966	10, R13 17)183	73, R14 87)6,365	32 72)2,304
3.	53, R26 56)2,994	62 79)4,898	55, R20 63)3,485	34 45)1,530	40, R8 98)3,928
4.	32, R10 95)3,050	93, R40 60)5,620	11 75)825	11, R18 36)414	14 67)938
5.	12 83)996	10, R2 54)542	92 90)8,280	11 16)176	42, R15 59)2,493
6.	13 47)611	20 28)560	24 49)1,176	53, R36 76)4,064	42 34)1,428

Set 29. Non-apparent quotients; 2-place quotients and divisors

Show any remainder with R.

1.	\$0.99, R4¢ 24)\$23.80	65, R4 13)849	89, R18 21)1,887	\$0.29 32)\$9.28	80, R37 47)3,797
2.	42 17)714	33, R20 28)944	77, R35 43)3,346	29, R43 64)1,899	24, R14 18)446
3.	57, R26 36)2,078	29, R50 54)1,616	25 39)975	36, R45 75)2,745	79 82)6,478
4.	92, R43 45)4,183	28 26)728	58, R4 62)3,600	46, R67 97)4,529	42 19)798
5.	96, R30 56)5,406	26 35)910	48, R33 87)4,209	29, R22 23)689	89, R86 92)8,274
6.	24 29)696	39, R70 71)2,839	89, R22 66)5,896	70, R35 37)2,625	89, R54 68)6,106

Set 30. Apparent and non-apparent quotient figures; 3-place quotients and 2-place divisors

Show any remainder with R.

	a	b	c	d
1.	305, R4 13)3,969	234, R18 21)4,932	300, R9 36)10,809	\$3.04 49)\$148.96
2.	243, R10 24)5,842	574 17)9,758	532 42)22,344	106, R17 57)6,059
3.	\$440 35)\$15,400	992, R20 64)63,508	369 95)35,055	\$2.07 28)\$57.96
4.	118 68)8,024	469 53)24,857	200, R14 19)3,814	386, R39 47)18,181
5.	497, R66 74)36,844	114, R60 86)9,864	\$2.06 75)\$154.50	139, R52 62)8,670
6.	\$3.23, R44¢ 83)\$268.53	102, R61 79)8,119	594, R22 38)22,594	105 92)9,660

Dividing by Hundreds

Meaning [O]

1. If the machines shown in the picture can fill 600 bags of cement in an hour, how long will it take to fill 1,800 bags? 1,800 ÷ 600 = ?

At the board, work these ways:

Find how many 600's you can subtract from 1,800. 3

Make division tables, as in A, with 6 as divisor and 600 as divisor. How many 600's in 1,800? 3

Divide as in box B, thinking of 1,800 as 18 hundreds. 18 ÷ 6 = 3, so 1,800 ÷ 600 = 3

Why must we write the quotient figure in one's place? 3 X 600 (ones) = 1,800 (ones)

[W]

Copy and divide, as in box B.

A

1	1
6)6	600)600
2	2
6)12	600)1,200
3	3
6)18	600)1,800

B

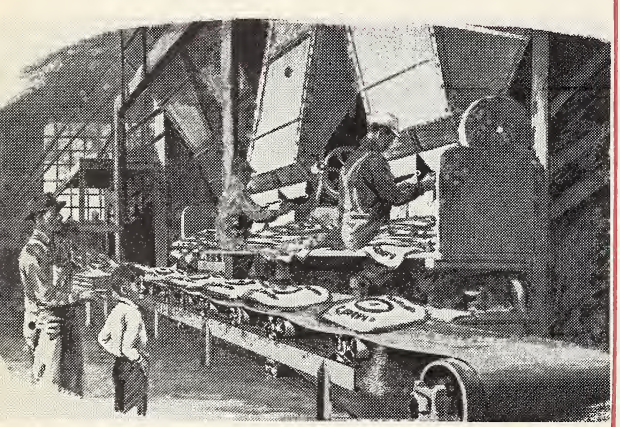
Side work

3 — 3

600)1,800 — 6)18

1 800 (3 × 600)

a	b	c	d
2. 300)1,500 (5)	500)3,500 (7)	400)1,600 (4)	700)6,300 (9)
3. 200)1,800 (9)	600)2,400 (4)	800)3,200 (4)	900)5,400 (6)



Dividing by Any 3-Place Divisor

[O]

1. At an average of 225 mi. a day, how many days will it take to drive 1,350 mi.?

To estimate the quotient figure, you use the first figure of the divisor and the first one or two figures of the partial dividend, just as you would for a 2-place divisor. For box A, *think*, "13 (hundreds) ÷ 2 (hundreds) = ?" 6, R1

Explain the division in box A.

A

6

225)1,350

1 350

B

67

231)15,477

13 86

1 617

1 617

Work these examples on the board:

2. 416)3,509 (8, R181)

3. 384)$26.88 ($0.07)

4. If 15,477 young trees are distributed equally among 231 towns, how many trees should each town receive? 154 hundreds not enough to give 1 hundred to each town.

Box B. Tell why 1,547 tens is the first partial dividend.

How many quotient figures will there be? 2

Work these examples on the board:

5. 320)4,800 (15)

6. 412)$124.47 ($0.30,887)

7. How many barrels of flour (196 lb.) can be made from 54,000 lb. of flour? 540 hundreds

Box C. What is the first partial dividend?

How do we know that there will be 3 figures in the quotient? There will be 3 partial dividends.

What is the last quotient figure? 5

C

275, R 100

196)54,000

39 2

14 80

13 72

1 080

980

100

D

205

384)78,720

76 8

1 920

1 920

8. Box D. There are no 384's in the second partial dividend, 192, so we write 0 in tens place in the quotient. 1920 ÷ 384 = 5

Work these examples on the board:

9. 144)80,784 (561)

10. 365)$642.40 ($1.76)

11. 112)34,496 (308)

12. 519)367,452 (708)

Teaching Pages 67 and 68

Pupil's Objective. To relearn the procedure for dividing by even hundreds and by any 3-place number.

Background. Pupils often have difficulty in placing quotient figures when dividing by 3-place numbers. The division tables suggested in box A and in the discussion beside the box have the advantage of suggesting to children that they may think of dividing hundreds by hundreds just as they divide ones by ones. Box A also shows how the trial figures (the red figures in the pupils' text) help in deciding what the quotient figure will be.

Box A on page 68 contains a division example that is not much more difficult than those shown on page 67, since there is only one figure in the quotient. However, box B shows a more difficult type of example as far as placing the first quotient figure is concerned. The discussion under Ex. 4

will help children decide where the first quotient figure should be placed.

It may be that *slower learners* will need to adopt a somewhat different procedure. Perhaps they should look at the figures in the dividend and think, "15 can't be divided by 231, 154 can't be divided by 231, 1,547 *can* be divided by 231, so the first quotient figure will be placed above the first 7 in the dividend." As for previous examples of this type, these pupils may place a small x above the first 7 if they feel that it is necessary to help them remember where to place the first quotient figure. Then they proceed with the division, using the 2 in the divisor 231 and the 15 in the partial dividend 1,547 as trial figures to help them decide what quotient figure to write above the x.

Note that in box D on page 68 trouble may be caused by a middle zero in a 3-place quotient, but this situation should be no more difficult than the work in box B on page 66 in the text.

Book Lesson (page 67). *Ex. 1:* Oral work. *Rows 2 and 3:* Written work.

Book Lesson (page 68). *Ex. 1–12:* Oral work.

1. For *slower learners*, encourage the use of a small x for locating the place of the first quotient figure until it can be safely discarded.

2. As some pupils work Ex. 2, 3, 5, 6, and 9–12 on the board, have the rest of the class do them at their desks.

Practice in Division

[W]

Divide and check. Show any remainder with R.

	a	b	c	d	e
1.	40, R6 8)326	47 6)282	300, R4 5)1,504	1,316, R4 7)9,216	$2.46 6)$14.76
2.	8, R4 52)420	4, R5 83)337	5 94)470	60, R2 41)2,462	$0.40 62)$24.80
3.	8, R5 35)285	9, R2 26)236	81, R37 47)3,844	60, R12 36)2,172	$0.35 24)$8.40
4.	604 48)28,992	702 15)10,530	800, R18 36)28,818	760, R7 14)10,647	$2.06 12)$24.72
5.	8 347)2,776	6 144)864	7 128)896	23 264)6,072	$4.59 18)$82.62
6.	308 216)66,528	7.	256 421)107,776	8.	$2.78 348)$967.44

Extra Practice. For more practice, work Set 31.

Using Division in Problems

Finding hidden question [O]

Some of these problems have two questions, one of which is hidden and must be answered first. Find each problem that has a hidden question and tell what the hidden question is.

1. A farmer received $76.80 for 160 chickens which averaged 4 lb. each in weight. How much did he get per pound? *How much did chickens weigh altogether? (640 lb.)* $0.12

2. If pears are sold 6 for 36¢, how much will 4 pears cost? *What does 1 pair cost? (6¢)* 24¢

3. If Mr. Mills distributes 272 maps equally among 34 schools, how many maps will each school have? 8

4. If 40 books cost $40.80, what does each book cost? $1.02

5. The cost of 13 pictures for the school at $3.75 each was shared equally by 25 parents. How much did each parent pay? *What was cost of all the pictures? ($48.75)* $1.95

6. Mr. Dow's car cost $2,664. He paid $664 down and the rest in 10 equal payments. How much was each payment? *What did he owe after the down payment? ($2,000)* $200

[W]

Now go back and write the work for each problem.

Teaching Pages 69 and 70

Pupil's Objectives. (*a*) To maintain recently learned skills in division involving 1-, 2-, and 3-place quotients and divisors; (*b*) to use division in solving problems which contain hidden questions; (*c*) to identify the correct process for solving specific problems; (*d*) to use round numbers in estimating answers prior to finding exact answers.

Background. Now that the reteaching program in division has been completed, you will want to discover what progress your pupils have made. The work on page 69 in the text will help you determine how well pupils are able to handle computation and problems which involve division of the kinds just reviewed.

The basic meanings of the four processes are summarized at the top of page 70 in the text. Pupils are to refer to these generalizations in telling what process to use for the problems on the remainder of the page. Note, also, that they are to use rounded numbers in estimating answers for these problems before they turn to the actual solving.

Book Lesson (page 69). *Ex. 1–8* (top): Written work. *Ex. 1–6:* Oral work. *Bottom of the page:* Written work. As an oral lesson preceding the written work at the top of the page, have pupils discuss the examples in column **a**, telling for each example the place in which the first quotient figure would be written and the trial quotient figures that would be used in finding its value.

Add? Subtract? Multiply? Divide?

Differentiating A., S., M., D. [O]

Remember: We add or multiply to find the total.
We subtract to find the remainder or the difference.
We divide to find how many times one group contains another group or to find the size of one of the equal parts of a group.

For each problem tell what process you will use and give the reason for your choice. Then use round numbers and estimate the answer.

1. On the average, Mr. Lane's car goes 16 mi. on a gallon of gasoline. At that rate, about how much gasoline would be used on a trip of 1,500 mi.? D.; (100); 93, R12, or 94 gal.

2. Mr. Lane's car cost $2,664. He paid $650 in cash for the first payment. How much more did he owe? S.; ($2,000); $2,014

3. If each shelf in the library holds 38 books, on the average, how many books will 22 shelves hold? M.; (800); 836

4. Rita made 16 sandwiches for the picnic, and her mother made 28. How many sandwiches did they make in all? A.; (45); 44

5. A farmer sold 8 doz. eggs at 51¢ a dozen. How much money did he receive for the eggs? M.; ($4); $4.08

6. A printer cuts 72 small cards from each large sheet. How many large sheets will he need for 2,232 small cards? D.; (30); 31

7. In the scrap-paper drive, the 6th-grade children collected 535 lb. of scrap paper, and the 5th grade collected 475 lb. How many pounds was that from the 5th and 6th grades? A.; (1,000); 1,010

8. A fish dealer paid the fishermen 63¢ a pound for fish which he sold for 95¢ a pound. How much more did the dealer charge per pound for the fish than he paid for it? S.; (30¢); 32¢

[W]

Now go back and solve problems 1–8.

Book Lesson (page 70). *Ex. 1–8:* Oral work. *Bottom of the page:* Written work.

1. Be sure pupils are thoroughly familiar with the generalizations in the box at the top of page 70. Let them use objects or semi-concrete symbols (such as dots or X's) to demonstrate each generalization. Then have them complete the chart which follows as a way of showing that they understand the meanings which are involved in the four basic operations.

a. When we *put together* smaller groups to make a larger group and
the smaller groups are unequal in size we _? _.
the smaller groups are equal in size we _? _.

b. When we *take apart* a larger group to make one or more smaller groups and
the smaller groups are unequal in size we _? _.
the smaller groups are equal in size we _? _.

2. Caution pupils regarding the use of cue words. For example, point out to them that the word "average" is used in both problems 1 and 3. See if they realize that the same process is not used in solving these two problems. Cite this as evidence that all problems must be read carefully.

3. Before pupils start the written work let them discuss the advantages of estimating answers using rounded numbers, of checking computation, and of checking the sense of the final answer.

Differentiations and Extensions

1. *More capable learners* may rewrite the 2-step problems at the bottom of page 69 in the text so that each has two questions. Ex. 1 might read:

a. 160 chickens averaged 4 pounds each in weight. What was their total weight? *b.* If a farmer received $76.80 for all the chickens, how much would he get per pound?

Slower learners may then solve these new problems. As you compare this work with that done in the text, you may discover which pupils are having difficulty because of hidden questions.

2. Emphasize with *all pupils* the importance of finding accurate answers in computation and problem-solving. At every opportunity during your activities, such as collecting milk money, or sending reports to the school office, have pupils check computations with you so that they will develop a feeling of responsibility for correct accounting.

3. **Extra Practice Set 31**, reproduced below, may be assigned for further practice in division.

Set 31. 3-place divisors; 1-, 2-, and 3-place quotients; apparent and non-apparent quotients

Show any remainder with R.

	a	b	c	d
1.	20, R18 124)2,498	34, R181 316)10,925	258 235)60,630	$2.39 403)$963.17
2.	905, R450 542)490,960	137, R450 651)89,637	$3.37 179)$603.23	36, R500 890)32,540
3.	901, R190 768)692,158	10, R106 982)9,926	260 625)162,500	$0.24 358)$85.92
4.	$1.24 963)$1,194.12	58 736)42,688	9, R105 195)1,860	85 847)71,995
5.	8, R32 462)3,728	209, R124 578)120,926	90, R493 819)74,203	379, R36 282)106,914

Teaching Pages 71, 72, and 73

Pupil's Objectives. (*a*) To take Inventory Test 9 to determine the level of understanding and ability so far developed in work with measurement numbers; (*b*) to compare counting numbers and measurement numbers; (*c*) to become reacquainted with the meaning of units of measure.

Background. The pages devoted to the reteaching of measurement are introduced by an inventory test which should serve to help you uncover the areas most seriously in need of review.

Strange as it may seem, many pupils never discover the difference between counting and measuring numbers. Begin by making the point that numbers are used to help describe quantitative situations. Then lead pupils to realize that if one should throw 8 cubes on the table, the situation could be described by counting the number of cubes and communicating the information using the number 8. However, if one were to ask the length of a table, it should become immediately apparent that there is nothing to be counted and, therefore, on the face of it, no convenient way of describing length in quantitative terms.

It took man a great many years to develop techniques for applying numbers to measurement situations. For example, he found he could refer to some object which was familiar to all and count the number of times that the object could be applied to the distance. Then, when this information was communicated, the recipient had some idea as to the length of the object being compared. It can be seen immediately that in order to describe quantitatively certain continuous situations, such as those that involve length, height, weight, etc., there must be some standard unit familiar to all which is useful in communicating information about size. It can also be seen that measurement numbers can never be accurate in the sense that counting numbers can be accurate. The person doing the measurement must make some sort of decision regarding how accurate he is going to be in applying the unit of measurement to the object that is to be measured.

If pupils understand the basic differences between measurement numbers and counting numbers, they will be less confused by the fact that measurement numbers are not based on ten (some, like pounds, being based on 16; others, like feet, being based on 12, etc.). They will also obtain a better feeling for the use of measurement numbers in problem situations.

Teacher's Preparation. You and your pupils may want to start collecting teaching aids that will be valuable, and in some cases essential, for the unit on measurement which begins in the text with this lesson. See Part IV of this Manual, pages 301–325, under *Equipment* (Measures). Also see under *Devices* (Area Squares), *Commercial Teaching Aids* (Films, Filmstrips, and Charts), and *Bibliography* (Reference Materials).

(*Continued on page 94, second column*)

When We Measure

Inventory Test 9 [W]

1. Write five units of length and their abbreviations.

2. There are 5,280 ft. in a mile. How many yards are there? 1,760

3. If 2 lb. of candy are shared equally by 4 girls, how many ounces of candy does each girl receive? 8

4. What do these abbreviations stand for? **a.** gal.; **b.** pk.; **c.** pt.; **d.** lb.; **e.** bu.; **f.** sec.; **g.** T.; **h.** da.; **i.** qt.; **j.** oz.

5. 1 gal. = 8 pt; 1 bu. = 32 qt.

6. Bob studies 45 min. each night. In 4 nights he studies 3 hr.

7. Which of these months have 30 days: January? June? April? October? September? May? November?

8. Four men were 64 in., 69 in., 70 in., and 73 in. tall. Their average height was 5 ft. 9 in.

9. Using the tables at the right and below, find the following:

a. 4 lb. 10 oz. are 74 oz. in all.
b. 3 T. = 6,000 lb.
c. 264 pt. = 33 gal.
d. 8 da. = 192 hr.
e. How much more are 3 bu. than 8 pk.? 1 bu.
f. How many hours and minutes is it from 11 A.M. to 2:15 P.M.? 3 hr. 15 min.
g. 3,860 lb. is how much less than 2 T.? 140 lb.
h. 2 rd. = 11 yd., so 10 rd. = 55 yd.

Linear Measure
12 in. = 1 ft.
3 ft. = 1 yd.
5½ yd. = 1 rd.
320 rd. = 1 mi.

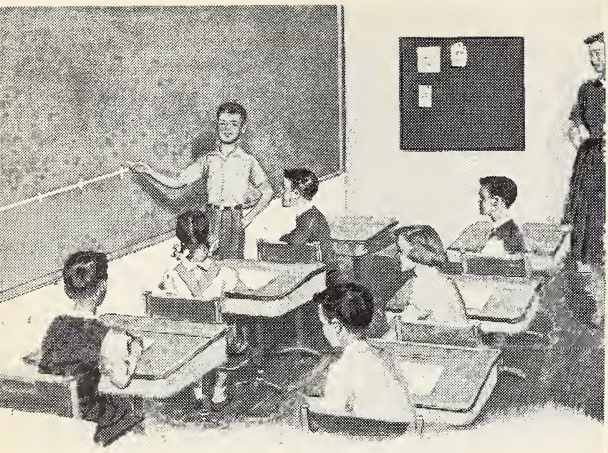
Liquid Measure
2 pt. = 1 qt.
4 qt. = 1 gal.

Weight
16 oz. = 1 lb.
2,000 lb. = 1 T.

Time
60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.

Dry Measure
2 pt. = 1 qt.
8 qt. = 1 pk.
4 pk. = 1 bu.

71



What Is Measuring?

Meaning of counting and measuring numbers [O]

1. How many children are shown in the picture? 8 How did you find out? by counting ✓ Is there more than one correct answer? No Explain. Counting numbers are exact. ✓

2. If you want to know the length of a line drawn on the board, is there anything that you can count? No First you must have a unit of measure, such as a foot rule, with which you can compare the length of the line. The line on the board above has been marked off in feet, so you can quickly tell that it is about 3 ft. long. Is that its *exact* length? No

3. Suppose three different people measured the line, one using a yardstick marked off in fourths of an inch, one using a steel tape marked off in tenths of an inch, and one using a ruler marked off in sixteenths of an inch. Do you think that all three would get *exactly* the same answer? No Give as many reasons for your answer as you can. ★Can we ever measure anything so that the answer is absolutely correct? No

Numbers that tell *how many* are **counting numbers.** Numbers that tell *how much* and that are found by comparing the size of something with a unit of measure are **measuring numbers.**

Counting numbers are accurate. Measuring numbers never can be expressed with entire accuracy.

Before we had our present measuring instruments, people often used parts of their bodies as units of measure. Do we ever do this today? Yes

4. Use the width of your palm to measure the length of your desk. How many palms long is your desk?

5. Using your foot as a unit of measure, find about how many feet long your classroom is.

6. Why would answers differ for Ex. 5? Body measurements differ in different people.

7. Why was it a good idea that people decided on standard units of measure like the inch and the ounce, that is, measures which are the same everywhere and for everybody? for agreement in communicating

8. Name 5 standard units of measure we use today. ✓ i.e., inch, ounce, pint, minute, pound, etc. [W]

Would you use a counting or a measuring number to tell

9. the length of a rope? measuring (inch, foot, or yard)
10. the weight of a package? measuring (ounce or pound)
11. the number of books on your desk? counting
12. the length of time you spend studying? measuring (minute or hour)
13. the number of quart bottles of milk on the doorstep? counting
14. the amount of grain in a bin? measuring (peck, bushel, or pound)

Now look again at Ex. 9–14. For each question that you answered with "A measuring number," tell what unit of measure you would use.

★Answers Not on Reproduced Page 72

3. Measurements given will differ according to degree of accuracy of measuring instrument, and also according to the care used by the person measuring. Moreover, the unit of measure used will affect the answers given. For example, the yardstick with $\frac{1}{4}$-inch divisions will allow measurement to nearest $\frac{1}{4}$ inch, or estimated measurement to nearest $\frac{1}{8}$ inch; the steel tape with $\frac{1}{10}$-inch divisions will allow measurement to nearest $\frac{1}{10}$ inch, or estimated measurement to nearest $\frac{1}{20}$ inch; and the ruler with $\frac{1}{16}$-inch divisions will allow measurement to nearest $\frac{1}{16}$ inch, or estimated measurement to nearest $\frac{1}{32}$ inch.

Pre-book Lesson. As you start to collect materials, allow pupils (particularly *slower learners*) to help. This organization, plus the responsibility of maintaining it, may provide many opportunities for learning.

Book Lesson (page 71). *Ex. 1–9:* Written work. Inventory Test 9. Before pupils start the test, refer to the tables of measures presented in the boxes, discussing each one as completely as you think necessary. Be sure pupils are familiar with all of the measurement abbreviations.

Book Lesson (pages 72 and 73). *Ex. 1–8:* Oral work. *Ex. 9–14:* Written work.

1. Have two or more pupils measure the lengths of some objects in the room. Select objects such that pupils will have to use the inch, foot, and yard units of measure. Expect careful measurement, but be sure pupils note discrepancies and the difficulty of obtaining exact agreement when the same object is measured by different pupils.

2. Remind pupils that *counting numbers* tell *how many*, and that *measurement numbers* tell *how much.*

Differentiations and Extensions

1. Give a few brief directions designed to ascertain if pupils can accurately read their rulers, providing help for *slower learners* as needed.

2. *More capable pupils* may:

a. Collect and organize information to present to others concerning the responsibility of the Federal Bureau of Standards and the State, County, or Municipal Sealers of Weights and Measures.

b. Use children's encyclopedias or other reference books to obtain information about the history of measures, the early measures used, and the outgrowth of standard units in our country and in other countries.

c. Investigate the meaning of, and define, the following terms: *cubit, meter, furlong, nautical mile, knot, fathom, light year, gram, carat.* Some pupils may be able to locate rulers showing inches and metric equivalents.

Teaching Pages 74 and 75

Pupil's Objectives. (*a*) To develop an assortment of reference measures; (*b*) to compare estimates to actual measurements.

Background. Measuring can be thought of as a comparison process. You can either estimate the length of a rope by comparing it with nearby objects with which you are familiar, or you may compare it with a tape measure in order to discover the number of standard units that would best describe its length. If I tell you that I have measured a piece of rope and found it to be "17 sticks" in length, you are completely in the dark as to its size. In the first place, since you cannot see the rope, you cannot compare it with other nearby objects in estimating its length, and, secondly, you do not know the size of the unit that I have used (the stick). These considerations show that much experience in making measurement comparisons is the key to facility in measurement, for experience provides familiarity with sizes of objects and sizes of standard units of measure.

The reference measures that have been introduced on pages 74 and 75 in the text are extremely useful. They are similar to the measures that have been used historically and that are, therefore, the basis for our present standard units of measure. Also, the use of these reference measures will help children develop skill in estimating heights, weights, capacities, and the like, when

(*Continued on page 96, second column*)

Reference Measures You Can Use

Estimating [O]

You often need to estimate how much of something you have when no measuring instrument, such as a yardstick or quart bottle, is at hand. These reference measures may help you:

Find a part of your finger that is about an **inch** long.
See if a man's shoe is about a **foot** long.
Find one of your books that weighs about a **pound.**
Weigh 4 new pencils to see if they weigh about an **ounce.**

Many familiar things make good reference measures. For example, most doors are about **7 ft.** high, and most tables are **30 in.** high. A bag of sugar weighs **5 lb.** or **10 lb.**, and a very large bag of potatoes weighs **100 lb.**

Tom thinks that the corn is 9 ft. tall because it seems 2 ft. taller than a door. Sue thinks the weeds by the pond are 60 in. tall because they seem twice as high as a table.

1. In this way estimate the length of your schoolroom.

2. By lifting a 10-pound bag of sugar and a pail of water, estimate the weight of the pail of water.

3. Pour sand into a can until you think you have put in 5 lb. Check by weighing.

4. Without a ruler, try to draw a 6-inch line. Check with your ruler. Then repeat the work to see if you improve.

5. As in Ex. 4, draw at the board a 2-foot line. Check it.

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6. What reference measure would you use to estimate the
a. length of your pencil? finger joint b. weight of a jackknife? 4 new pencils
c. weight of meat bought? book or bag of sugar d. height of your room? door

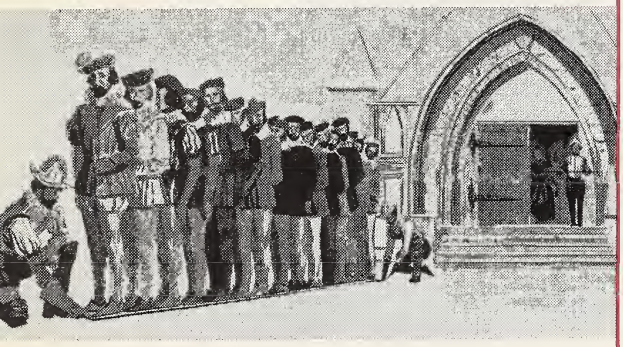
7. Give reference measures you might use for these units: ★

a. a mile	b. a yard	c. a rod	d. a second
e. an hour	f. a gallon	g. a minute	h. a quart

Use a reference measure if you need to and tell what goes in each blank.

8. The box of cereal weighs 10 ounces.
9. Jack weighs 120 pounds. His baby brother weighs 120 ounces.
10. Sam's belt is 30 inches long.
11. A small car ran about 100 mi. on 3 gallons of gasoline.
12. Barbara went down to the store and back in 20 minutes.

Some measures of length were crudely established one Sunday morning in the 16th century at the decree of a king. The total length of the left feet of the first 16 men coming out of church was declared to be the lawful rod, and the sixteenth part of it was declared to be the lawful foot.



★Answers Not on Reproduced Page 75

7. These answers are among various possibilities.

a. Mile. Distance between two local landmarks known to be about a mile.

b. Yard. Distance from tip of nose to end of outstretched arm.

c. Rod. Width of street about $16\frac{1}{2}$ feet wide.

d. Second. Time taken to say "one thousand one."

e. Hour. Time taken by an hour-long program, or time to walk a distance known to take about an hour.

f. Gallon. Amount of liquid required to fill a gallon can, or to fill four quart bottles.

g. Minute. Time taken in counting slowly to 60.

h. Quart. Amount of liquid contained in a quart milk bottle, or capacity of a quart berry box.

approximations, not exact measurements, are the most useful form for answers.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7–12:* Written work.

1. Have each pupil measure his arm, or part of his arm, in order to find reference measures approximating a foot and a yard. Pupils should realize that these reference measures will have to be changed as growth takes place.

2. Extend Ex. 6 to include other objects of varying lengths.

3. Let pupils discuss the historical significance of the scene pictured at the bottom of page 75 in the text.

Differentiations and Extensions. *All pupils* may:

a. Make charts, possibly illustrated, with information about reference measures and standard measures, both for classroom display and for individual arithmetic notebooks.

b. Fill identical containers with contents varying in respect to weight. Experiences in lifting them and in weighing them will help pupils differentiate between capacity and weight. Use as many different kinds of balances or scales as possible.

c. Compare labels on commercial products distinguishing between net and gross measures. Help them become aware of methods of packaging that make the contents appear to be more than they actually are.

d. Engage in contests to determine who can most closely approximate actual measures.

e. Determine the amount of oil stored in oil tanks at home or at school and in automobile gasoline tanks.

f. Enjoy hearing a Sealer of Weights and Measures talk about his work. (The owner of any store which uses scales can tell you who the Sealer of Weights and Measures is.)

Teaching Pages 76 and 77

Pupil's Objective. To relearn the meaning of, and to obtain practice in, changing larger units of measure to smaller units and smaller units of measure to larger units.

Background. Pupils are already used to thinking of tens in smaller units (ones) or larger units (hundreds, etc.). Now this same idea will be

Changing to Other Units of Measure

Larger units to smaller; meaning [O]

1. One hot day in August, the Jackson Ice Cream Stand sold 43 gallons of ice cream. This was how many quarts?

The picture above shows 43 gallon containers against the wall and the picture on page 77 shows the quart containers needed for the same amount of ice cream. Each time you look at one of the gallon containers, *think*, "4 quarts." For 43 gallons there will be 43 of these 4's. 43×4, or 4×43, = 172

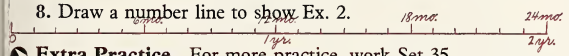
Because each larger unit of measure equals a fixed number of smaller units, we multiply to change from larger units to smaller units. The smaller the unit used, the greater is the number of units.

[W]

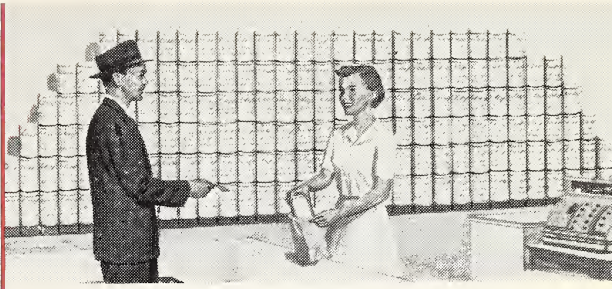
Make these changes. Use page 71 if you need help.

2. 2 years to 24 months
3. 8 tons to 16,000 pounds
4. 5 lb. 4 oz. to 84 ounces
5. 4 miles to 21,120 feet
6. 7 bu. 3 pk. to 31 pecks
7. 6 hr. 48 min. to 408 minutes

8. Draw a number line to show Ex. 2.



Extra Practice. For more practice, work Set 35.



Smaller units to larger; meaning [O]

9. Suppose Ex. 1 had started this way: One hot day in August, the Jackson Ice Cream Stand sold 172 quarts of ice cream. This was how many gallons?

Whenever you look at 4 of the quart containers, you can *think*, "1 gallon." Then for every 4 of the 172 there will be 1 gallon. How can you find the number of 4's in 172? Study the work in the box. Explain what the quotient means. 43 qt.

```
  43
4)172
 16
 12
 12
```

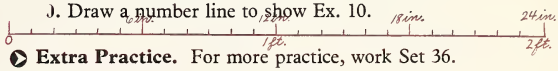
Because a fixed number of smaller units of measure equals a larger unit, we divide to change from smaller units of measure to larger units. The larger the unit used, the smaller is the number of units.

[W]

Make these changes. Use page 71 if you need help.

10. 24 inches to 2 feet
11. 48 months to 4 years
12. 1,095 days to 3 years
13. 300 pecks to 75 bushels
14. 360 minutes to 6 hours
15. 93 oz. to 5 pounds and 13 ounces
16. 80 ft. to 26 yards and 2 feet
17. 24 da. to 3 weeks and 3 days
18. 72 qt. to 18 gallons
19. 86 pt. to 43 quarts

J. Draw a number line to show Ex. 10.



Extra Practice. For more practice, work Set 36.

carried over and used in reviewing the changing of units of measure to larger or smaller units (see the dark print on pages 76 and 77 in the text). The only added source of difficulty in working with measures is that, unlike our decimal system which has a base of 10, the base for measurement units changes according to the measure being used. For example, when we are changing feet to inches, or inches to feet, we must think in terms of the base 12; when we are changing pounds to ounces, or ounces to pounds, we must think in terms of the base 16; while in changing days to weeks, or weeks to days, we use the base 7.

It may be noted here that if pupils are familiar with the idea behind changing units of measure, the information required for the actual computation may always be found in tables of measures. Pupils should, however, be encouraged to learn those tables of measure most commonly used.

Pre-book Lesson. Provide materials (such as gallon and quart containers, yardsticks and foot

rulers, etc.) which pupils can use in discovering the equivalents in the more common tables of measures. The equivalents thus found may be drawn together in table form and preserved for future use.

Book Lesson. *Ex. 1:* Oral work. *Ex. 2–8:* Written work. *Ex. 9:* Oral work. *Ex. 10–20:* Written work. If necessary, *slower learners* may use side work in the way shown below for Ex. 2 and Ex. 10 in the text.

Ex. 2	*Side Work*
1 year = 12 months	12
2 years = _?_ months	× 2
	24

Ex. 10	*Side Work*
12 in. = 1 ft.	2
24 in. = _?_ ft.	12)24
	24

Differentiations and Extensions

1. *All pupils* may:

a. Experiment with a balance, making 1 large weight equal 2 smaller weights, etc.

b. Write sentences containing descriptive information about themselves which require changing larger units to smaller units, as illustrated below.

I am _?_ years and _?_ months old, or _?_ months.
I weigh _?_ pounds and _?_ ounces, or _?_ ounces.
My belt is _?_ inches long, or _?_ feet and _?_ inches.

2. Assign **Extra Practice Sets 35 and 36** as needed.

Set 35. Changing larger units of measure to smaller units

	a	b	c
1.	2 ft. = 24 in.	1 qt. = 2 pt.	2 pk. = 16 qt.
2.	2 T. = 4,000 lb.	5 gal. = 20 qt.	2 hr. = 7,200 sec.
3.	3 bu. = 96 qt.	1 yd. = 36 in.	2 gal. = 16 pt.
4.	3 mi. = 15,840 ft.	2 da. = 48 hr.	3 lb. = 48 oz.

Set 36. Changing smaller units of measure to larger units

	a	b
1.	75 sec. = 1 min. 15 sec.	20 oz. = 1 lb. 4 oz.
2.	17 pt. = 8 qt. 1 pt.	21 da. = 3 wk. 0 da.
3.	3,000 lb. = 1 T. 1,000 lb.	18 pk. = 4 bu. 2 pk.
4.	80 min. = 1 hr. 20 min.	2,800 lb. = 1 T. 800 lb.

Finding the Total

A. of denominate numbers [O]

1. Find the total weight of tomatoes Mrs. Hanes is buying.

In box A, adding ounces and pounds is somewhat like adding ones and *tens*. Why do we not have to carry a 1 from the 12 ounces to the pound's column? Explain the addition. *It takes 16 oz. to make 1 lb.*

A
1 lb. 8 oz.
+1 lb. 4 oz.
2 lb. 12 oz.

2. Mrs. Hanes bought 2 lb. 6 oz. of ham and 1 lb. 12 oz. of steak. How much meat in all did Mrs. Hanes buy (box B)?

Work Ex. 2 at the board. *Think*, "6 oz. + 12 oz. = 18 oz. 18 oz. = 1 lb. 2 oz." You write "2 oz." in ounce's column and carry 1 (the 1 pound) to pound's column.

B		C	
1	Side work	1	Side work
2 lb. 6 oz.	16 oz. = 1 lb.,	12 yr. 7 mo.	12 mo. = 1 yr.,
+1 lb. 12 oz.	so 18 oz. =	+2 yr. 8 mo.	so 15 mo. =
4 lb. 2 oz.	1 lb. 2 oz.	15 yr. 3 mo.	1 yr. 3 mo.

3. Sue is 12 yr. 7 mo. old. Kay is 2 yr. 8 mo. older. How old is Kay? Explain Ex. C. How could you check it? *Add again.*

[W]

Add. Think carefully about the carrying.

4.	5.	6.
6 bu. 3 pk.	5 gal. 2 qt.	7 ft. 9 in.
+8 bu. 2 pk.	+7 gal. 3 qt.	+5 ft. 6 in.
15 bu. 1 pk.	*13 gal. 1 qt.*	*13 ft. 3 in.*

Extra Practice. For more practice, work Set 33.

78

Teaching Pages 78 and 79

Pupil's Objective. To relearn how to add and subtract measurement numbers, without and with carrying and borrowing.

Background. Addition and subtraction of denominate numbers is no more difficult than addition and subtraction of whole numbers, except that the basis for regrouping (carrying and borrowing) is determined by the measures being used. Since pupils have already had experience in adding and subtracting whole numbers using bundles

of sticks, the best way to make crystal clear the importance of the regrouping for measurement numbers is to use bundles grouped for specific units of measures (see the Pre-book Lesson).

Pre-book Lesson

1. Present a problem such as the following:

How much must be cut from a board 3 feet 2 inches in length in order to make a shelf 1 foot 9 inches long?

Lead pupils to the realization that they must bundle together three groups of 12 sticks each to stand for the 3 feet and that they must leave 2 sticks loose to stand for the 2 inches remaining. They will quickly realize that they cannot take 9 sticks from the two separate sticks. Basing their thinking on previous experiences with whole-number operations of this kind, they should suggest borrowing a bundle of 12 sticks in order to make enough from which to borrow. Point out emphatically that they are borrowing 12 sticks rather than 10. When the regrouping is completed, they will have 14 sticks from which 9 may be taken, leaving 5 sticks. If they then take away one bundle to stand for the 12 feet in the length of the shelf, the remainder will be one bundle and 5 separate sticks, or 1 foot 5 inches. Show the written form as below so that pupils will see its relation to the work with bundle numbers.

$$\begin{array}{r} {\scriptstyle 2}\ \ {\scriptstyle (14)} \\ 3 \text{ feet } \not{2} \text{ inches} \\ -\ 1 \text{ foot } 9 \text{ inches} \\ \hline 1 \text{ foot } 5 \text{ inches} \end{array}$$

Proceed in the same way using other units of measure so that children will realize how important knowing the equivalents in the tables of measures is in working with measures.

2. Give a short check to determine if pupils know the equivalents in the tables shown on page 71 in the text.

Illustration

Write the missing number in Ex. *a–f*.

a. 1 ft. = _? _ in.	*d.* 8 qt. = _? _ pk.
b. 1 yd. = _? _ ft.	*e.* 1 T. = _? _ lb.
c. 1 qt. = _? _ pt.	*f.* 1 gal. = _? _ qt.

In Ex. *g–n*, which is more, and how much more?

g. 1 hour or 70 min.	*k.* 1 yd. or 27 in.
h. 7 pecks or 1 bu.	*l.* 18 oz. or 1 lb.
i. 1 pt. or 1 qt.	*m.* 1 ton or 1,000 lb.
j. 2 ft. or 1 yd.	*n.* 1 gal. or 7 qt.

Finding the Remainder or Difference

S. of denominate numbers [O]

1. A can marked "1 lb. 4 oz." weighs how much less than one marked "1 lb. 8 oz."?

Explain the subtraction in box A.

A

1 lb. 8 oz.
−1 lb. 4 oz.
4 oz.

2. A book weighing 3 lb. 4 oz. is how much heavier than a book weighing 1 lb. 8 oz.?

In box B, can you subtract 8 oz. from 4 oz.? No What do you borrow from 3 lb.? 1 lb. How is this different from borrowing a ten? 1 ten = 10 ones; 1 lb. = 16 oz. 1 lb. or 16 oz.

Explain all the work in box B.

B

	Side work
2 lb. (20 oz.)	Borrow 1 lb., or 16 oz.
~~3 lb.~~ ~~4 oz.~~	4 oz.
−1 lb. 8 oz.	+ 16 oz.
1 lb. 12 oz.	20 oz.

3. Today Sue played her violin for 50 min. Kay played the piano for 2 hr. 10 min. Kay has played how much longer than Sue?

In box C you borrow an hour and change it to 60 min. Why? You cannot subtract 50 from 10.

Explain the subtraction in box C. Then explain how you would check the work. Add remainder and subtrahend to see if it equals minuend.

C

	Side work
1 hr. (70 min.)	Borrow 1 hr., or 60 min.
~~2 hr.~~ ~~10 min.~~	10 min.
− 50 min.	+ 60 min.
1 hr. 20 min.	70 min.

[W]

Subtract. Think how many of the smaller units of measure there are in the 1 larger unit you have to borrow.

	a	b	c
4.	6 yd. 1 ft.	12 lb. 4 oz.	10 min.
	−2 yd. 2 ft. (3 yd. 2 ft.)	−7 lb. 6 oz. (4 lb. 14 oz.)	−3 min. 48 sec. (6 min. 12 sec.)
5.	1 gal. 1 qt.	5 T. 300 lb.	5 da. 2 hr. 12 min.
	−3 qt. (2 qt.)	−1,500 lb. (4 T. 800 lb.)	−3 da. 5 hr. 30 min. (1 da. 20 hr. 42 min.)

Extra Practice. For more practice, work Set 34.

Book Lesson (page 78). *Ex. 1–3:* Oral work. *Ex. 4–6:* Written work. Have *all pupils* do the side work at first so that you can follow their thinking and help them to establish a pattern. *More capable pupils* may soon discard it.

Book Lesson (page 79). *Ex. 1–3:* Oral work. *Rows 4 and 5:* Written work. Have *all pupils* consider, in broad terms, the activities of a typical 24-hour day. Let each pupil summarize his own report, making sure that the total is 24 hours. Assuming that the reporting is fairly accurate, the results should be significant as a personality study of each individual and in providing practice in arithmetic.

Differentiations and Extensions. Extra Practice Sets 33 and 34, reproduced below, may be assigned for further practice.

Set 33. Addition of denominate numbers

Show side work and write each sum in the best form.

	a	b	c
1.	2 hr. 42 min. + 1 hr. 35 min. **4 hr. 17 min.**	6 da. 15 hr. + 2 da. 12 hr. **9 da. 3 hr.**	1 T. 1,200 lb. + 2 T. 1,800 lb. **4 T. 1,000 lb.** **(or $4\frac{1}{2}$ T.)**
2.	3 pt. 1 c. 2 pt. 1 c. + 1 pt. 1 c. **3 qt. 1 pt. 1 c.** **(or 7 pt. 1 c.)**	1 bu. 3 pk. 2 bu. 1 pk. + 3 pk. **4 bu. 3 pk.**	3 qt. 1 pt. 1 qt. 1 pt. + 1 qt. 1 pt. **1 gal. 2 qt. 1 pt.** **(or 6 qt. 1 pt.)**
3.	1 yd. 2 ft. 8 in. 2 ft. 10 in. + 2 yd. 1 ft. 5 in. **5 yd. 11 in.**	2 gal. 3 qt. 1 pt. 1 gal. 2 qt. 1 pt. + 1 qt. 1 pt. **4 gal. 3 qt. 1 pt.**	5 bu. 3 pk. 2 qt. 1 bu. 1 pk. 3 qt. + 2 bu. 3 pk. 5 qt. **10 bu. 2 qt.**

Set 34. Subtraction of denominate numbers

Show side work for each example.

1.	6 yd. 1 ft. − 2 yd. 2 ft. **3 yd. 2 ft.**	10 min. − 3 min. 48 sec. **6 min. 12 sec.**	1 gal. 1 qt. − 3 qt. **2 qt.**
2.	5 hr. 10 min. − 2 hr. 40 min. **2 hr. 30 min.** **(or $2\frac{1}{2}$ hr.)**	4 mi. 3,000 ft. − 4,000 ft. **3 mi. 4,280 ft.**	8 lb. − 3 lb. 9 oz. **4 lb. 7 oz.**
3.	3 pk. 2 qt. − 1 pk. 3 qt. **1 pk. 7 qt.**	9 ft. − 6 ft. 9 in. **2 ft. 3 in.** **(or $2\frac{1}{4}$ ft.)**	7 qt. 1 pt. − 2 qt. 1 pt. **5 qt.** **(or 1 gal. 1 qt.)**
4.	3 qt. 1 pt. 1 c. − 1 qt. 1 pt. 1 c. **2 qt.**	2 yd. 2 ft. 2 in. − 1 yd. 2 ft. 8 in. **2 ft. 6 in.** **(or $2\frac{1}{2}$ ft.)**	9 bu. 3 pk. 6 qt. − 7 bu. 3 pk. 7 qt. **1 bu. 3 pk. 7 qt.**

NOTES

Teaching Pages 80, 81, and 82

Pupil's Objective. To relearn how to multiply and divide measurement numbers.

Background. We multiply measurement numbers by parts just as we multiply whole numbers by parts. The parts are made up of the units of measure being multiplied, and when the multiplication produces enough of the smaller units to make a larger unit, there is carrying.

To highlight the similarities and differences, compare the multiplication of 2 pounds 9 ounces by 3 with the corresponding whole-number example, 3 times 29.

Note that in the denominate-number example (Ex. *a* below) a group of 16 ounces is carried to the next column, while in the whole-number example (Ex. *b*) two groups of 10 ones are carried to the next column. Note, also, that the final answer for the denominate-number example shows a 2-place number in the place-value column for ounces.

a. 2 lb. 9 oz.
× 3
7 lb. 11 oz.

Side work

3 × 9 oz. = 27 oz.
27 oz. ÷ 16 oz. = 1 lb. 11 oz.
(Write "11 oz." in ounce's place and add 1 lb. to the product in pound's column.)

b.

Tens	Ones
2	9
×	3
8	7

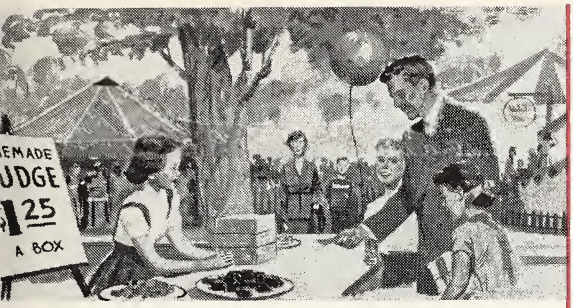
What we think

3 × 9 = 27, or 2 tens and 7 ones. (Write "7" in one's column and add 2 tens to the product in ten's column.)

Dividing denominate numbers is also done by parts. When the largest unit cannot be divided, as in box B on page 81 in the text (4 yd. cannot be divided by 7), the denominate number is changed into smaller units so that the division may be carried out. When there is a remainder after dividing the first and largest unit, as in box C on page 81 in the text, it is changed to the smaller unit and combined with whatever other smaller units there are to make the second partial dividend.

Book Lesson (page 80). *Ex. 1:* Oral work. *Ex. 2–10:* Written work. Have pupils determine whether the changing of units after the first multiplication is similar to the work on page 76 or on page 77.

Book Lesson (page 81). *Ex. 1–3:* Oral work. *Rows 4–6:* Written work.



The Candy Counter at the Fair

M. of denominate numbers [O]

1. Each box of fudge weighs 2 lb. 6 oz. Mr. Cox is buying how much in 3 boxes?

3 × 2 lb. 6 oz. = ?

3 × 6 oz. = 18 oz.

There are 16 oz. in 1 lb., so 18 oz. = 1 lb. 2 oz.

	Side work
2 lb. 6 oz.	3 × 6 oz. = 18 oz.
×3	18 oz. = 1 lb. 2 oz.
7 lb. 2 oz.	

2 oz. 1 lb.

What do we write in ounce's column? What do we carry?

We multiply 2 lb. by 3 and add the 1 lb. carried. What do we write in the pound's column? 7 lb.

Why can we check the product by addition? M. is a quick way of adding equal numbers.

[W]

Multiply the following measuring numbers. You will need to think carefully about the carrying.

2. 3 qt. 1 pt. ×5 — 17 qt. 1 pt.
3. 5 gal. 3 qt. ×3 — 17 gal. 1 qt.
4. 5 ft. 7 in. ×6 — 33 ft. 6 in.
5. 4 bu. 3 pk. ×9 — 42 bu. 3 pk.
6. 4 yd. 2 ft. ×8 — 37 yd. 1 ft.
7. 3 min. 40 sec. ×5 — 18 min. 20 sec.
8. 4 hr. 30 min. ×6 — 27 hr.
9. 2 lb. 10 oz. ×7 — 18 lb. 6 oz.
10. 4 yd. 20 in. ×2 — 9 yd. 4 in.

Extra Practice. For more practice, work Set 37.

Finding Equal Parts

D. of denominate numbers [O]

1. Bob raised 12 pk. 4 qt. of beans from his 4 rows. How much did each row average? $\frac{1}{4}$ of 12 pk. 4 qt. = ?

Box A: 12 pk. ÷ 4 = 3 pk. Write "3 pk." in peck's column.

4 qt. ÷ 4 = 1 qt. Write "1 qt." in quart's column. Now check.

A

3 pk. 1 qt.
4)12 pk. 4 qt.
12 pk.
4 qt.
4 qt.

Check
4 × 3 pk. 1 qt. = ?

2. A rope 4 yd. 2 ft. long is cut into 7 equal parts. How long is each part?

$\frac{1}{7}$ of 4 yd. 2 ft. = 4 yd. 2 ft. ÷ 7 (box B). Explain the work.

Why was 4 yd. 2 ft. changed to 14 ft.? 4 couldn't be divided by 7.

B

7)4 yd. 2 ft. = 7)14 ft.
14 ft.

	Side work
2 ft.	1 yd. = 3 ft.
	4 × 3 ft. = 12 ft.
	12 ft. + 2 ft. = 14 ft.

3. $\frac{1}{5}$ of 13 ft. 4 in. = ? Explain the work in box C.

C

5)13 ft. 4 in. = 2 ft. 8 in.
10 ft.
3 ft. 4 in. = 40 in.
40 in.

Side work
1 ft. = 12 in.
3 × 12 in. = 36 in.
36 in. + 4 in. = 40 in.

[W]

Copy, divide, and check.

a	b	c
4. 3)6 ft. 9 in. — 2 ft. 3 in.	5)10 lb. 15 oz. — 2 lb. 3 oz.	6)4 bu. 2 pk. — 3 pk.
5. 8)5 hr. 20 min. — 40 min.	3)5 gal. 1 qt. — 1 gal. 3 qt.	7)3 T. 300 lb. — 900 lb.
6. 6)41 ft. 6 in. — 6 ft. 11 in.	2)3 wk. 3 da. — 1 wk. 5 da.	4)15 min. 36 sec. — 3 min. 54 sec.

Extra Practice. For more practice, work Set 38.

1. The heading at the top of the page in the text—"Finding Equal Parts"—refers, of course, to fractional-part division. Be particularly aware of labels for the quotients on this page, keeping in mind the fact that in fractional-part division, the dividend and quotient must have the same label.

2. Encourage the use of *side work* on a temporary basis only.

3. *More capable children* may be permitted to omit the oral part of this lesson and proceed directly to the Ex. 4–6.

4. Using boxes A, B, and C, and the examples in rows 4, 5, and 6, have a few pupils identify examples that illustrate the phrases below. Then ask that each phrase be completed to express a generalization about changing the units in fractional-part division.

a. The divisor is contained evenly in all units of the dividend (box A).

b. The divisor is not contained even once in the larger units of the dividend (box B).

c. The divisor is contained in the first partial dividend with a remainder (box C).

Dividing Measuring Numbers

[O]

1. How many 4-ounce jars can be filled from a can containing 3 lb. 8 oz. of mustard?

Why should you change 3 lb. 8 oz. to ounces before dividing? 3 lb. not divisible by 4.

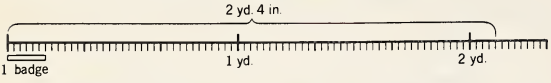
Explain all the work in the box.

$$\begin{array}{r} 14 \\ 4\overline{)56} \\ 4 \\ \hline 16 \\ 16 \\ \hline \end{array}$$

Side work
1 lb. = 16 oz.
3 × 16 oz. = 48 oz.
48 oz. + 8 oz. = 56 oz.

2. Find how many 6-inch badges you can cut from a ribbon 2 yd. 4 in. long.

Use the number line below to find the answer. Do you work with yards and inches or just with inches? Explain.



Are all 76 inches used up in making 12 badges? No How many inches are left over? 4 These extra inches are the remainder.

On the board, work the example with figures. Explain how you would check your work.

[W]

Divide the following:

3. 5 in.)4 ft. 3 in. 10, R1 in.
4. 10 min.)1 hr. 50 min. 11
5. 3 qt.)4 gal. 2 qt. 6
6. 2 qt.)4 pk. 7 qt. 19, R1 qt.
7. 2 in.)1 ft. 4 in. 8
8. 2 min.)1 hr. 20 min. 40

Extra Practice. For more practice, work Set 39.

Book Lesson (page 82). *Ex. 1 and 2:* Oral work. *Ex. 3–8:* Written work.

1. The frequent use of a number line in connection with measurement division will often increase understanding.

2. Draw to your pupils' attention the fact that in measurement division the quotient has no label.

Differentiations and Extensions

1. *All pupils*

a. Be aware of opportunities to apply learnings, either in real or in simulated activities. Consider the present needs of your pupils and also the extension of their understandings. For example, urban pupils may have had fewer experiences with dry measures than rural pupils, but, nonetheless, they may have, from a consumer's point of view, just as great a need for understanding. Consider measurements needed on farms, in dairies, markets, express and freight occupations, etc., particularly as associated with your present units of study.

b. Survey hobbies in the class in a search for experiences requiring the use of measures. The information derived may be used to develop realistic situations which will require denominate-number computation.

2. **Extra Practice Sets 37, 38, and 39** may be assigned as needed.

Set 37. Multiplication of denominate numbers

	a	b	c
1.	3 qt. 1 pt. × 5 = 17 qt. 1 pt.	2 ft. 5 in. × 3 = 7 ft. 3 in.	4 hr. 25 min. × 4 = 17 hr. 40 min.
2.	1 yd. 2 ft. × 6 = 10 yd.	7 min. 10 sec. × 9 = 64 min. 30 sec.	6 lb. 8 oz. × 7 = 45 lb. 8 oz.
3.	1 mi. 700 ft. × 8 = 9 mi. 320 ft.	2 bu. 1 pk. × 7 = 15 bu. 3 pk.	1 ft. 4 in. × 6 = 8 ft.
4.	3 yd. 2 ft. × 3 = 11 yd.	2 gal. 3 qt. × 2 = 5 gal. 2 qt.	4 pk. 5 qt. × 5 = 23 pk. 1 qt.

Set 38. Fractional-part division with denominate numbers

1.	2 T. 300 lb. 2)4 T. 600 lb.	10 oz. 5)3 lb. 2 oz.	2,740 ft. 2)1 mi. 200 ft.
2.	2 ft. 8 in. 5)13 ft. 4 in.	2 hr. 10 min. 3)6 hr. 30 min.	2 in. 8)1 ft. 4 in.
3.	3 pk. 2)1 bu. 2 pk.	1 gal. 2 qt. 5)7 gal. 2 qt.	1 pk. 2 qt. 6)7 pk. 4 qt.
4.	2 ft. 7)4 yd. 2 ft.	3 pk. 6)4 bu. 2 pk.	1 pt. 5)2 qt. 1 pt.
5.	1 c. 5)1 qt. 1 c.	2 da. 14 hr. 2)5 da. 4 hr.	1 min. 9 sec. 9)10 min. 21 sec.

Set 39. Measurement division with denominate numbers

The first number is how many times the second? To find out, divide the first number by the second.

1. 4 ft. 2 in.; 5 in. 10
2. 2 hr. 20 min.; 1 hr. $2\frac{1}{3}$
3. 6 qt.; 2 qt. 1 pt. $2\frac{2}{5}$
4. 8 qt.; 1 gal. 2 qt. $1\frac{1}{3}$
5. 4,500 lb.; 1 T. 500 lb. $1\frac{4}{5}$
6. 1 bu. 1 pk.; 2 pk. $2\frac{1}{2}$

(*Set 39 continues on page 103*)

The first number is what part of the second? To find out, divide the first number by the second.

7. 1 ft. 9 in.; 27 in. $\frac{7}{9}$

8. 1 hr. 20 min.; 2 hr. $\frac{2}{3}$

9. 2 qt.; 3 qt. 1 pt. $\frac{4}{7}$

10. 7 qt.; 8 gal. 3 qt. $\frac{1}{5}$

11. 40 sec.; 4 min. 40 sec. $\frac{1}{7}$

12. 1 yd. 2 ft.; 10 ft. $\frac{1}{2}$

Teaching Page 83

Pupil's Objective. To relearn the procedure for finding a missing factor in multiplication or division.

Background. On page 23 in the text, pupils learned how to find a missing addend or a missing minuend represented by the letter *n*. Now they will learn how to find a missing factor in multiplication examples and a missing divisor or quotient in division examples.

Pupils are already familiar with the idea that the quotient times the divisor should equal the dividend. This check for division involves multiplication and is the reason that the divisor and quotient may be considered factors of the product. Once this idea has been reviewed on page 83, pupils see the two kinds of examples in multiplication and the two kinds of examples in division (boxes C and D) in which it is necessary to find the missing factor, *n*, by dividing. By Ex. 6 they have been led to discover for themselves the generalization that is shown in dark print at the bottom of the page, and they use it in doing the written work for Ex. 7–15.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7–15:* Written work.

1. For Ex. 2, be sure that pupils see how the fact that the divisor 4 and the quotient 8 are factors is related to the division check.

2. For boxes C and D, have pupils tell whether *n* represents a missing factor. Then lead them to see that in a more general way *n* represents a missing factor in both multiplication and division.

Differentiations and Extensions

1. Let *slower learners* tell in an oral lesson just what they would do to find the missing number represented by *n* in Ex. 7–15. Allow time for them to work out each example after they have decided on the process to use.

Finding n

The missing factor [O]

A	B
Multiplication	Division
$4 \times 8 = 32$	$32 \div 4 = 8$
factors *product*	*dividend* *divisor* *quotient*

1. In box A the factors are 4 and 8, the numbers that are multiplied together to give the product.

2. Although box B does not show multiplication, what two numbers can be multiplied together to get 32? *4 and 8*

3. From box B, which two of these can be thought of as factors: dividend, divisor, quotient? Explain.

Finding Missing Factor in Multiplication and Division

C		D	
$n \times 8 = 32$	$4 \times n = 32$	$32 \div n = 8$	$32 \div 4 = n$
$n = 32 \div 8$	$n = 32 \div 4$	$n = 32 \div 8$	$n = 32 \div 4$
$n = 4$	$n = 8$	$n = 4$	$n = 8$

4. In the multiplication examples in box C, what is done each time to find **n**, the missing factor? *product is divided by known factor*

5. In the division examples in box D, what is done to find **n**, the missing divisor? **n**, the missing quotient? *dividend divided by known number*

6. Then in both multiplication and division, what must we do to find **n**, a missing factor? *divide*

[W]

Write your work to find **n**, the missing factor.

7. $n \times 9 = 72$ (8)
8. $45 \div n = 5$ (9)
9. $8 \times n = 48$ (6)
10. $95 \div 5 = n$ (19)
11. $27 \times n = 108$ (4)
12. $n \times 21 = 84$ (4)
13. $568 \div n = 8$ (71)
14. $n \times 8 = 136$ (17)
15. $100 \div 4 = n$ (25)

In a multiplication or a division example, we find the missing factor by dividing by the known factor.

83

2. Let *all pupils* suggest a way in which they might check their written work. They should see that they may substitute the number they have found for *n* in the original example and do the work indicated by the signs to see if they obtain a correct answer.

3. *More capable children* may attempt to make up problems involving the examples in the written work. The following is a problem that might be used for Ex. 7 in the text: "There are 9 tomato plants in each row in my garden. Altogether I have 72 tomato plants. How many rows of tomato plants are there?"

Practice with Measuring Numbers

[W]

Add, subtract, multiply, or divide, as the signs tell you.

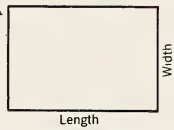
	a	b	c
1.	20 yd. 1 ft. −6 yd. 2 ft. 13 yd. 2 ft.	10 hr. 40 min. ×7 74 hr. 40 min.	32 lb. 11 oz. +27 lb. 8 oz. 60 lb. 3 oz.
2.	15 gal. 2 qt. ×8 124 gal.	25 lb. 9 oz. −17 lb. 12 oz. 7 lb. 13 oz.	7 yd. 2 ft. 5 in. +3 yd. 2 ft. 7 in. 11 yd. 2 ft.
3.	2 pk. 7 qt. +6 qt. 3 pk. 5 qt.	7 T. 670 lb. ×6 44 T. 20 lb.	13 wk. 4 da. −12 wk. 6 da. 5 da.
4.	9)14 ft. 3 in. 1 ft. 7 in.	5)3 gal. 1 pt. 5 pt.	3)4 T. 1,750 lb. ✓ 1 T. 1,250 lb.

Perimeter and Area of a Rectangle

Meaning [O]

1. Diagram A is a **rectangle.** It has 4 sides and 4 square corners. Are all the sides the same length? No Are any of the sides the same length? Yes Check by measuring with your ruler.

A



2. Either of the long sides of a rectangle is called its **length,** and either of the short sides is called the **width.** The length of rectangle A is about 1¼ in., and the width is about ⅞ in. The length and the width of a rectangle are its **dimensions.**

3. "A rectangle 3 in. by 5 in." means a rectangle with the dimensions 3 in. and 5 in. A shorter way to write this is: 3 in. × 5 in., or 3″ × 5″. The sign ″ means inches

4. The distance around a rectangle is called its **perimeter.** Tell the perimeter of a rectangle 3″ × 5″. 16″ See if you can think of more than one way of finding the perimeter. add 4 dimensions. 2 × sum of length and width.

5. Are all squares rectangles? Yes Are all rectangles squares? No Tell how a square is different from other rectangles. A square has four equal sides.

We often need to know not only the dimensions of a rectangle but also the amount of surface it contains, or its **area.** We measure area in area units just as we measure length in linear units. One unit of area is the **square inch (sq. in.),** or the surface contained in a square with 1-inch sides. Is diagram B a square inch? Yes

B

6. To find the area of a rectangle 4 in. × 6 in., Mike measured it with a square inch cut out of cardboard. On the board draw a rectangle with dimensions 4″ × 6″ and do what Mike did.

Mike measured with his square inch to find how many square inches there would be in a row along the long side of the rectangle. This diagram shows how he began. Finish the row in your diagram. One row contains 6 sq. in.

Then Mike thought how many of these rows it would take to fill the whole rectangle. The rectangle is 4 inches wide, so there would be 4 rows.

Mike multiplied 6 sq. in., the number of square inches in one row, by the number of rows, 4. 4 × 6 sq. in. = 24 sq. in. The area of the rectangle is 24 square inches.

7. In any rectangle, would there be as many square inches in one row as there are inches in the length? yes Would there always be as many rows as there are inches in the width? Yes

To find the area of a rectangle, think of the number of area units in one row and then multiply by the number of rows.

[W]

Multiply to find the area of each of these rectangles. If you need help with any example, draw a diagram. (Ans. in square in.)

8. 2″ × 3″ 6 9. 4″ × 5″ 20 10. 3″ × 4″ 12 11. 7″ × 10″ 70
12. 3″ × 8″ 24 13. 5″ × 13″ 65 14. 9″ × 9″ 81 15. 6″ × 15″ 90

Teaching Pages 84, 85, 86, and 87

Pupil's Objectives. (*a*) To obtain mixed practice in work involving denominate numbers; (*b*) to study the meaning of square measure; (*c*) to develop understanding of the following terms: *dimensions, perimeter, rectangle, square inch, square foot, square yard, acre, square mile;* (*d*) to find areas.

Background. The examples in rows 1-4 at the top of page 84 in the text will give pupils mixed practice with denominate numbers. If the results of the work show that some pupils are having difficulty, reteach whichever of the preceding pages are appropriate for the difficulty encountered.

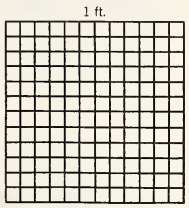
The rest of page 84 and all of page 85 deal with the meaning of finding perimeters and areas. In order to avoid language difficulties that may obscure the arithmetical aspects of finding perimeters and areas, be sure the terms shown in heavy type are thoroughly understood.

Pupils should be pleased to know that finding the perimeter of a rectangle simply means finding the distance around it. This information is reviewed in Ex. 4. Pupils are also asked in this same exercise to tell more than one way of finding a perimeter. If they suggest multiplying both the width and the length by 2, they will, without knowing it, have developed and used a formula.

More Area Units

Square foot; square yard; acre [O]

Another area unit, 1 **square foot (sq. ft.)**, is a square with 1-foot sides.



1. Draw a square foot on the board and divide it into square inches as at the right.

2. There are 12 sq. in. in one row. There are 12 rows. Then in all there are 12 × 12 sq. in., or 144 sq. in., in 1 sq. ft.

3. A **square yard (sq. yd.)** is a unit with each side 1 yard long. On the board, draw a square yard and divide it into square feet.

4. A square yard contains 3 × 3 sq. ft., or 9 sq. ft. Explain how you know.

5. Look up and report to the class about how men used to measure an acre of land.

6. From the table at the foot of the page, tell which is larger, 1 acre or 1 square mile.

[W]

7. Find the area of **a.** a rug 6 ft. by 9 ft.; **b.** a floor 12′ by 15′ (the sign ′ means "foot" or "feet"); **c.** a card 5″ × 8″. 54 sq. ft. 180 sq. ft. 40 sq. in.

8. A desk 24 in. by 36 in. is 6 sq. ft. in area. (Change 24 in. and 36 in. to feet before you multiply.)

9. A card table 36″ × 36″ is 9 sq. ft. in area.

10. Find the area of a piece of carpet 3 yd. × 8 yd. 24 sq. yd.

11. What is the area of a tennis court (a rectangle) 78′ × 36′? 2,808 sq. ft.

12. Find the area of a football field (a rectangle) 360′ by 160′. 57,600 sq. ft.

144 sq. in. = 1 sq. ft. **43,560 sq. ft. = 1 acre (A.)**
9 sq. ft. = 1 sq. yd. **640 A. = 1 square mile (sq. mi.)**

Finding Areas

Rectangles and squares [W]

Draw a picture to help you solve each problem.

1. Find the area of a piece of screening 6 feet by 2 feet. 12 sq. ft.

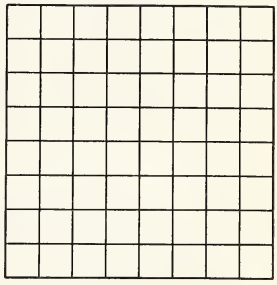
2. Jane's cookbook says that a cake may be baked either in a square pan 9 inches by 9 inches or in an oblong pan 8 inches by 12 inches. Which pan would make a cake of larger area? (81 sq. in.) (96 sq. in.) Oblong

3. Mr. Ordway figures that a quart of paint will cover about 100 square feet. Will he need more than 1 quart of paint for a porch floor that is 14 feet wide and 7 feet long? No (7×14=98 sq. ft.)

4. Ann is taking a course in refinishing furniture. The catalogue says, "Practice on a piece of your furniture that takes floor space not more than 27 inches by 27 inches." Ann wants to work on a footstool that takes a floor space of 24 inches by 30 inches. Is this a smaller area or a larger area than the catalogue suggests? Smaller (27×27=729 sq. in.; 30×24=720 sq. in.)

5. Mrs. Walker bought a rubber pad 6 feet by 9 feet to put under a rug. The pad was priced by the square yard. How many square yards did Mrs. Walker buy? 6 sq. yd.

6. Ed planned to make a checkerboard for his mother's birthday gift. According to official rules, the board must be a square not less than 14 in. on a side nor more than 16 in., marked off into 64 smaller squares. Ed drew this plan for his board. Each small square was 2 in. on a side. How long was each side of the board? 16 in. How many of the small squares were there in all? 64 Did Ed's board fit the rules? yes Find the area of the board. 256 sq. in.



Most pupils engage in few activities that necessitate the finding of areas. This is one of the reasons finding areas sometimes causes unnecessary difficulty. If you present a realistic problem which entails the finding of an area (such as finding the area of the top of a table in order to know how much paint to buy) and allow pupils to measure the area by using squares of cardboard cut exactly 1 square foot, understanding is much more likely to result. Moreover, pupils will probably be able to state, without further guidance, the generalization shown in dark print near the bottom of page 85 in the text. At any rate, the meaning of this generalization will be clear if a practical illustration is used.

After the meaning of measuring a surface is developed on page 85, the square foot, square yard, acre, and square mile are introduced on page 86 and, finally, on page 87, pupils solve problems that involve finding areas of rectangles and squares.

Book Lesson (top of page 84). *Rows 1–4:* Written work.

Pre-book Lesson (bottom of page 84 and all of page 85). Discuss with pupils the measurements of a piece of paper, possibly one used in construction. Help pupils determine the situations which require knowing dimensions (length and

width), and the situations which require knowing the amount of surface, or area.

Book Lesson (bottom of page 84 and all of page 85). *Ex. 1–7.* Oral work. *Ex. 8–15:* Written work. As you meet particular terms, have pupils identify them by referring to rectangles and squares in the classroom.

Book Lesson (page 86). *Ex. 1–6:* Oral work. *Ex. 7–12:* Written work.

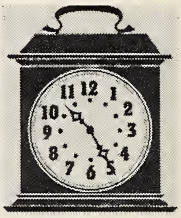
Book Lesson (page 87). *Ex. 1–6:* Written work. Have pupils examine problems for hidden questions. Then let them draw on the board one or two pictures related to specific problems as a way of clarifying problem situations.

Differentiations and Extensions. Help *all pupils* to develop a knowledge of some of the standard sizes met in buying household, school, and business supplies. You may include sizes of filing cards, paper, towels, sheets, rugs, and tables. School inventory or order lists may suggest many other possibilities. Have pupils compare surface areas of some of the smaller and larger sized objects within the same classification.

Units for the Measuring of Time

Minute, day, month, year, decade, century [O]

Everyone needs to know what time it is; that is, the year, the month, the day, the hour, the minute, and sometimes the second.



1. What year is this? What month?

2. What is the date today? What day of the week is it?

3. What time does this clock face show? Let's call this the right time now. 10:25

4. Using your answers for Ex. 1, 2, and 3 in one statement, tell the time at this minute.

We also may need to measure the length of time.

5. How can you tell how long it takes you to reach school from home? By clocks or a watch

6. How can you tell how long it is since Christmas? By a calendar

7. How long ago did Columbus discover America? Subtract 1492 from present date.

8. What units for measuring time do we use in Ex. 5–7? Ex. 5: minute or hour; Ex. 6: month, week, day; Ex. 7: year

The chief time units are the day, the month, and the year.

The day. Think of a ball of yarn as the earth and the needle through its center as its axis. The earth turns on its axis from left to right, so the sun seems to move from east to west.

9. Show this with a ball and a needle.

A day is the length of time the earth takes for a full turn on its axis. Hours, minutes, and seconds are parts of the day, and a week is 7 times the length of a day.

10. A day = 1440 minutes. 11. A week = 168 hours.

Teaching Pages 88 and 89

Pupil's Objective. To learn more about the units for measuring time.

Background. The units for measuring time do not exist in the same way other measurement units exist. However, they can be experienced by pupils and they are so much a part of our everyday experiences that their abstract nature is not a matter for undue alarm.

The standard units for measuring time have been used for hundreds of years with very little change. For the most part they are based on natural phenomena, and pupils may enjoy a historical preview of the development of some of the more common units.

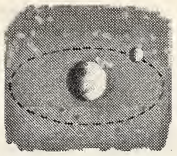
The work on pages 88 and 89 is concerned with identifying useful units of time. Also, an attempt is made, as in Ex. 5 on page 88, to develop reference measures (if pupils know that it takes them ten minutes to reach school from home, they can use this duration as a reference measure for determining or estimating other durations). Other reference measures (such as counting "1,000; 1,001; 1,002" for estimating the passing of seconds) can and should be developed by pupils.

Book Lesson. *Ex. 1–14:* Oral work. *Ex. 15–19:* Written work. A chart for the bulletin board or for individual notebooks may develop as you discuss the relationships between natural sciences and measures of time.

Differentiations and Extensions

1. *All pupils* should learn some of the methods used in recording time (e.g., quarter past one; 1:15 P.M.; June 1, 1954; 6/1/54).

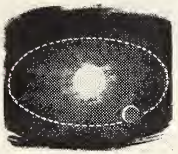
The month. The moon goes around the earth once in about 30 days. The month is based on that fact.



12. Do all the months in our calendar have the same number of days? No See if you can find out why they do not.

13. On the board, add the number of days in all 12 months. How many days in a common year? 365

The year. The earth goes around the sun, or *revolves* around it. The time of a full revolution is a year of 365 days. But the year is not really an exact number of times the moon-month or an exact number of times the earth-day. So, to give to our years the extra length they need, we have agreed that one year of every four shall be a leap year with an extra day in it.



14. Ten years are often called a **decade**, and 100 years, a **century**. A century contains how many decades? 10

[W]

15. Days, months, and years have names. Write the names of the days of the week.

16. Write the names of the months and their abbreviations.

17. Years are numbered. In what year were you born?

18. Which months have 30 days? April, June, September, November which 31 days? January, March, May, July, August, October, December

19. How many days has February? 28 or 29

By the moon,
1 mo. = about 30 da.

By agreement,
1 mo. = 31, 30, 29, or 28 da.
12 mo. = 1 yr.

By the sun,
1 yr. = about 365 da.

By agreement,
1 yr. = 365 or 366 da.
= 52 wk.
10 yr. = 1 decade
100 yr. = 1 century

b. Study word roots and other derivatives for the names of the months and for other words, such as *decades*, *calendar*, *century*, *annual*, and the like.

c. Explain the use of standard time and comment on its importance to the people in radio, television, and transportation.

d. Report on topics such as the following: Clocks, Then and Now; Primitive Methods of Keeping Time; How the Calendar Has Been Changed; Suggestions for Changing the Calendar in the Future; How the Sailor Tells Time; Daylight-Saving Time.

NOTES

2. *Slower learners*

a. Minimum skills would seem to be ability to read time within a five-minute period, and preferably within a one-minute period; ability to read the date from a calendar; ability to spell and list in order the days of the week and months of the year.

b. These children may list the measuring units in order of size from the smallest (second), to the largest (century).

3. *More capable pupils* may:

a. Study more thoroughly the vocabulary and the information related to the rotation of the earth, the revolution of the earth, and the phases of the moon.

The Calendar

Days, months, year [O]

1. Below is part of the calendar for a recent year. How many more months are left in the year? 9

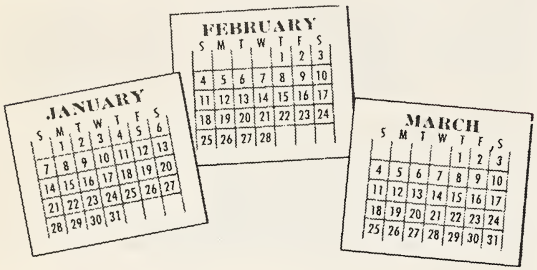
2. Suppose today is Feb. 10. Find today on the calendar. What day of the week is it? Saturday A week later is what date? Feb. 17

3. Why is the year of this calendar a **common year**? There are 28 days in Feb., not 29 as in leap years. [W]

4. Bob's birthday is Jan. 9, and Pat is 12 days younger than Bob. What is the date of Pat's birthday? Jan. 21

5. From Feb. 22 to Mar. 17 is how many weeks and days? 3 wk. 2 da.

6. How many months is it from Mar. 15 to Sept. 15? 6 mo.



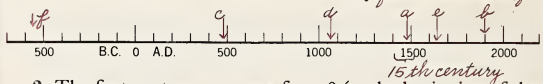
Years are numbered from the birth of Christ. Those since his birth are labeled A.D. (Anno Domini, which means "in the year of our Lord"). Those before his birth are labeled B.C. (before Christ). A year written without A.D. or B.C. is thought of as A.D. We write A.D. before the number of the year, and B.C. after it.

7. The Pilgrims landed at Plymouth Rock in 1620. Write the date with the correct label, A.D. or B.C. A.D. 1620

The Landing of Columbus

Centuries on a number line [O]

1. The number line below (called a *time line*) indicates the centuries. The birth of Christ is at 0. Why? Years are numbered from the birth of Christ



2. The first century A.D. runs from 0 (at the beginning of the year 1) through the year 100. Point to that space.

3. Point to the space for the 15th century. It is from the beginning of 1401 through 1500.

4. The scene above took place in 1492, in the 15th century.

5. What century are we in now? 20th When did it begin? Jan. 1, 1901 When will it end? Dec. 31, 2000

The number of hundreds in the last year of a century names the century.

6. Copy the above time line. On it, place these events: (See points a to f on line above.) [W]

a. Columbus landed, 1492
b. Lindbergh's birth, 1902
c. Fall of Rome, 476
d. Norman Conquest, 1066
e. Pilgrims landed, 1620
f. Battle of Marathon, 490 B.C.

Teaching Pages 90 and 91

Pupil's Objectives. (*a*) To become more familiar with the calendar; (*b*) to learn how to show time on a time line.

Book Lesson (page 90). *Ex. 1–3:* Oral work. *Ex. 4–7:* Written work.

Book Lesson (page 91). *Ex. 1–5:* Oral work. *Ex. 6:* Written work. Emphasize the fact that the time line in the text depicts centuries and that it extends to the left of 0 as well as to the right of it.

Differentiations and Extensions

1. Using the partial calendar on page 90 in the text, have pupils tell the dates for the seven consecutive Wednesdays beginning January 3; the date on which the first Sunday falls in February and in March; the number of Mondays in January; and other such information.

2. *More capable children* may refer to children's encyclopedias for information about the historical development of the calendar. They may report their findings to the class.

Problems in Measurement

[W]

Write all your work.

1. Find the average number of days per month for the last 6 months of a calendar year. $30\frac{2}{3}$

2. For the last four days, it has taken Joe 11 min., 9 min., 10 min., and 12 min. to walk to school. What was his average? $10\frac{1}{2}$

3. On the average, Mr. Fox carries 2 T. of coal in his truck. How many trips will he make to carry 40 T.? 20

4. An eighth of a mile is sometimes called "a furlong." How many feet are there in a furlong? 660

5. A grocer puts 25 bu. of potatoes into 1-peck bags for sale. How many bags does he need? 100

6. How many quart bottles will 20 gal. of honey fill? 80

7. John bought 2 lb. 8 oz. of bacon, 12 oz. of dry cereal, and 1 lb. of cocoa. The three things weighed how much? 4 lb. 4 oz.

Extra Practice. For practice, work Set 40.

Do You Understand?

Test of Information and Meaning 2

1. How could you find the number of 32's in 725 without dividing? continually subtracting 32's from 725

2. When you find $\frac{1}{8}$ of 736, are you using measurement division or fractional-part division?

3. What table number helps you to find the quotient for $7\overline{)51}$? 49 for $4\overline{)35}$? 32 for $32\overline{)83}$? 64

4. If there are two partial dividends, how many quotient figures will there be? 2

5. If you divide tens, where will you write the quotient figure? in ten's place

6. What is the largest quotient figure you can ever try? 9

7. How much more is 1 gallon than 1 quart? 3 qt.

8. In a rectangle 8′ by 5′, the numbers 8′ and 5′ are the dimensions.

9. When we find the number of square units in a rectangle, we are finding its area.

10. How many square inches are there in 1 sq. ft.? 144 How many square feet in 1 sq. yd.? 9

11. In what century is the year 1630? 17th the year 1955? 20th

12. Which months have 31 days? Jan., March, May, July, Aug., October, December

Do You Make Mistakes?

Diagnostic Test 2

				Study Pages	Practice Sets
1.	$8\overline{)96}$ 12	$5\overline{)278}$ 55, R3	$4\overline{)8,104}$ 2,026	49-53	23, 24
2.	$60\overline{)240}$ 4	$73\overline{)368}$ 5, R3	$56\overline{)170}$ 3, R2	58-59	25
3.	$38\overline{)190}$ 5	$18\overline{)162}$ 9	$39\overline{)275}$ 7, R2	60-62	26, 27
4.	$25\overline{)575}$ 23	$47\overline{)\$0.85}$	$25\overline{)13,325}$ 533	64-66	28-30
5.	$400\overline{)2,400}$ 6	$395\overline{)10,275}$ 26, R5	$427\overline{)157,136}$ 368	67-68	31
6.	6 ft. 3 in. +2 ft. 4 in. 8 ft. 7 in.	8 lb. 5 oz. −4 lb. 10 oz. 3 lb. 11 oz.	3 gal. 2 qt. +5 gal. 3 qt. 9 gal. 1 qt.	78-79	33, 34
7.	a 4 yd. 2 ft. ×3 14 yd.	b 3 lb. 14 oz. ×4 15 lb. 8 oz.		80	37
8.	$5\overline{)8}$ hr. 15 min. 1 hr. 39 min.	$2\text{ ft.})\overline{3}$ yd. 1 ft. 5		81, 82	38-40

Teaching Pages 92, 93, 94, and 95

Pupil's Objectives. (*a*) To solve problems involving denominate numbers; (*b*) to obtain mixed practice in whole-number computation; (*c*) to take the four regular end-of-chapter tests.

Book Lesson (top of page 92). *Ex. 1–7:* Written work.

1. Let children refer to the tables of measures on page 327 in the text if necessary. Encourage them to commit to memory the more common tables.

2. Use the results of this work to determine areas needing strengthening for either groups or individuals. Try to make provisions for planning the help needed.

Book Lesson (bottom of page 92 and top of page 93). *Ex. 1–12:* Written work. Test of Information and Meaning 2.

Book Lesson (bottom of page 93). *Rows 1–8:* Written work. Diagnostic Test 2. Explain again the function of this kind of test and show how it should be taken. Demonstrate the use of study pages and practice sets. Practice sets with answers are reproduced on previous pages in the Manual. For their location, refer to the heading *Practice* in the index for this Manual.

Mixed Practice

[W]

Copy, and do as the signs say. Check your answers.

	a	b	c	d	e	f
1.	273	8,963	755	3,806	$95.57	$63.07
	5,786	3,826	4,513	7,712	84.26	5.73
	958	859	1,668	59	0.75	98.95
	+675	+407	+268	+942	+4.65	+2.61
	7,692	14,055	7,204	12,519	$185.23	$170.36
2.	768	4,059	20,061	70,350	$73.42	$40.06
	−397	−2,673	−2,581	−3,876	−50.79	−2.79
	371	1,386	17,480	66,474	$22.63	$37.27
3.	630	9,841	2,308	8,002	$6.37	$8.05
	×27	×76	×706	×410	×28	×27
	17,010	747,916	1,629,448	3,280,820	$178.36	$217.35

	a	b	c	d	e
4.	81 8)648	509 7)3,563	782 4)3,128	1,814 5)9,070	$1.56 6)$9.36
5.	670,R2 7)4,692	$2.03 9)$18.27	60 60)3,600	$0.48 43)$20.64	197,R20 193)38,041
6.	75,R84 95)7,209	59,R36 72)4,284	52,R8 86)4,480	$0.76,R56¢ 64)$49.20	229,R123 347)79,586

How Well Can You Figure?

Computation Test 2

	a	b	c	d
1.	13 6)78	22,R3 8)179	316 5)1,580	8 80)640
2.	7,R1 17)120	7 21)147	8,R35 75)635	3,R31 35)136
3.	$0.75 46)$34.50	7 700)4,900	75 128)9,600	86 625)53,750

4. 5 gal. 3 qt. + 2 gal. 1 qt. = 8 gal.

5. 8 hr. 20 min. − 3 hr. 40 min. = 4 hr. 40 min.

6. 8 ft. 5 in. × 6 = 50 ft. 6 in., or 16 yd. 2 ft. 6 in.

7. 4 bu. 3 pk. × 4 = 19 bu.

8. 5 oz.)3 lb. 2 oz. = 13 da., or 1 wk. 6 da.

9. $\frac{1}{4}$ of 7 wk. 3 da. = 10

10. $\frac{1}{8}$ of 5 yd. 1 ft. = 2 ft.

11. $\frac{1}{3}$ of 1 yr. 3 mo. = 5 mo.

Can You Solve Problems?

Problem Test 2

1. On a city bus there is a mark on the door with the sign, "40 in., full fare." Jack is 3 ft. 6 in. tall. Would he have to pay full fare? Yes

2. At $1.25 a square yard, find how much it would cost to have a 9′ by 12′ rug cleaned. $15.00

3. The Braves Baseball Club was organized in Boston in 1876. The Club transferred to Milwaukee in 1953. How long were the Braves in Boston? 77 yr.

4. The hardware store advertised 24-quart wastebaskets. How many pecks would such a wastebasket hold? 3 pk.

5. A farm magazine advertisement offered 10 extra chicks with every 100 chicks ordered. If Mr. Barr ordered 200 chicks at $15.40 per 100, what was the average cost per chick for the chicks he received? $0.14

6. An ordinary watch ticks 5 times a second, and a hall clock ticks 60 times a minute. Does the watch tick faster or more slowly than the clock? Watch ticks faster

7. A manufacturer claims that his pencil sharpener can sharpen 36,000 pencils before it gets dull. If 30 pencils are sharpened twice a day, in how many days may a new sharpener get dull? 600 da.

8. How many years was it from 1492 to 1776? 284 yr. Draw a time line to show the centuries from the birth of Christ to the year 2000 and on it mark the approximate positions of the years 1492 and 1776.

9. How many gallons will a 12-quart pail hold? 3 gal.

10. Mrs. Walton paid 72¢ for a package of Canadian bacon which contained 24 slices. Allowing 3 slices per serving, find how much one serving of bacon cost. 9¢

Book Lesson (top of page 94). *Rows 1–6:* Written work. Use this mixed practice as relief from the testing program, or assign selected portions of it on the basis of the results of the Diagnostic Test.

Book Lesson (bottom of page 94). *Ex. 1–11:* Written work. Computation Test 2. If necessary, this test may be used for still further practice, as there is an alternate computation test on page 302 of this Manual. The table of per cents on page 111 applies to both computation tests.

Book Lesson (page 95). *Ex. 1–10:* Written work. Problem Test 2.

Differentiations and Extensions

1. All tests may be used for teaching as well as evaluation purposes. Tabulate errors item by item and discuss with pupils their errors and reasons for them. In using the problems for teaching purposes, try to make the problem situations as real as possible by using objects or by dramatizing the situations. Do not proceed to Chapter 3 until all children have attained a degree of mastery of this chapter's content that is commensurate with their ability.

2. Discuss your arithmetic program with the class. They may have valuable suggestions for changes or improvements. Make certain that the

everyday real arithmetic problems of your children are recognized and solved. Be sure, too, that your arithmetic program is integrated with your whole teaching program.

3. Assign, as needed, **Extra Practice Set 40**, mentioned on page 92 in the text.

Set 40. Changing units of measure

	a	b
1.	54 in. = 4 ft. 6 in.	2 bu. 3 pk. = 1 bu. 7 pk.
2.	3 lb. 5 oz. = 53 oz.	1 yd. 4 ft. = 2 yd. 1 ft.
3.	2 yd. 7 ft. = 4 yd. 1 ft.	2 pk. 1 qt. = 17 qt.
4.	1 qt. 1 pt. = 3 pt.	4 yd. 3 ft. = 3 yd. 6 ft.
5.	4 qt. 1 c. = 17 c.	4 hr. 75 min. = 5 hr. 15 min.
6.	2 bu. 3 pk. = $2\frac{3}{4}$ bu.	200 sec. = 3 min. 20 sec.

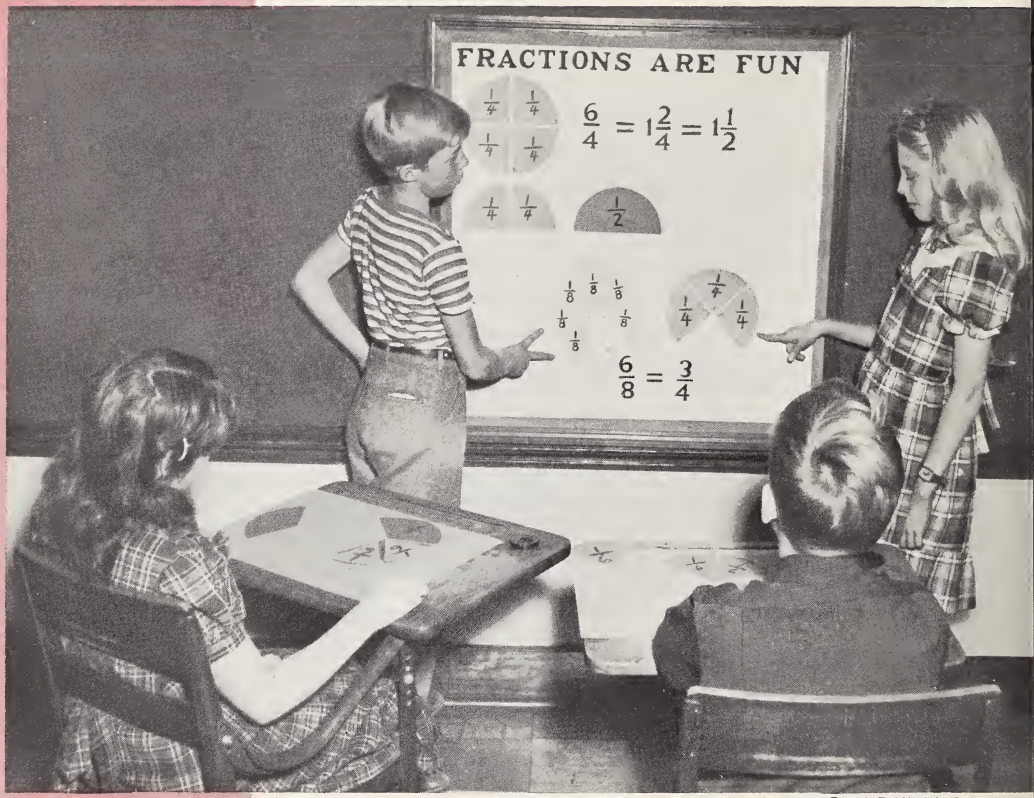
4. Use the table below to determine per cents for entry on the test record cards mentioned on page 63 of the Manual. Scores, of course, should be a matter for the individual pupil only.

TABLE OF PER CENTS FOR CHAPTER 2 SCORES

Score	Per Cents for Problem Test 2
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100

Score	Per Cents for Computation Test 2
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50

Score	Per Cents for Computation Test 2
11	55
12	60
13	65
14	70
15	75
16	80
17	85
18	90
19	95
20	100



Detroit Public Schools

Fractional parts of equal circles help pupils understand the changes involved in writing a fraction in best form.

Teaching Chapter 3 of Grade 6

Introduction

The reteaching program continues in Chapter 3. The principal purpose is to provide a foundation of basic fraction meanings and ability in adding and subtracting like- and unlike-fractions and mixed numbers.

I. Learning Outcomes in Chapter 3

1. Understanding of fractions, including ability to change form and to identify equivalents
2. Ability to add and subtract like- and unlike-fractions, without and with carrying and borrowing
3. Ability to solve problems
4. Understanding of various specific concepts, generalizations, relationships, and skills
5. Desirable emotionalized responses (attitudes, appreciations, values)

Understanding of Fractions, Including Ability to Change Form and to Identify Equivalents (pages 98–109)

Listed below are the main topics included under this heading.

a. Meaning of fractions, including an understanding of the terms and ability to read and write fractions
b. The meaning of fractional unit
c. Mixed numbers and proper and improper fractions.
d. The meaning of a fraction as a number
e. Using equivalents in comparing fractions
f. The division meaning of a fraction
g. Showing the remainder in division in a fraction
h. Changing the form of a fraction: to higher, to lower, and to lowest terms; changing improper fractions to best form

All of the work referred to in the topics above is review, but a thorough reteaching program is desirable, since inability to work with fractions will affect ability to work with decimals later on in grade 6.

While the terminology "fractional unit" may be new to you, it is important that you develop the concept carefully and emphasize its meaning in use whenever you are dealing with fractions. Just as in working with measures we think of a basic unit (such as inch, foot, pound, etc.), so in working with fractions we must think of a basic unit that is the size of one of the equal parts. Since the denominator tells the number of equal parts in a whole, it also indicates the size of the fractional unit. The 8 in $\frac{5}{8}$ tells us that there are 8 equal parts in the whole. We therefore know that the fractional unit is the size of one of these equal parts, or $\frac{1}{8}$. Pupils already know that they cannot add unlike-numbers. When they come to adding fractions, they will realize that $\frac{5}{8}$ and $\frac{5}{16}$ cannot be added because they do not have the same fractional unit. Therefore, both fractionals must be changed so that they have a common fractional unit.

In comparing unlike-fractions (fractions with different fractional units), pupils discover that the larger the denominator the smaller is the size of the fractional unit. Changing a fraction to an equal fraction with higher terms can be taught meaningfully if pupils see that the larger numbers for the terms mean smaller fractional units and more units.

Proper fractions, improper fractions, and mixed numbers are shown on a number line in being reintroduced to pupils. The number line helps show how fractions are related to whole numbers. It also helps in comparing fractions, for number lines are very useful in showing equivalents (such as $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{3}$ and $\frac{4}{6}$, etc.). The number line also shows quite clearly why a fraction equals 1 when the part shown by the numerator equals the number of parts in the whole.

Probably the most familiar meaning of a fraction is that it stands for a part of a whole. After this idea is reviewed, pupils learn that a fraction may also mean a part of a group, as $\frac{3}{8}$ of 24 candles. Another meaning is presented on page 104 in the text—that a fraction may mean division and indicate that the numerator is to be divided by the denominator. The extension of this idea permits pupils to write the remainder in division examples in a fraction, as shown on page 105 in the text.

Ability to Add and Subtract Like- and Unlike-Fractions (pages 110–121)

The sequence of presentation for this material is as follows:

a. Addition and subtraction of like-fractions
b. Addition and subtraction of mixed numbers without and with carrying and borrowing

c. Addition and subtraction of unlike-fractions
Common denominator present—first without, then with, carrying and borrowing
Common denominator not present—first with common fractions, then with mixed numbers involving both carrying and borrowing

If the fractions $\frac{3}{8}$, $\frac{3}{4}$, and $\frac{3}{16}$, all referring to parts of a pound, are to be added, we must first think of a fractional unit which will measure each part exactly. Because $\frac{1}{8}$ lb. is exactly twice as large as $\frac{1}{16}$ lb. and $\frac{1}{4}$ lb. is exactly 4 times as large as $\frac{1}{16}$ lb., we can use $\frac{1}{16}$ lb. as the unit for measuring eighths, fourths, and sixteenths of a pound. We may demonstrate these relationships, if necessary, by using parts of circles of the same size to show that a part $\frac{1}{16}$ in size can be applied 2 times to $\frac{1}{8}$ and 4 times to $\frac{1}{4}$. Hence, we call $\frac{1}{16}$ the common fractional unit. The authors of this series believe that the concept of a common fractional unit is necessary to an understanding of a common denominator.

In developing the Golden Rule of Fractions, the concept of a fractional unit is used to promote understanding. When $\frac{5}{8}$ is changed to $\frac{10}{16}$, the factor 2 by which both terms of $\frac{5}{8}$ must be multiplied shows that the fractional unit $\frac{1}{16}$ is half as large as $\frac{1}{8}$, and therefore, to maintain the equality, there must be twice as many sixteenths. The Golden Rule of Fractions is extended in the work devoted to adding and subtracting fractions. Pupils are led to realize that they always change to the smallest fractional unit (largest denominator) when a common denominator is present. If the common denominator is not present, pupils develop and use the following generalization:

When no given denominator is a common denominator, we can find the smallest common denominator by multiplying the largest given denominator first by 2, then by 3, and so on, until we get a number that contains all the given denominators an even number of times.

Ability to Solve Problems (pages 117, 121, 123–126)

In addition to discovering the meanings of a fraction as an aid in solving problems, pupils also differentiate addition and subtraction in problems; solve problems involving addition and subtraction of unlike-fractions, telling why certain fictitious pupils made the errors they did; and, finally, learn several different kinds of helps that can be used in problem-solving. For example, pupils learn to decide: (*a*) what is asked; (*b*) whether groups are put together or separated; (*c*) which process must be used in solving; (*d*) what a reasonable answer would be. They also identify problems having too many or too few numbers. In the first case, they must disregard the unnecessary information, while in the second case they must supply information in order to solve the problems.

This type of practice sharpens the pupils' sense of discrimination. They learn that in problem-solving, they must read thoroughly and accurately. The judgment involved in exercises of this nature is an attribute which should be fostered and strengthened whenever possible, since it will play an important role in dealing with practical affairs.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

best form (for fractions)
common denominator
common fractional unit
cup (c.)
denominator
fraction
fractional unit
Golden Rule of Fractions
higher terms (in a fraction)
improper fraction
level full (in cooking measures)
like-fractions
lower and lowest terms (in a fraction)
measuring cup
mixed number
numerator
reduce (fractions)
smallest common denominator
tablespoon (tbsp.)
teaspoon (tsp.)
unlike-fractions

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 3 will be found on the following pages in the text, and on the corresponding reproduced pages in this Manual: pages 98, 99, 102, 104, 106, 107, 109, 110, 114, 116, 120, 122.

3. *Understandings and skills beyond those listed:*

a. Understanding of, and ability to use, cooking measures (page 97)

b. Ability to round fractions and mixed numbers in estimating (page 122)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

Are you giving heed to the social aim of arithmetic? Are you calling your pupils' attention to the many practical uses of arithmetical skills in number situations both in and out of school? More important, are they calling *your* attention to such uses? Are your pupils anxious to understand each new thing they learn? Are they developing some appreciation of the logic and order that are part of arithmetic? Do they see enough values in what they learn so that they want to learn more? Your answers to these questions will help you decide how successfully your program satisfies this outcome.

Can You Use Cooking Measures?

Inventory Test 10 [O]

To help Cathy learn to use cooking measures correctly, her mother made the large table of measures shown on the opposite page.

Wheat Muffins	
2 c. flour	3 tbsp. sugar
$\frac{1}{2}$ tsp. salt	3 tbsp. butter
4 tsp. baking powder	1 egg
	1 c. milk

Loaf Cake	
$\frac{2}{3}$ c. sugar	1 tsp. vanilla
1 c. milk	2 c. flour
1 egg	5 tbsp. butter
3 tsp. baking powder	$\frac{1}{2}$ tsp. salt

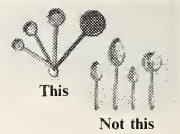
1. Which units of measure will Cathy use in making the loaf cake? cup, teaspoon, tablespoon

Cathy uses a measuring cup, not a teacup. She also uses measuring spoons. She fills the spoons and the cup level full, not heaping.

2. Cathy has no tablespoon. How can she use a teaspoon to measure the sugar for the muffins? (1 tbsp. = 3 tsp.)

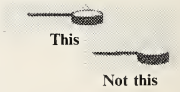
3. On a measuring cup, the $\frac{3}{4}$ mark is higher than the $\frac{2}{3}$ mark. Then is $\frac{2}{3}$ c. of sugar more or (less) than $\frac{3}{4}$ c.?

4. The muffins take 3 tbsp. of butter and the cake takes 5 tbsp. So the muffins take $\frac{3}{5}$ as much butter as the cake.



5. What part of a cup is 3 tbsp.? $\frac{3}{16}$

6. The 1 c. of milk for the cake is what part of a quart? $\frac{1}{4}$ of a pint? $\frac{1}{2}$



7. About how much more sugar does the cake need than the muffins? About $\frac{1}{2}$ cup more ($7\frac{1}{2}$ tbsp.)

8. Tell some other ways in which fractions are used.

Teaching Pages 96, 97, and 98

Pupil's Objectives. (*a*) To take Inventory Test 10 as a way of discovering how much of the previous work with fractions has been retained; (*b*) to relearn the meaning of unit and multiple fractions and the meaning of the words *numerator*, *denominator*, and *terms*; (*c*) to use diagrams as the basis for writing fractions both in words and in figures.

Background. The Inventory Test on page 97 in the text has a dual purpose. It will help you decide how well your pupils understand and can use cooking measures while uncovering the extent of their ability to work with fractions.

Familiarity with the various kinds of cooking measures and ability to use them correctly is, of course, important, since it represents a kind of learning that is extremely practical. The phase of the test that deals with fractions is also important, since most of Chapter 3 is devoted to reteaching fractions, and you will want to know how best to plan your program to meet individual needs.

Notice that the review on page 98 is to be done as written work since by now the ideas presented there should be pretty completely understood. However, you may want to make sure after the book lesson that pupils are thoroughly familiar with the words *numerator* and *denominator* and their meanings since any confusion will retard progress in later work.

Teacher's Preparation. An understanding of fractions is best developed, in the early stages, by the use of some of the concrete and semi-concrete materials described under "Devices" in Part IV of this Manual (pages 310–315). Particularly recommended are the following: calculating strips for fractions; fraction slide rule; fractional pieces (some prepared for use with a flannel board). Other desirable materials are area squares, bead lines, a flannel board, an n-stand, discs or other quantity objects, ribbon or tape, and graph paper. See also "Commercial Teaching Aids" (pages 320–322) for further suggestions concerning films, devices, and games that may be purchased.

At this time you may want to consider possibilities for pupil participation in planning, developing, and organizing the construction, use, and display of other learning aids.

Book Lesson (pages 96 and 97). *Ex. 1–8:* Oral work. Inventory Test 10.

1. Constantly refer to the pictures and recipes on these two pages during the discussion.

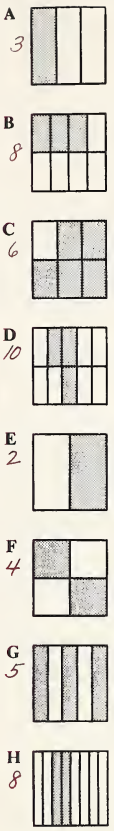
2. If possible, use measuring cups, tablespoons, teaspoons, and quart and pint containers to demonstrate the various relationships brought out on page 97, but be ever on the alert for concepts and abilities that will need reteaching.

Pre-book Lesson (page 98). You may want to use diagrams and charts on the board to bring out clearly fractional parts of a whole. *Slower learners* may also benefit from activities involving

(*Continued on page 118*)

The Numbers Called Fractions

Meaning; writing fractions; terms of fraction [W]



Squares A to H are the same size. Each of the squares is divided into equal parts.

1. On your paper write the letters, A, B, C, and so on, in a column. After your letter A write the number of equal parts in square A; after B, the number of equal parts in square B; and so on, to H. (*See diagrams.*)

2. Square A has 3 equal parts. Think of one of the parts—the one that is colored. What part of the square is this? Write your answer in words and in figures. *one third, or* $\frac{1}{3}$

The **terms** of the fraction you wrote are 1 and 3. The names of the terms are given below:

$\frac{1}{3}$ *numerator*
denominator

The **denominator** helps you think about the size of each equal part by showing you the number of equal parts in the whole square. The **numerator** tells you that you are to think about *one* of the three equal parts.

3. Think of 2 of the 3 equal parts of square A (the white part). Write in words and in figures the fraction that stands for this part of square A. Then write what the denominator and the numerator mean. *two thirds* $\frac{2}{3}$ *3* *2* *2 of the 3 equal parts of a whole*

4. After B, C, D, and so on, write in words and in figures the fraction of each square that is colored and the fraction that is white. ★

A fraction is a number which means one or more equal parts of a whole.

98

★Answers Not on Reproduced Page 98

4.

Square	Colored	White
A	one third, $\frac{1}{3}$	two thirds, $\frac{2}{3}$
B	three eighths, $\frac{3}{8}$	five eighths, $\frac{5}{8}$
C	five sixths, $\frac{5}{6}$	one sixth, $\frac{1}{6}$
D	three tenths, $\frac{3}{10}$	seven tenths, $\frac{7}{10}$
E	one half, $\frac{1}{2}$	one half, $\frac{1}{2}$
F	two fourths, $\frac{2}{4}$	two fourths, $\frac{2}{4}$
G	three fifths, $\frac{3}{5}$	two fifths, $\frac{2}{5}$
H	two eighths, $\frac{2}{8}$	six eighths, $\frac{6}{8}$

cutting various geometric shapes (e.g., rectangles, circles, triangles, etc.) into fractional parts. These may be pasted to larger sheets, labeled, and put on display.

Book Lesson (page 98). *Ex. 1–4:* Written work.

Differentiations and Extensions

1. *All children:*

a. May help collect some of the materials mentioned under Teacher's Preparation. *More capable pupils* may act as group leaders, helping to see that directions are followed and that measurements are correct.

b. May bring from home for discussion recipes containing fractions.

2. *More capable pupils:*

a. May use the following information in writing statements or questions that will bring out fractional relationships:

Standard sizes for ladles used in serving are $\frac{1}{3}$ cup, $\frac{3}{4}$ cup, and 1 cup.

Standard sizes for scoops are $\frac{2}{3}$ cup, $\frac{1}{2}$ cup, $\frac{2}{5}$ cup, $\frac{1}{3}$ cup, and $\frac{1}{4}$ cup.

b. May draw diagrams to show in as many ways as they can the meaning of $\frac{1}{4}$, or any other fractional part of a whole.

Teaching Pages 99 and 100

Pupil's Objectives. (*a*) To develop increased understanding of the fractional-unit idea; (*b*) to show proper and improper fractions and mixed numbers on a number line.

Background. The concept of a fractional unit was introduced in Grade 5 and is retaught on page 99 in the text. This is an exceedingly important concept, since it introduces the idea of size which does not usually occur when pupils are confined to thinking about denominators or the number of equal parts in a whole. To tell the fractional unit for the fraction $\frac{2}{3}$, one refers to the denominator to discover how many equal parts there are in the whole (3). One of these equal parts ($\frac{1}{3}$) names the fractional unit. However, to get a feeling for the *size* of the fractional unit, one must know the size of the whole to which the fraction $\frac{2}{3}$ refers.

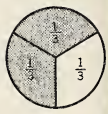
Since the meaning of the term "fractional unit" carries the idea of size, it is more enlightening to state the usual generalization *Only like-fractions can be added or subtracted* in the form *Only fractions with the same fractional unit can be added or subtracted.* The former generalization is permissible, but pupils generally interpret like-fractions to be those with like-denominators, yet $\frac{1}{3}$ of an orange and $\frac{2}{3}$ of a melon cannot be added even though the denominators in $\frac{1}{3}$ and $\frac{2}{3}$ are the same.

The concept of fractional unit then is as important as the idea that there are different kinds of measurement units. Just as a pupil must think of the measurement unit 1 inch when trying to estimate the length of a book, so must he think of a piece the size of one of the three equal parts

What Is a Fractional Unit?

Meaning: size [O]

Just as we can think of a yard as being made up of 3 smaller units called feet or of a pound as being made up of 16 smaller units called ounces, we can think of any whole as being made up of a number of equal parts. We call the size of one of these equal parts the **fractional unit** for that whole thing. The fractional unit for circle A is $\frac{1}{3}$.

A 

1. For each of circles B–D, tell what the fractional unit is and give the denominator of the fraction that names the fractional unit. (See diagrams.)

B

2. Circles A, B, C, and D are equal in size. Does the size of the fractional unit get larger or smaller as the denominator gets larger? Why? more but smaller equal parts

3. In these equal circles, which is larger, $\frac{1}{3}$ or $\frac{1}{4}$? $\frac{1}{6}$ or $\frac{1}{5}$? $\frac{1}{5}$ or $\frac{1}{8}$? $\frac{1}{6}$ or $\frac{1}{4}$?

C

4. In circle A, 2 of the equal parts are green. We can think of the green part of circle A as two $\frac{1}{3}$'s, or $\frac{2}{3}$. What does the numerator, 2, tell? That 2 of the 3 parts are colored.

5. In circle B, 4 equal parts are green. What fraction tells how much of circle B is green? $\frac{4}{5}$ Which term of the fraction tells the number of equal parts that are green? numerator Which term shows the size of the fractional unit? denominator

D

6. In C, how many $\frac{1}{4}$'s in $\frac{3}{4}$? 3 In D, how many $\frac{1}{6}$'s in $\frac{5}{6}$? 5

The denominator shows the size of the fractional unit, and the numerator tells the number of equal parts in the fraction.

[W]

7. For each of these fractions, write the fractional unit and the number of equal parts in the fraction: $\frac{4}{5}$; $\frac{2}{7}$; $\frac{7}{10}$; $\frac{1}{2}$; $\frac{8}{100}$. $\frac{1}{5}$, 4; $\frac{1}{7}$, 2; $\frac{1}{10}$, 7; $\frac{1}{2}$, 1; $\frac{1}{100}$

99

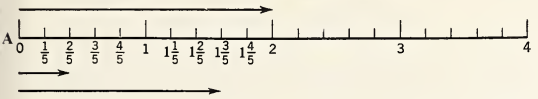
Fractions on a Number Line

Proper and improper fractions; mixed numbers [O]

A whole number, such as 6 or 52 or 1,280, tells how many wholes there are. Proper and improper fractions, such as $\frac{7}{8}$, $\frac{4}{5}$, and $\frac{5}{3}$, tell how many equal parts there are. So a mixed number, such as $2\frac{3}{4}$, is the sum of wholes and equal parts. $2\frac{3}{4} = 2 + \frac{3}{4}$.

1. Explain what these proper fractions mean: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{16}$; these improper fractions: $\frac{9}{4}$, $\frac{12}{7}$, $\frac{11}{6}$; these mixed numbers: $7\frac{9}{10}$, $8\frac{2}{3}$, $5\frac{1}{4}$.

Whole numbers, fractions, and mixed numbers may be shown on a number line like either A or B below. On line A, the fractions represented by the small divisions are fifths.

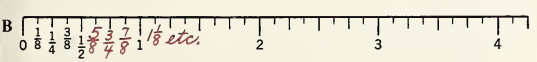


[W]

2. On line A, the whole number 2 stands for the distance from 0 to 2. In the same way, the fraction $\frac{2}{5}$ stands for the distance from 0 to $\frac{2}{5}$. Which arrow shows this distance? the first arrow under line A

3. What mixed number stands for the distance shown by the second arrow under line A? $1\frac{3}{5}$

4. Copy line A as well as you can and under it show with arrows these distances: $\frac{4}{5}$; $2\frac{2}{5}$; 3.



5. Line B shows the edge of a ruler. With your ruler, draw a line like this. Label the equal parts of each inch. (See line B.)

6. Above your line B, show with arrows a distance of $\frac{7}{8}$ in.; of $1\frac{1}{4}$ in.; of $3\frac{5}{8}$ in. Label each arrow.

7. Below your line, show with an arrow the distance $2 + 1$; $\frac{1}{4} + \frac{3}{4}$; $1 + \frac{3}{8}$; $1\frac{1}{2} + 2\frac{1}{2}$.

of a whole when trying to estimate $\frac{2}{3}$ of a circle. Moreover, just as pupils need many experiences in working with actual measurement units, they also need much experience in working with parts of real objects and groups if they are to become proficient in work with fractions.

Proper and improper fractions and mixed numbers are retaught on page 100. Pupils are asked to show these various kinds of numbers on a number line; then the parallel between a ruler and a number line is drawn and pupils are asked to show $\frac{7}{8}$ of an inch, $1\frac{1}{4}$ inch, and $3\frac{5}{8}$ inch. Finally, in Ex. 7, they are prepared for a review of adding fractions by being asked to show distances such as $\frac{1}{4} + \frac{3}{4}$ and $1 + \frac{3}{8}$ on a number line.

Teacher's Preparation. Have the following materials available: prepared circles, squares, and other units to show fractional parts (or materials for making them); cardboard cut to represent candy bars; and, possibly, a number line similar to line B on page 100 in the text which would be suitable for classroom display.

Pre-book Lesson (page 99). Give pupils experience in using fractional units to measure whole units. They may do this by using fractional parts cut from oak tag the size of fractions drawn on the board to be measured. *Slower learners* may also benefit from making their own parts and units to keep in individual boxes for further practice work.

Book Lesson (page 99). *Ex. 1–6:* Oral work. *Ex. 7:* Written work. For Ex. 4, 5, and 6, emphasize the idea that the larger the denominator, the more equal parts there are in the whole, and, therefore, the smaller is each equal part (hence, also, the fractional unit). Let two, then three, then more pupils share a piece of oak tag representing a candy bar. From this experience, pupils should come to realize that the more parts into which something is divided, as indicated by the denominators, the smaller each part will be.

Book Lesson (page 100). *Ex. 1:* Oral work. *Ex. 2–7:* Written work.

★Answers Not on Reproduced Page 100

1. *Proper fractions:* $\frac{3}{4}$ means 3 of the 4 equal parts of a whole; $\frac{2}{3}$ means 2 of the 3 equal parts of a whole; etc.

Improper fractions: $\frac{9}{4}$ means 9 of the 4 equal parts of a whole; $\frac{12}{7}$ means 12 of the 7 equal parts of a whole; etc.

Mixed numbers: $7\frac{9}{10}$ means $7 + \frac{9}{10}$, or 7 wholes and 9 of the 10 equal parts of a whole; $8\frac{2}{3}$ means $8 + \frac{2}{3}$, or 8 wholes and 2 of the 3 equal parts of a whole; etc.

Differentiations and Extensions

1. *All children*

a. Have them write in their notebooks any terms which they feel they need to record for further study. Definitions, diagrams, and illustrations may also be entered.

b. Have them show improper fractions and mixed numbers using various sizes and shapes as well as a number line.

2. *Slower learners* will need many experiences in manipulating, identifying, reproducing, and comparing fractional parts, improper fractions, and mixed numbers. If possible, allow them to keep fractional pieces and whole units in individual boxes so that they will be accessible for small-group work. Encourage them to use these for study during spare time. Attempt to supervise individual study of this kind as lessons are developed so that the practice will be profitable.

Teaching Page 101

Pupil's Objective. To review the meaning of fractions which refer to groups rather than wholes.

Background. A fraction can be used to describe a group in the same way that it may be used to describe a whole. Pupils have already encountered this idea in earlier grades in the form of fractional-part division ($\frac{1}{7}$ of 28 = 4). They have also previously encountered the idea that one of a number of objects in a homogeneous group equals one of the equal parts of that group. If, as in Ex. 1 on page 101 in the text, there are 7 children in a group, one child represents 1 of the 7 equal parts and 4 children equal $\frac{4}{7}$ of the whole. This idea is not difficult, but it is important for children to have as many experiences in finding parts of groups as they have had in finding parts of wholes.

Pre-book Lesson. Divide a number of objects (pencils, books, discs) into equal stacks. Have pupils tell how many parts in all and the name of the fractional unit represented by one of the stacks. Then have them find $\frac{2}{3}$, $\frac{3}{4}$, or $\frac{7}{8}$ of a group until they are able with ease to find more than one of the equal parts of a group.

Book Lesson. *Ex. 1 and 2:* Oral work. *Ex. 3–7:* Written work. Before pupils start the written work, have at least the *slower learners* determine the fractional unit for the fractions in each exercise.

Differentiations and Extensions

1. *All pupils* may obtain practice in finding fractional parts of a group by solving problems based on the information in the next column.

Equal Parts of a Group

[O]

You have been thinking about equal parts of a single thing. You can also find equal parts of a group.

1. At Ann's birthday party, 4 of the 7 children are girls. What part of the group is girls? $\frac{4}{7}$

Each girl is 1 of the 7 equal parts of the group, or $\frac{1}{7}$ of it. So 4 girls are 4 - $\frac{1}{7}$'s, or $\frac{4}{7}$, of the group at the party.

2. Two of the 5 apples in a basket are red. How many equal parts are red? 2 What part of the apples is red? $\frac{2}{5}$ What part is not red? $\frac{3}{5}$

[W]

Write answers for the following:

3. Jack has read 7 of the 12 pages of a story. What part of the story has he read? $\frac{7}{12}$

4. Sue used 3 of her 10 stamps for mailing letters. What part of her stamps did she use? $\frac{3}{10}$

5. Helen saved \$5 of the \$12 that she got for her birthday. What part of her money did she save? $\frac{5}{12}$

6. Draw four bananas and color $\frac{3}{4}$ of them.

7. Draw nine candles and color $\frac{4}{9}$ of them.

101

Articles	*Saved*	*Spent or Used*
\$25.00	$\frac{1}{5}$	$\frac{4}{5}$
50¢	$\frac{1}{2}$	$\frac{1}{2}$
20 crayons	$\frac{1}{4}$	$\frac{3}{4}$
48 cookies	$\frac{5}{6}$	$\frac{1}{6}$
35 apples	$\frac{2}{7}$	$\frac{5}{7}$

2. *Slower learners* may use calculating strips for common fractions, fractional pieces and a flannel board, and pictures or flat objects with an n-stand as an aid in increasing their understanding of the meaning of fractions that apply to groups.

Teaching Pages 102 and 103

Pupil's Objective. To compare fractions by means of a number line, thus discovering equivalents and relationships.

Background. The number lines at the top of page 102 in the text are extremely useful. They show the most commonly-used fractions and you may, therefore, wish to have pupils insert a replica of the chart in their arithmetic notebooks for ready reference.

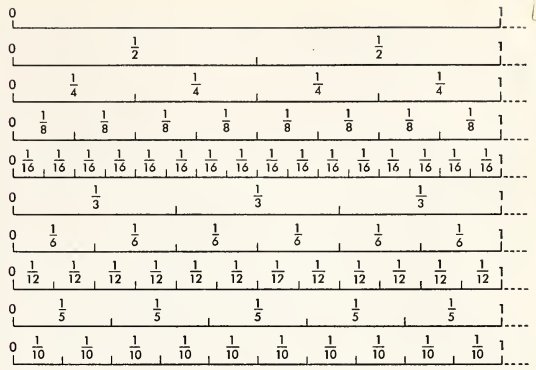
In the work in the text pupils are to use the number lines to make comparisons and discover relationships. For example, they derive the generalization shown in dark print on page 102. They compare unit fractions, thereby setting the stage for discovering that the fewer equal parts there are in a whole, the larger is each part, or the smaller the denominator the larger is the fraction. They then compare equal fractions as a basis for changing to higher or lower terms and for learning equivalents. Finally, they use the number line to help them tell which fractions are equal to $\frac{1}{2}$, which are less than $\frac{1}{2}$, more than $\frac{1}{2}$, and which are equal to 1 whole. This type of practice is given so that they will be proficient in estimating with fractions later on.

Notice that in all work on these two pages pupils are to refer to the number lines. On pages 106 and 107 in the text they will be introduced to the Golden Rule of Fractions and will review the changing of fractions to higher or lower terms without the use of the number lines.

Teacher's Preparation. Compare using the chart on this page with using calculating blocks and strips for common fractions (see page 312 of this Manual). The principles involved in these devices are basically the same, for all three demonstrate relationships existing among fractions. It is, of course, advantageous to have the chart, a set of strips, and a set of blocks available so that you may provide a variety of experiences in comparing fractions.

Book Lesson. *Ex. 1–13:* Oral work. *Ex. 14–30:* Written work.

1. Record fractional equivalents for a whole (Ex. 1–7) on the board, allowing pupils to state a generalization similar to the one in the middle of the page.



Fractions We Use Often

Comparing fractions [O]

These number lines, picturing the distance from 0 to 1, show the fractions we use most often.

1. Find by counting on the second line how many $\frac{1}{2}$'s = 1. 2

2. Find how many $\frac{1}{8}$'s = 1. 8

3. How many $\frac{1}{10}$'s = 1? 10

4. $\frac{5}{5} = 1$

5. $\frac{3}{3} = 1$

6. $\frac{12}{12} = 1$

7. $\frac{4}{4} = 1$

A fraction equals 1 when the number of parts shown by the numerator equals the number of parts in the whole.

8. Using the number lines, tell which is more (longer):
a. $\frac{1}{3}$ or $\frac{1}{2}$; **b.** $\frac{1}{6}$ or $\frac{1}{8}$; **c.** $\frac{1}{10}$ or $\frac{1}{12}$; **d.** $\frac{3}{8}$ or $\frac{1}{4}$; **e.** $\frac{3}{10}$ or $\frac{1}{8}$.

9. By comparing lengths on the lines, find the missing numerators: $\frac{2}{8} = \frac{1}{4}$; $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{4}{12}$.

Name the largest (*l.*) and the smallest (*s.*) fraction in each group:

10. $\frac{1}{2}$ *l.*, $\frac{1}{4}$, $\frac{1}{6}$ *s.*

11. $\frac{1}{8}$, $\frac{1}{3}$ *l.*, $\frac{1}{5}$ *s.*

12. $\frac{1}{16}$ *s.*, $\frac{1}{10}$, $\frac{1}{12}$ *l.*

13. $\frac{1}{6}$ *l.*, $\frac{1}{10}$, $\frac{1}{8}$ *s.*

2. For Ex. 22, suggest a line either 4 inches or 8 inches long.

3. If pupils correct either their own or their partners' papers in a developmental lesson such as this is, you will have an added opportunity for detecting confusions and for reinforcing understandings.

Differentiations and Extensions. For reference, place on the board the answers for Ex. 27 and 30. Verify some of the answers with semi-concrete materials of various shapes and sizes, possibly on a flannel board. Also, compare the equivalents with measurement numbers ($\frac{3}{3}$ yd. = 1 yd.; $\frac{2}{4}$ hr. = $\frac{1}{2}$ hr.;

[W]

For each fraction below, use number lines on page 102 and find the missing numerator.

14. $\frac{1}{3} = \frac{2}{6}$ 15. $\frac{3}{12} = \frac{1}{4}$ 16. $\frac{2}{8} = \frac{4}{16}$ 17. $\frac{4}{10} = \frac{2}{5}$

18. $\frac{6}{8} = \frac{3}{4}$ 19. $\frac{6}{16} = \frac{3}{8}$ 20. $\frac{9}{12} = \frac{3}{4}$ 21. $\frac{3}{5} = \frac{6}{10}$

22. Now draw a long number line and divide it into 32ds. On the same line

a. show $\frac{1}{2}$'s with red division marks;
b. show $\frac{1}{4}$'s with green division marks;
c. show $\frac{1}{8}$'s with blue division marks;
d. show $\frac{1}{16}$'s with brown division marks.

23. Copy the following fractions and write the missing numerators. Use the line you drew for Ex. 22 to help you.

$\frac{1}{2} = \frac{16}{32}$; $\frac{1}{4} = \frac{8}{32}$; $\frac{1}{8} = \frac{4}{32}$; $\frac{1}{2} = \frac{4}{8}$; $\frac{3}{8} = \frac{6}{16}$; $\frac{6}{8} = \frac{3}{4}$.

Use the number lines on page 102 to help you find answers for the following:

24. Which of these fractions equal $\frac{1}{2}$?

$\frac{8}{16}$ $\frac{4}{12}$ $\frac{6}{10}$ $\frac{4}{8}$ $\frac{2}{3}$ $\frac{5}{10}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{3}{5}$

25. Which of these fractions are less than $\frac{1}{2}$?

$\frac{1}{3}$ $\frac{2}{12}$ $\frac{4}{10}$ $\frac{3}{5}$ $\frac{5}{16}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{5}{6}$ $\frac{5}{12}$

26. Which of these fractions are more than $\frac{1}{2}$?

$\frac{5}{6}$ $\frac{2}{3}$ $\frac{7}{16}$ $\frac{3}{4}$ $\frac{3}{10}$ $\frac{4}{5}$ $\frac{7}{8}$ $\frac{3}{12}$ $\frac{9}{16}$

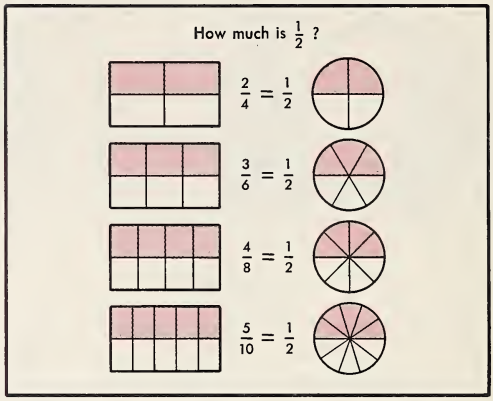
27. Make a list of all fractions on the number lines equal to $\frac{1}{2}$. ★

28. Make another list of all fractions on the number lines which are less than $\frac{1}{2}$. ★

29. Make still another list of all fractions on the number lines which are greater than $\frac{1}{2}$. ★

30. Now make a list of all the fractions on the number lines which are equal to 1. ★

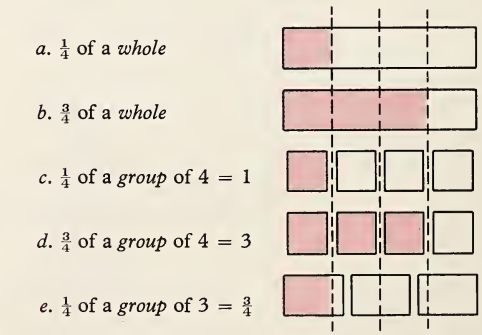
etc.). A chart showing equivalents for $\frac{1}{2}$ (see top of next column) will allow you to ask pupils to compare the size of the fractional unit with the size of the fraction colored. This practice is good readiness work for the later development of rounding mixed numbers.



Teaching Pages 104 and 105

Pupil's Objectives. (*a*) To review the idea that a fraction may mean division; (*b*) to see again how a remainder may be shown in a fraction; (*c*) to obtain practice in division when remainders are to be written in fractions.

Background. Some of the meanings of a fraction are shown in *a* to *e* below. The two cases shown in *a* and *b* deal with wholes, whereas *c*, *d*, and *e* deal with groups. The meanings in *a* to *d* have already been reviewed. Notice that in *c* and *d* the size of the group (4) is large enough to give 1 whole to each of the 4 equal parts into which the group is to be divided.



A fraction may also mean division. This meaning is shown in *e*, above, and is covered on page 104 in the text. Notice in *e* that the fraction $\frac{3}{4}$ means

★Answers Not on Reproduced Page 103

27. *Fractions equal to* $\frac{1}{2}$: $\frac{2}{4}, \frac{4}{8}, \frac{8}{16}, \frac{3}{6}, \frac{6}{12}, \frac{5}{10}$

28. *Fractions less than* $\frac{1}{2}$: $\frac{1}{4}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{1}{16}, \frac{2}{16}, \frac{3}{16}, \frac{4}{16}, \frac{5}{16}, \frac{6}{16}, \frac{7}{16}, \frac{1}{3}, \frac{1}{6}, \frac{2}{6}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}, \frac{1}{5}, \frac{2}{5}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$

29. *Fractions greater than* $\frac{1}{2}$: $\frac{2}{2}, \frac{3}{4}, \frac{4}{4}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}, \frac{9}{16}, \frac{10}{16}, \frac{11}{16}, \frac{12}{16}, \frac{13}{16}, \frac{14}{16}, \frac{15}{16}, \frac{16}{16}, \frac{2}{3}, \frac{3}{3}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}, \frac{12}{12}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$

30. *Fractions equal to 1:* $\frac{2}{2}, \frac{4}{4}, \frac{8}{8}, \frac{16}{16}, \frac{3}{3}, \frac{6}{6}, \frac{12}{12}, \frac{5}{5}, \frac{10}{10}$

that 3 whole objects are to be divided into 4 equal parts but that there are not enough in the group of 3 to give at least one whole to each of the 4 equal groups.

On page 105 pupils learn how to show a remainder in division in a fraction. In many cases, such as in Ex. 1 in the text, this is the only way of handling the remainder that makes sense as far as the problem situation is concerned. In other cases, such as in the problem below, it would *not* make sense to show the remainder in a fraction.

Nine children were to be divided into two groups. How many were in each group?

In the problem just given, an answer of $4\frac{1}{2}$ would be quite inappropriate since the quotient, including the fraction $\frac{1}{2}$, refers to children. Therefore, the remainder should be shown with R and in referring to the problem situation, pupils would have to say that there would be one child left over, or that one team would have 4 children and another would have 5.

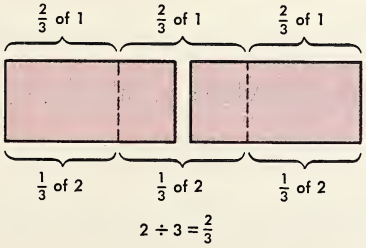
Notice that the fraction in box A on page 105 represents the first meaning of a fraction discussed on the previous Manual page. If the problem had been to divide 9 doughnuts among 5 people, the fraction $\frac{4}{5}$ would represent the meaning shown in *e*.

Teacher's Preparation. For the pre-book lesson related to page 104, have ready cardboard representing candy bars, and some tape, ribbon, or string.

Pre-book Lesson (page 104)

1. Cut one cardboard representing a candy bar to show $\frac{2}{3}$.

2. Ask how two candy bars could be shared equally among 3 children. Draw a picture on the board to represent the solution.



3. Have a pupil try to divide 3 yards of tape, ribbon, or string into 4 equal parts. If anyone suggests $\frac{3}{4}$ yd. as the answer, let him prove it by cutting the tape into 4 equal parts.

A Fraction May Mean Division

[O]

You have been thinking of the fractions $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{11}{5}$ and so on as numbers meaning one or more of the equal parts of something. But a fraction has other meanings too.

You know these two ways of writing a division example: $3\overline{)8}$ and $8 \div 3$. You could also write this example as the improper fraction $\frac{8}{3}$. Any fraction, proper or improper, may stand for a division. The fraction $\frac{3}{8}$ means $3 \div 8$. To see how this works, try to show by a diagram how 3 candy bars could be divided equally among 8 boys. Would each boy get $\frac{3}{8}$ of a bar? Yes

So, depending on its use, $\frac{3}{8}$ may be

a. a number and mean 3 of the $\frac{1}{8}$'s of something; or

b. a division and mean that the dividend, 3, is to be divided by the divisor, 8.

1. Tell two meanings for each of these fractions:

$\frac{11}{8}$ eleven $\frac{1}{8}$'s, $11 \div 8$; $\frac{9}{7}$ nine $\frac{1}{7}$'s, $9 \div 7$; $\frac{7}{10}$ seven $\frac{1}{10}$'s, $7 \div 10$; $\frac{5}{9}$ five $\frac{1}{9}$'s, $5 \div 9$; $\frac{2}{3}$ two $\frac{1}{3}$'s, $2 \div 3$; $\frac{4}{5}$ four $\frac{1}{5}$'s, $4 \div 5$; $\frac{7}{4}$ seven $\frac{1}{4}$'s, $7 \div 4$; $\frac{6}{6}$ six $\frac{1}{6}$'s, $6 \div 6$

A fraction may be a number and mean one or more equal parts; or it may stand for the division of the numerator by the denominator.

[W]

2. Write the following fractions:

a. the number meaning "seven $\frac{1}{8}$'s." $\frac{7}{8}$

b. the division "9 divided by 4." $\frac{9}{4}$

c. the division "5 divided by 6." $\frac{5}{6}$

d. the number meaning "eight $\frac{1}{3}$'s." $\frac{8}{3}$

e. the division "10 divided by 7." $\frac{10}{7}$

3. Write four proper fractions, that is, fractions which have numerator smaller than denominator and mean less than 1.

4. Write four improper fractions, that is, fractions which have numerator equal to or greater than denominator and are equal to 1 or more than 1.

Book Lesson (page 104). *Top of page and Ex. 1:* Oral work. *Ex. 2–4:* Written work.

1. Add labels to one of the fractions in Ex. 1. For example, say $\frac{7}{10}$ mi. may mean 7 of the tenths in a mile, or 7 miles divided into 10 equal parts.

2. Discuss the generalization in the middle of the page and relate it to the work with concrete materials and with written fractions.

3. Before pupils start the written work, see if they realize that all parts of Ex. 2 may be expressed as fractions. Be sure, too, that pupils understand the definitions given in Ex. 3 and 4.

Showing a Remainder in a Fraction

Meaning [O]

1. If 9 doughnuts were to be divided among 4 people, how many whole doughnuts would each of the 4 people have? 2 also what part of a doughnut? $\frac{1}{4}$

In box A, explain why the fraction, $\frac{1}{4}$, means that the 1 doughnut left over is divided among the 4 people. The remainder, 1, means 1 doughnut, and $\frac{1}{4}$ means $1 \div 4$

A
$2\frac{1}{4}$
$4\overline{)9}$
8
1

Why would you not write the remainder, 1, in a fraction if you were dividing marbles? marble can-not be divided easily into 4 equal parts.

2. In box B, explain why the fraction $\frac{4}{5}$ is used beside the quotient, 10. To show that the remainder, 4, is to be divided by 5.

B
$10\frac{4}{5}$
$5\overline{)54}$
5
4

3. Make up a problem to fit the example in box B. ★

4–5. Boxes C and D show the work for $59 \div 7$. Make a problem for Ex. C for which you would need to show the remainder with R. ★ Make another problem for Ex. D in which you could show the remainder in a fraction which means division. ★

C	D
8, R3	$8\frac{3}{7}$
$7\overline{)59}$	$7\overline{)59}$
56	56
3	3

6. Make up a rule about when you can show the remainder in a fraction in a division example. when the requirements of the problem make it sensible to do so.

[W]

Work the following and write any remainder in a fraction.

	a	b	c	d	e
7.	$4\overline{)25}$ $6\frac{1}{4}$	$3\overline{)23}$ $7\frac{2}{3}$	$5\overline{)242}$ $48\frac{2}{5}$	$6\overline{)929}$ $154\frac{5}{6}$	$25\overline{)78}$ $3\frac{3}{25}$
8.	$6\overline{)19}$ $3\frac{1}{6}$	$7\overline{)45}$ $6\frac{3}{7}$	$8\overline{)327}$ $40\frac{7}{8}$	$4\overline{)763}$ $190\frac{3}{4}$	$46\overline{)465}$ $10\frac{5}{46}$
9.	$5\overline{)42}$ $8\frac{2}{5}$	$4\overline{)19}$ $4\frac{3}{4}$	$6\overline{)499}$ $83\frac{1}{6}$	$9\overline{)578}$ $64\frac{2}{9}$	$34\overline{)795}$ $23\frac{13}{34}$
10.	$8\overline{)51}$ $6\frac{3}{8}$	$6\overline{)35}$ $5\frac{5}{6}$	$7\overline{)213}$ $30\frac{3}{7}$	$8\overline{)989}$ $123\frac{5}{8}$	$63\overline{)4,663}$ $74\frac{1}{63}$
11.	$5\overline{)37}$ $7\frac{2}{5}$	$9\overline{)65}$ $7\frac{2}{9}$	$4\overline{)387}$ $96\frac{3}{4}$	$3\overline{)769}$ $256\frac{1}{3}$	$79\overline{)2,985}$ $37\frac{62}{79}$

Pre-book Lesson (page 105). Have pupils try to divide the following materials equally among 4 pupils:

a. 6 circles of paper representing cookies
b. 10 inches of ribbon or string
c. 7 pieces of paper
d. 11 pennies
e. 9 pupils for games

If at first they are content to leave *a*, *b*, and *c* as remainders written with R, ask if it is possible to continue division so that all units will be used, but still shared equally. The absurdity of having fractions as remainders for *d* and *e* should be obvious.

Book Lesson (page 105). *Ex. 1–6:* Oral work. *Rows 7–11:* Written work.

Differentiations and Extensions

1. Ask *all pupils* to differentiate between the meanings of fractions recently reviewed by suggesting another illustration for each of the meanings given below.

a. One or more equal parts of a whole

$\frac{3}{4}$ of a pie

_____?_____

b. One or more equal parts of a group

$\frac{3}{4}$ of 20 marbles equals 15 marbles.

_____?_____

c. Numerator divided by the denominator

3 hours divided into 4 equal periods of time is $\frac{3}{4}$ hour.

_____?_____

d. A remainder expressed as a fraction

The average weight of meat for 6 pounds, 5 pounds, 4 pounds, and 6 pounds is $5\frac{1}{4}$ pounds.

_____?_____

2. *More capable pupils* may draw diagrams or pictures to represent the new meaning of a fraction developed on these pages. They should add them to the display which showed the meanings previously studied.

★Answers Not on Reproduced Page 105

3. Mr. Farwell wrapped 54 lb. of salt codfish in 5 packages of equal weight. How much did each package of fish weigh?

4. (Ex. C) If 59 children are divided into 7 equal teams, how many children will be on each team?

5. (Ex. D) How many weeks are there in 59 days?

Teaching Pages 106 and 107

Pupil's Objectives. (*a*) To review the Golden Rule of Fractions; (*b*) to relearn the procedure for, and to practice, changing fractions to higher, lower, and lowest terms.

Background. Pupils have already experienced using number lines on pages 102 and 103 in the text to change the terms of a fraction. In this lesson pupils will learn how to change fractions by using the Golden Rule of Fractions, since this

presents a way of changing without referring to pictures of fractions.

To understand why the Golden Rule of Fractions applies, pupils must realize what happens when the terms of a fraction are increased or decreased. When the fraction $\frac{1}{2}$ is written as the equivalent fraction $\frac{2}{4}$, what has really happened is that we have taken twice as many parts that are $\frac{1}{2}$ as large. In the same way, when we reduce $\frac{2}{4}$ to $\frac{1}{2}$, we are taking $\frac{1}{2}$ as many parts that are twice as large. When working with the numbers in a fraction, this can be accomplished by multiplying or dividing the numerator and the denominator of a fraction by the same number.

One other aspect of changing to higher or lower terms is that pupils must distinguish, as on page 107 in the text, between reducing to *lower* terms and reducing to *lowest* terms. To be proficient in reducing to lowest terms, pupils must be very familiar with the division facts so they will realize when both terms of a fraction can no longer be divided by any number except 1. This idea is stated in the generalization near the middle of page 107 in the text.

Teacher's Preparation. Have available measures, calculating strips for fractions, fractional pieces, and other materials that may be used for comparing fractional equivalents.

Pre-book Lesson

1. Ask pupils to suggest ways, using both concrete and semi-concrete materials, that may be used to review the understanding of equivalent fractions.

2. Place examples such as the following on the board for purposes of discussion:

$\frac{2}{4} = \frac{1}{2}$ $\quad$ $\frac{3}{6} = \frac{1}{2}$ $\quad$ $\frac{4}{8} = \frac{1}{2}$

$\frac{2}{6} = \frac{1}{3}$ $\quad$ $\frac{4}{6} = \frac{2}{3}$ $\quad$ $\frac{12}{16} = \frac{3}{4}$

Lead the discussion in the direction of drawing the following conclusions from an analysis of some of the equivalents above:

a. One of the pair of fractions is in higher terms—that is, the denominator means smaller parts and the numerator means more parts than in the other fraction of the pair.

b. Usually it is more convenient to consider a fraction that is in the lowest terms possible. (Here an analogy may be made by referring to carrying many small articles bundled together to make a few large packages.)

Changing the Form of a Fraction

[O]

1. From the number lines on page 102, you found that $\frac{6}{8} = \frac{3}{4}$; that is, six $\frac{1}{8}$'s = three $\frac{1}{4}$'s. You can quickly change $\frac{6}{8}$ to $\frac{3}{4}$ by dividing both the 6 and the 8 by 2. $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$.

Change the fractions in Ex. 2–4 by dividing as in Ex. 1.

2. $\frac{6}{12} = \frac{1}{2}$ $\quad$ 3. $\frac{4}{12} = \frac{1}{3}$ $\quad$ 4. $\frac{6}{9} = \frac{2}{3}$

5. From the number lines, you found also that $\frac{1}{2} = \frac{4}{8}$; that is, one $\frac{1}{2}$ = four $\frac{1}{8}$'s. Without using the diagram, by what can you multiply both terms of $\frac{1}{2}$ to get $\frac{4}{8}$? 4 $\frac{4 \times 1}{4 \times 2} = \frac{4}{8}$.

Change the fractions in Ex. 6–8 by multiplying as in Ex. 5.

6. $\frac{1}{4} = \frac{4}{16}$ $\quad$ 7. $\frac{3}{8} = \frac{6}{16}$ $\quad$ 8. $\frac{5}{6} = \frac{10}{12}$

The Golden Rule of Fractions is this: Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

Changing to higher terms

9. Study the work in box A.

In this mixed number, the fraction has been changed to another fraction of the same value with a larger denominator and so with a smaller fractional unit. It has been changed **to higher terms.**

A

$2\frac{3}{4} = 2\frac{?}{20}$

$\frac{3}{4} = \frac{5 \times 3}{5 \times 4} = \frac{15}{20}$

$2\frac{3}{4} = 2\frac{15}{20}$

10. Why is $\frac{1}{20}$ a smaller fractional unit than $\frac{1}{4}$ even though the number 20 is larger than the number 4? The more equal parts a whole is divided into, the smaller each part is. [W]

Change each fraction below to higher terms as required.

	a	b	c	d	e
11.	$\frac{3}{4} = \frac{9}{12}$	$\frac{5}{6} = \frac{10}{12}$	$\frac{4}{5} = \frac{8}{10}$	$\frac{1}{4} = \frac{4}{16}$	$\frac{1}{3} = \frac{2}{6}$
12.	$\frac{1}{6} = \frac{3}{18}$	$\frac{2}{3} = \frac{6}{9}$	$\frac{5}{8} = \frac{15}{24}$	$\frac{5}{8} = \frac{10}{16}$	$\frac{3}{5} = \frac{12}{20}$
13.	$3\frac{2}{3} = 3\frac{4}{6}$	$1\frac{3}{8} = 1\frac{6}{16}$	$2\frac{3}{11} = 2\frac{6}{22}$	$3\frac{1}{7} = 3\frac{2}{14}$	$6\frac{2}{9} = 6\frac{4}{18}$

3. Ask a pupil to volunteer to show the "short way" used during the previous year to find the lowest terms of a fraction. Be sure that all pupils understand that dividing the numerator and the denominator by the same number does not change the value of the fraction.

Book Lesson. *Ex. 1–10:* Oral work. *Rows 11–13:* Written work. *Ex. 14–18:* Oral work. *Rows 19–22:* Written work.

1. For Ex. 5, bring out the fact that it will occasionally be necessary to change a fraction to higher terms. Because this operation is the reverse of changing to lower terms, a reverse procedure (that of multiplying instead of dividing) must

Changing to lowest terms [O]

14. Three players are what part of a ball team of 9 players?

One boy is $\frac{1}{9}$ of the team, so 3 boys are $\frac{3}{9}$. By the Golden Rule,

$$\frac{3 \div 3}{9 \div 3} = \frac{1}{3}.$$

This is reducing $\frac{3}{9}$ **to lower terms.**

15. At Sue's party, 18 out of 24 cookies were eaten. Why can you say that $\frac{18}{24}$ were eaten? 18/24 means 18 of a group of 24 equal things.

16. In box B, $\frac{18}{24}$ is reduced first to $\frac{9}{12}$, but $\frac{9}{12}$ is still not the best form. Why? Both terms can be divided by 3.

17. When $\frac{9}{12}$ is reduced to $\frac{3}{4}$, is the fraction $\frac{3}{4}$ in lowest terms? yes HELPER: Can both terms of $\frac{3}{4}$ be divided by any number other than 1? No

18. In box C, we divide both terms of $\frac{18}{24}$ by 6 and get $\frac{3}{4}$ at once. Why is this better than dividing first by 2 and then by 3? Quicker

B

$$\frac{18}{24} = \frac{?}{?}$$

$$\frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

C

$$\frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

To be in best form, fractions in answers should be in lowest terms.

A fraction is in lowest terms when no number except 1 will exactly divide both its terms.

[W]

Reduce to best form if fraction not already in lowest terms.

	a	b	c	d	e	f
19.	$\frac{4}{6}$ 2/3	$\frac{3}{9}$ 1/3	$\frac{10}{12}$ 5/6	$\frac{3}{12}$ 1/4	$\frac{4}{7}$	$8\frac{8}{16}$ 8 1/2
20.	$\frac{7}{14}$ 1/2	$\frac{10}{15}$ 2/3	$\frac{8}{12}$ 2/3	$\frac{7}{10}$	$\frac{8}{24}$ 1/3	$\frac{4}{20}$ 1/5
21.	$\frac{6}{18}$ 1/3	$\frac{5}{12}$	$\frac{6}{20}$ 3/10	$\frac{8}{18}$ 4/9	$\frac{15}{24}$ 5/8	$\frac{7}{28}$ 1/4
22.	$2\frac{6}{10}$ $2\frac{3}{5}$	$5\frac{4}{8}$ $5\frac{1}{2}$	$8\frac{1}{3}$	$7\frac{8}{20}$ $7\frac{2}{5}$	$4\frac{9}{12}$ $4\frac{3}{4}$	$1\frac{6}{24}$ 1/4

Extra Practice. Work Sets 32 and 41.

for multiplying the terms. $\frac{3}{4} = \frac{?}{20}$. $n \times 4 = 20$. $n = 5$. We multiply the numerator by 5.

3. Before assigning the written work on page 106, let pupils determine by what number each term of the first fraction in **a–e** of row 11 must be multiplied.

4. Before assigning the written work on page 107, ask pupils to suggest ways to determine by what number each term is to be divided in reducing fractions. Record a summary of their suggestions, being sure that at least these points are made:

a. For $\frac{6}{12}$, first try dividing by the numerator. 6 will divide both the numerator and the denominator.

b. For $\frac{8}{12}$, first try dividing by the numerator. 8 will not divide both terms. Try 7, 6, 5, etc., until a number that will divide both terms is reached.

c. For $\frac{7}{10}$, no number, other than 1, will divide both terms. Therefore $\frac{7}{10}$ is in lowest terms.

Differentiations and Extensions

1. *All pupils* may record in their notebooks pertinent facts about the Golden Rule of Fractions, with typical examples, after the model plan has been organized and written on the board.

2. *More capable pupils* may be able to determine the divisor for reducing a fraction by considering the factors of the numerator. The only numbers that will evenly divide the numerator of $\frac{9}{12}$ are 9, 3, and 1. 9 will not divide the denominator, but 3 will. Therefore, the divisor is 3.

3. For further practice in changing the terms of fractions, assign **Extra Practice Sets 32 and 41.**

be used. Have pupils substantiate this conclusion by comparing fractions and the equivalents derived by means of materials and diagrams.

2. Some help may be needed, particularly for *slower learners* in determining the number to use

Set 32. Changing fractions to higher and lower terms

Copy, and write the missing numerators.

	a	b	c	d	e	f
1.	$\frac{1}{2} = \frac{2}{4}$	$\frac{3}{6} = \frac{6}{12}$	$\frac{8}{16} = \frac{2}{4}$	$\frac{1}{4} = \frac{3}{12}$	$\frac{3}{4} = \frac{6}{8}$	$\frac{1}{2} = \frac{5}{10}$
2.	$\frac{3}{4} = \frac{9}{12}$	$\frac{3}{5} = \frac{6}{10}$	$\frac{3}{4} = \frac{12}{16}$	$\frac{10}{16} = \frac{5}{8}$	$\frac{1}{2} = \frac{3}{6}$	$\frac{2}{3} = \frac{8}{12}$
3.	$\frac{2}{5} = \frac{4}{10}$	$\frac{4}{6} = \frac{2}{3}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{1}{2} = \frac{8}{16}$	$\frac{1}{4} = \frac{4}{16}$	$\frac{1}{2} = \frac{4}{8}$

Set 41. Reducing fractions to lowest terms

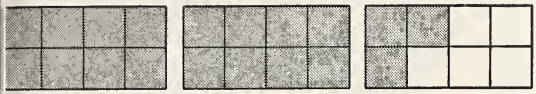
Reduce these fractions to lowest terms.

	a	b	c	d	e	f	g	h	i	j	k	l
1.	$\frac{2}{4}$ 1/2	$\frac{3}{9}$ 1/3	$\frac{2}{8}$ 1/4	$\frac{5}{20}$ 1/4	$\frac{16}{32}$ 1/2	$\frac{4}{36}$ 1/9	$\frac{4}{6}$ 2/3	$\frac{7}{28}$ 1/4	$\frac{4}{12}$ 1/3	$\frac{16}{24}$ 2/3	$\frac{4}{10}$ 2/5	$\frac{6}{16}$ 3/8
2.	$\frac{3}{24}$ 1/8	$\frac{5}{30}$ 1/6	$\frac{8}{32}$ 1/4	$\frac{3}{6}$ 1/2	$\frac{4}{8}$ 1/2	$\frac{2}{12}$ 1/6	$\frac{3}{15}$ 1/5	$\frac{4}{16}$ 1/4	$\frac{25}{40}$ 5/8	$\frac{4}{32}$ 1/8	$\frac{9}{36}$ 1/4	$\frac{8}{10}$ 4/5
3.	$\frac{6}{8}$ 3/4	$\frac{15}{20}$ 3/4	$\frac{24}{36}$ 2/3	$\frac{3}{18}$ 1/6	$\frac{4}{24}$ 1/6	$\frac{8}{48}$ 1/6	$\frac{6}{9}$ 2/3	$\frac{6}{10}$ 3/5	$\frac{9}{12}$ 3/4	$\frac{6}{30}$ 1/5	$\frac{8}{24}$ 1/3	$\frac{6}{18}$ 1/3
4.	$\frac{15}{40}$ 3/8	$\frac{16}{48}$ 1/3	$\frac{2}{10}$ 1/5	$\frac{6}{12}$ 1/2	$\frac{5}{15}$ 1/3	$\frac{8}{12}$ 2/3	$\frac{8}{16}$ 1/2	$\frac{32}{48}$ 2/3	$\frac{10}{15}$ 2/3	$\frac{4}{20}$ 1/5	$\frac{14}{16}$ 7/8	$\frac{10}{30}$ 1/3

Changing Improper Fractions to Best Form

[O]

1. If there are 19 people at Jack's Halloween party and each of the 19 people is served $\frac{1}{8}$ of a cake, how much cake will be eaten?



In the drawing of the 3 cakes, the colored pieces are the $\frac{1}{8}$'s to be served. Count to see how many $\frac{1}{8}$'s are colored. 19

Without counting the $\frac{1}{8}$'s in the picture, tell how many $\frac{1}{8}$'s there are in 1 cake; 8 in 2 cakes. 16 Then how many $\frac{1}{8}$'s will be eaten from the third cake to make $\frac{19}{8}$? 3 (eighths)

How can you tell from the drawing that $\frac{19}{8}$ cakes and $2\frac{3}{8}$ cakes are the same? Yellow shows both 19 and 2 3/8 shaded What kind of fraction is $\frac{19}{8}$? What kind of number is $2\frac{3}{8}$? The improper fraction $\frac{19}{8}$ is in best form when it is changed to the mixed number $2\frac{3}{8}$.

2. If only 16 people are at the party, $\frac{16}{8}$, or 2 cakes, will be eaten. Is 2 a whole number? yes Why is $\frac{16}{8}$ an improper fraction? The improper fraction, $\frac{16}{8}$, is in best form when it is changed to a whole number. numerator is larger than denominator.

3. We usually change an improper fraction to best form by using figures instead of drawings.

To find how many wholes, (that is, how many $\frac{8}{8}$'s) and how many extra $\frac{1}{8}$'s there are in $\frac{19}{8}$, *think*, "$\frac{8}{8}$ = 1 whole, $\frac{16}{8}$ = 2 wholes. Then in $\frac{19}{8}$ there are 2 wholes and $\frac{3}{8}$ more, or $2\frac{3}{8}$." $\frac{19}{8} = 2\frac{3}{8}$

$\frac{19}{8}$ = ?
$\frac{8}{8}$ = 1 whole
$\frac{16}{8}$ = 2 wholes
$\frac{19}{8}$ = 2 + $\frac{3}{8}$

You know that $\frac{19}{8}$ means 19 ÷ 8. Then a quick way to change the fraction to best form is to do the division, writing the remainder in a fraction. $19 \div 8 = 2\frac{3}{8}$.

4. Change the fraction, $\frac{22}{4}$, to a mixed number. $22 \div 4 = 5$ In the box, why is $5\frac{2}{4}$ changed to $5\frac{1}{2}$? better form

$\frac{22}{4} = 4\overline{)22} = 5\frac{2}{4} = 5\frac{1}{2}$

To change an improper fraction to a whole or a mixed number, divide the numerator of the fraction by the denominator.

Answers are in best form when improper fractions have been changed to whole or mixed numbers and when proper fractions are in lowest terms.

[W]

Change to best form:

	a	b	c	d	e	f	g	h
5.	$\frac{15}{8}$ $1\frac{7}{8}$	$\frac{12}{4}$ 3	$\frac{17}{6}$ $2\frac{5}{6}$	$\frac{5}{5}$ 1	$\frac{24}{6}$ 4	$\frac{222}{3}$ 74	$\frac{26}{7}$ $3\frac{5}{7}$	$\frac{42}{5}$ $8\frac{2}{5}$
6.	$\frac{9}{9}$ 1	$\frac{32}{6}$ $5\frac{1}{3}$	$\frac{45}{8}$ $5\frac{5}{8}$	$\frac{30}{6}$ 5	$\frac{28}{8}$ $3\frac{1}{2}$	$\frac{31}{9}$ $3\frac{4}{9}$	$\frac{25}{5}$ 5	$\frac{27}{6}$ $4\frac{1}{2}$

Extra Practice. Work Set 42.

Oral Exercises

1. Change to best form: $\frac{22}{6}$ $3\frac{2}{3}$; $\frac{16}{4}$ 4; $\frac{19}{5}$ $3\frac{4}{5}$; $3\frac{4}{12}$ $3\frac{1}{3}$ Review of meanings
2. What part of an hour is 45 min.? $\frac{45}{60} = \frac{3}{4}$
3. What fraction means 7 of the $\frac{1}{10}$'s of a circle? $\frac{7}{10}$
4. Is $\frac{3}{8}$ more or less than $\frac{1}{2}$? less $(\frac{1}{2} = \frac{4}{8})$
5. What does the number below the fraction line tell? number and size of the equal parts in the whole
6. Give three fractions which are equal to $\frac{3}{4}$. $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}$, etc.
7. What is the fractional unit of $\frac{7}{8}$? $\frac{1}{8}$ $\frac{5}{6}$? $\frac{1}{6}$ $\frac{2}{3}$? $\frac{1}{3}$ $\frac{11}{12}$? $\frac{1}{12}$ $\frac{3}{4}$? $\frac{1}{4}$
8. Why does $\frac{3}{12} = \frac{1}{4}$? $\frac{3 \div 3}{12 \div 3} = \frac{1}{4}$
9. What number contains 59 thousands, 1 hundred, 7 ones? 59,107
10. What is the missing numerator? $\frac{7}{8} = \frac{28}{32}$
11. When would $\frac{1}{3}$ be larger than $\frac{1}{2}$? $\frac{1}{3}$ of a larger thing can be larger than $\frac{1}{2}$ of a smaller thing.
12. In the number 5,042 there are 504 tens in all.

Teaching Pages 108 and 109

Pupil's Objectives. (*a*) To relearn the procedure for changing an improper fraction to its best form and to practice this skill; (*b*) to review orally certain meanings, particularly those relating to fractions.

Background. The key to changing an improper fraction to best form is the number of parts in a whole. This is best shown with objects or with pictures (see page 108 in the text). The reasoning with figures is given after Ex. 3 on page 108, and the quick method for working with figures is given in the generalization at the top of page 109.

Pre-book Lesson. You may review briefly the meaning of an improper fraction by asking pupils to tell which of the following fractions mean as much as, or more than, a whole:

$\frac{3}{3}$ $\frac{2}{3}$ $\frac{7}{6}$ $\frac{5}{4}$ $\frac{8}{8}$ $\frac{9}{10}$ $\frac{4}{2}$ $\frac{5}{6}$ $\frac{6}{6}$ $\frac{5}{3}$

Verify a few answers, particularly where doubt seems to exist, by having pupils manipulate fractional parts or draw diagrams.

Book Lesson (page 108 and top of page 109). *Ex. 1–4:* Oral work. *Rows 5 and 6:* Written work. For the written work, remind *all pupils* to examine answers as a check to make sure that they are in *lowest* terms.

Book Lesson (bottom of page 109). *Ex. 1–12:* Oral work.

Differentiations and Extensions

1. Ask *all pupils* to draw diagrams to represent the improper fractions in rows 5 and 6 on page 109.

2. *Slower learners* may make and play games that require matching equivalent fractions in lower and higher terms or matching improper fractions and equivalent mixed numbers. See Part IV of this Manual for directions for the following appropriate games: "Zooks," "Old Hat," "Single Search," "Over Orange," "Clover."

3. *More capable pupils* may test their understanding by reducing more difficult fractions such as the ones below.

$\frac{25}{50}$	$\frac{30}{60}$	$\frac{24}{60}$	$\frac{12}{24}$	$\frac{15}{60}$	$\frac{30}{90}$
$\frac{84}{24}$	$\frac{50}{20}$	$\frac{60}{48}$	$\frac{60}{24}$	$\frac{150}{60}$	$\frac{100}{40}$

4. Assign **Extra Practice Set 42** as needed.

Set 42. Changing improper fractions

Change to whole numbers or to mixed numbers in best form.

1. a. $\frac{10}{2}$ 5 b. $\frac{21}{9}$ $2\frac{1}{3}$ c. $\frac{58}{8}$ $7\frac{1}{4}$ d. $\frac{35}{16}$ $2\frac{3}{16}$
   e. $\frac{12}{5}$ $2\frac{2}{5}$ f. $\frac{60}{24}$ $2\frac{1}{2}$ g. $\frac{19}{6}$ $3\frac{1}{6}$ h. $\frac{37}{8}$ $4\frac{5}{8}$
   i. $\frac{49}{5}$ $9\frac{4}{5}$ j. $\frac{28}{12}$ $2\frac{1}{3}$ k. $\frac{74}{10}$ $7\frac{2}{5}$ l. $\frac{144}{48}$ 3

2. a. $\frac{32}{3}$ $10\frac{2}{3}$ b. $\frac{52}{10}$ $5\frac{1}{5}$ c. $\frac{30}{6}$ 5 d. $\frac{72}{20}$ $3\frac{3}{5}$
   e. $\frac{18}{12}$ $1\frac{1}{2}$ f. $\frac{73}{5}$ $14\frac{3}{5}$ g. $\frac{45}{25}$ $1\frac{4}{5}$ h. $\frac{83}{3}$ $27\frac{2}{3}$
   i. $\frac{35}{20}$ $1\frac{3}{4}$ j. $\frac{96}{48}$ 2 k. $\frac{35}{2}$ $17\frac{1}{2}$ l. $\frac{63}{4}$ $15\frac{3}{4}$

3. a. $\frac{15}{4}$ $3\frac{3}{4}$ b. $\frac{65}{2}$ $32\frac{1}{2}$ c. $\frac{24}{18}$ $1\frac{1}{3}$ d. $\frac{40}{36}$ $1\frac{1}{9}$
   e. $\frac{54}{3}$ 18 f. $\frac{85}{8}$ $10\frac{5}{8}$ g. $\frac{32}{24}$ $1\frac{1}{3}$ h. $\frac{46}{12}$ $3\frac{5}{6}$
   i. $\frac{75}{16}$ $4\frac{11}{16}$ j. $\frac{100}{25}$ 4 k. $\frac{72}{9}$ 8 l. $\frac{58}{16}$ $3\frac{5}{8}$

Teaching Pages 110 and 111

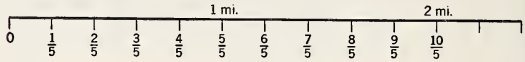
Pupil's Objective. To review and to practice the addition and subtraction of like-fractions as they occur in computation and in problems.

Background. It is really not difficult to add or subtract like-fractions. In fact, it is no more difficult than adding whole numbers, for 2 *sevenths* and 3 *sevenths* equal 5 *sevenths* ($\frac{5}{7}$) just as 2 *pencils* and 3 *pencils* equal 5 *pencils*. This idea is developed in Ex. 1 and 2 on page 110 and stated in the generalization at the bottom of the page. The work in the problems on page 111 also involves like-fractions and gives extended practice in adding and subtracting these fractions when they occur in problem situations. On page 110, pupils are instructed to refer to a number line as an aid in determining answers. For Ex. 7–11 on page 111, they are asked to check answers by drawing number lines.

Adding and Subtracting Like-Fractions

[O]

1. Carl walks $\frac{2}{5}$ mi. to the school-bus stop. Then he rides $\frac{4}{5}$ mi. in the bus. How far does he go on his way to school?



Find the answer for Ex. 1 by using the number line above.

Why do you know that $\frac{2}{5}$ and $\frac{4}{5}$ are like-fractions? They have same fractional unit. HELPER. Do they have the same fractional unit? Yes

A
$\frac{2}{5}$
$+\frac{4}{5}$
$\frac{6}{5} = 1\frac{1}{5}$

In box A, two $\frac{1}{5}$'s plus four $\frac{1}{5}$'s = six $\frac{1}{5}$'s. Why is $\frac{6}{5}$ changed to $1\frac{1}{5}$? Improper fraction in best form when changed to whole or mixed number.

2. Jane rides $\frac{7}{8}$ mi. and Sue $\frac{5}{8}$ mi. in the school bus. How much farther does Jane ride than Sue?

Find the answer by using your ruler as the number line.

In box B, explain the subtraction. Why is $\frac{2}{8}$ changed to $\frac{1}{4}$? better form

B
$\frac{7}{8}$
$-\frac{5}{8}$
$\frac{2}{8} = \frac{1}{4}$

We can add or subtract like-fractions, that is, fractions with the same fractional unit.

The numerators tell how many equal parts are to be added or subtracted. The denominators show the size of the fractional unit.

110

[W]

Copy and add or subtract. Give answers in best form.

3.	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{5}{9}$	$\frac{3}{4}$
	$+\frac{2}{3}$	$+\frac{5}{6}$	$+\frac{5}{8}$	$+\frac{1}{5}$	$+\frac{1}{2}$	$+\frac{9}{10}$	$+\frac{7}{9}$	$+\frac{3}{4}$
	1	1	$1\frac{1}{2}$	$\frac{4}{5}$	1	$1\frac{1}{5}$	$1\frac{1}{3}$	$1\frac{1}{2}$
4.	$\frac{5}{16}$	$\frac{6}{7}$	$\frac{5}{12}$	$\frac{5}{6}$	$\frac{3}{8}$	$\frac{2}{15}$	$\frac{11}{12}$	$\frac{7}{10}$
	$\frac{3}{16}$	$\frac{5}{7}$	$\frac{7}{12}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{3}{10}$
	$+\frac{7}{16}$	$+\frac{2}{7}$	$+\frac{1}{12}$	$+\frac{1}{6}$	$+\frac{7}{8}$	$+\frac{2}{15}$	$+\frac{7}{12}$	$+\frac{9}{10}$
	$\frac{10}{16}$	$1\frac{2}{7}$	$1\frac{1}{2}$	$\frac{5}{6}$	$1\frac{3}{4}$	$\frac{3}{5}$	$1\frac{1}{2}$	$1\frac{9}{10}$
5.	$\frac{5}{8}$	$\frac{4}{5}$	$\frac{9}{10}$	$\frac{7}{8}$	$\frac{7}{12}$	$\frac{7}{10}$	$\frac{7}{9}$	$\frac{7}{16}$
	$-\frac{1}{8}$	$-\frac{2}{5}$	$-\frac{7}{10}$	$-\frac{3}{8}$	$-\frac{5}{12}$	$-\frac{3}{10}$	$-\frac{4}{9}$	$-\frac{3}{16}$
	$\frac{4}{8}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{4}{8}$	$\frac{7}{12}$	$\frac{4}{10}$	$\frac{3}{9}$	$\frac{4}{16}$
6.	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{6}{7}$	$\frac{9}{10}$	$\frac{5}{12}$	$\frac{9}{11}$	$\frac{5}{6}$	$\frac{15}{16}$
	$-\frac{1}{8}$	$-\frac{1}{3}$	$-\frac{2}{7}$	$-\frac{7}{10}$	$-\frac{1}{12}$	$-\frac{3}{11}$	$-\frac{1}{6}$	$-\frac{9}{16}$
	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{4}{7}$	$\frac{1}{5}$	$\frac{4}{12}$	$\frac{6}{11}$	$\frac{4}{6}$	$\frac{7}{16}$

Work Ex. 7–11. Give your answers in best form. Then prove each answer by drawing a number line and using it.

7. Sam and Tom started for the ball game at the same time. Sam arrived in $\frac{1}{4}$ hr. but Tom needed $\frac{3}{4}$ hr. How much longer did it take Tom than it took Sam? $\frac{1}{2}$ hr.

8. Betty bought $\frac{3}{8}$ lb. of chocolate peppermints, and Sue bought $\frac{7}{8}$ lb. How much candy did they buy in all? $1\frac{1}{4}$ lb.

9. Lois had $\frac{7}{12}$ dozen candy pumpkins, Janet had $\frac{5}{12}$ dozen, and Barbara had $1\frac{1}{12}$ dozen. How many dozen candy pumpkins did these girls have together? $1\frac{11}{12}$ doz.

10. Bob's fish weighed $\frac{15}{16}$ pound and John's, $\frac{7}{16}$ pound. How much more did Bob's fish weigh than John's? $\frac{1}{2}$ lb.

11. Jane walked downtown in $\frac{3}{10}$ hr., spent $\frac{7}{10}$ hr. shopping, and rode home with her father in $\frac{1}{10}$ hr. How much time did the whole trip take? $1\frac{1}{10}$ hr., or 1 hr. 6 min.

Extra Practice. For more practice with fractions, work Sets 43 and 44.

Pre-book Lesson

1. Establish a feeling of need for learning the skills embodied in this lesson. Elicit suggestions concerning circumstances when it is necessary to add and subtract fractions and mixed numbers. Lead pupils to realize how often fractions occur in working with measures of time, length, weight, and area.

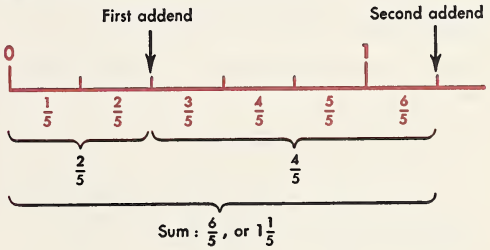
2. Let pupils prove that fractional units must be *alike* for adding and subtracting. Using pieces from wholes of the same size, let them put together fractional pieces that are alike ($\frac{1}{6} + \frac{2}{6}$) and some that are different ($\frac{1}{3} + \frac{1}{2}$). With those that are different, point out that they cannot be added until some change is made so that the pieces put together are all the same size. Assure the class that addition and subtraction of unlike-fractions will be reviewed later following the review of the meaning of a common denominator.

Book Lesson. *Ex. 1 and 2:* Oral work. *Ex. 3–11:* Written work.

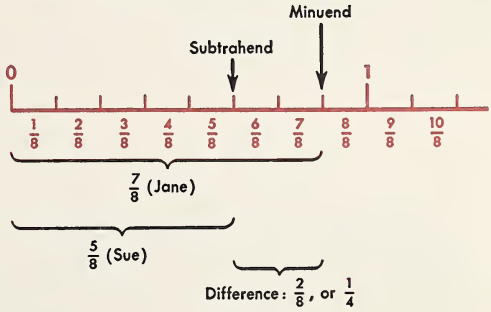
Differentiations and Extensions

1. Have the number lines for Ex. 1 and 2 drawn on the board so that practice in showing addition and subtraction with them will be obtained. This procedure will be used again with the problems on page 111.

Addition. (Ex. 1) To find the sum, count to the right from the first addend.



Subtraction. (Ex. 2) To find the difference (or a remainder), count to the left from the minuend.



2. Observe pupils as they start the written work on page 111. Withdraw those pupils who are having difficulty so that you may help them in a small group. These *slower learners* may need help in remembering to add just the numerators. Allow them to manipulate fractional pieces, then show

the written form for one of the exercises using both words and figures, as illustrated below.

one third	$\frac{1}{3}$
+ two thirds	$+\frac{2}{3}$
three thirds $= 1$	$\frac{3}{3} = 1$

3. Whereas *more capable pupils* may be able to change most answers to best form in one step, *slower learners* may frequently need to use two steps, as shown below.

$\frac{14}{8} = ?$ $\qquad$ $\frac{14}{8} = 1\frac{6}{8} = 1\frac{3}{4}$

4. Pupils who need more practice may work **Extra Practice Sets 43 and 44.**

Set 43. Addition of like-fractions

Add. Reduce answers to lowest terms.

	a	b	c
1.	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ $1\frac{1}{2}$	$\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$ $1\frac{7}{8}$	$\frac{3}{10}+\frac{7}{10}+\frac{9}{10}$ $1\frac{9}{10}$
2.	$\frac{3}{4}+\frac{1}{4}+\frac{3}{4}$ $1\frac{3}{4}$	$\frac{5}{16}+\frac{3}{16}+\frac{11}{16}$ $1\frac{3}{16}$	$\frac{2}{3}+\frac{2}{3}+\frac{1}{3}$ $1\frac{2}{3}$
3.	$\frac{1}{12}+\frac{5}{12}+\frac{11}{12}$ $1\frac{5}{12}$	$\frac{2}{9}+\frac{5}{9}+\frac{7}{9}$ $1\frac{5}{9}$	$\frac{3}{20}+\frac{7}{20}+\frac{19}{20}$ $1\frac{9}{20}$
4.	$\frac{7}{15}+\frac{9}{15}+\frac{2}{15}$ $1\frac{1}{5}$	$\frac{5}{6}+\frac{5}{6}+\frac{1}{6}$ $1\frac{5}{6}$	$\frac{2}{5}+\frac{1}{5}+\frac{4}{5}$ $1\frac{2}{5}$
5.	$\frac{9}{32}+\frac{15}{32}+\frac{19}{32}$ $1\frac{11}{32}$	$\frac{25}{48}+\frac{9}{48}+\frac{17}{48}$ $1\frac{1}{16}$	$\frac{8}{25}+\frac{18}{25}+\frac{4}{25}$ $1\frac{1}{5}$

Set 44. Subtraction of like-fractions

Subtract. Give all answers in best form.

	a	b	c	d	e
1.	$\frac{3}{8}-\frac{1}{8}$ $\frac{1}{4}$	$\frac{7}{10}-\frac{5}{10}$ $\frac{1}{5}$	$\frac{5}{6}-\frac{1}{6}$ $\frac{2}{3}$	$\frac{4}{5}-\frac{3}{5}$ $\frac{1}{5}$	$\frac{15}{16}-\frac{7}{16}$ $\frac{1}{2}$
2.	$\frac{7}{8}-\frac{5}{8}$ $\frac{1}{4}$	$\frac{19}{32}-\frac{15}{32}$ $\frac{1}{8}$	$\frac{7}{12}-\frac{1}{12}$ $\frac{1}{2}$	$\frac{39}{48}-\frac{27}{48}$ $\frac{1}{4}$	$\frac{7}{8}-\frac{3}{8}$ $\frac{1}{2}$
3.	$\frac{5}{16}-\frac{3}{16}$ $\frac{1}{8}$	$\frac{2}{3}-\frac{1}{3}$ $\frac{1}{3}$	$\frac{13}{20}-\frac{7}{20}$ $\frac{3}{10}$	$\frac{8}{15}-\frac{4}{15}$ $\frac{4}{15}$	$\frac{9}{10}-\frac{3}{10}$ $\frac{3}{5}$
4.	$\frac{5}{8}-\frac{3}{8}$ $\frac{1}{4}$	$\frac{7}{16}-\frac{3}{16}$ $\frac{1}{4}$	$\frac{3}{10}-\frac{1}{10}$ $\frac{1}{5}$	$\frac{11}{12}-\frac{5}{12}$ $\frac{1}{2}$	$\frac{13}{16}-\frac{3}{16}$ $\frac{5}{8}$

Adding Mixed Numbers

Without and with carrying [O]

1. Jean had $1\frac{5}{8}$ yd. of ribbon and bought $2\frac{1}{8}$ yd. more. How much ribbon did she have then?

$1\frac{5}{8}$ means $1 + \frac{5}{8}$. What does $2\frac{1}{8}$ mean? $2 + \frac{1}{8}$

We are to find $1 + \frac{5}{8} + 2 + \frac{1}{8}$. Why can we write this "$1 + 2 + \frac{5}{8} + \frac{1}{8}$"? Order of addends does not affect sum.

Box A shows how you can write these mixed numbers to add first the fractions and then the whole numbers. Explain. Check by adding upward.

A

$1\frac{5}{8}$
$+2\frac{1}{8}$
$3\frac{6}{8} = 3\frac{3}{4}$

Why was $3\frac{6}{8}$ changed to $3\frac{3}{4}$? better form

2. On the board find the sum of $3 + \frac{3}{8} + 2\frac{1}{8}$. $5\frac{1}{2}$

3. Tom sold two chickens. One of them weighed $4\frac{3}{4}$ lb. and the other $3\frac{3}{4}$ lb. What was the total weight of the chickens?

B

$4\frac{3}{4}$
$+3\frac{3}{4}$
$7\frac{6}{4} = 8\frac{1}{2}$

Side work
$\frac{6}{4} = 1\frac{2}{4}$, or $1\frac{1}{2}$
$7\frac{6}{4} = 7 + 1\frac{1}{2}$, or $8\frac{1}{2}$

In box B, how do we get $7\frac{6}{4}$? $\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$; $4+3=7$

Explain the side work. How is this like carrying? $\frac{4}{4}$, or 1 whole is carried to one's column.

4. On the board find the sum of $3\frac{3}{8} + \frac{4}{8} + 2\frac{3}{8}$. $5\frac{10}{8}$, or 7

[W]

Copy and add downward. Check by adding upward.

	a	b	c	d	e	f	g
5.	$6\frac{5}{8}$	1	$3\frac{5}{6}$	$3\frac{2}{8}$	$1\frac{1}{2}$	$2\frac{7}{10}$	$3\frac{2}{5}$
	$+3$	$+2\frac{1}{4}$	$+4$	$+5\frac{3}{8}$	$+\frac{1}{2}$	$+2\frac{9}{10}$	$+1\frac{3}{5}$
	$9\frac{5}{8}$	$3\frac{1}{4}$	$7\frac{5}{6}$	$9\frac{5}{8}$	2	$5\frac{3}{5}$	5
6.	$6\frac{2}{7}$	$3\frac{5}{6}$	$4\frac{2}{9}$	$3\frac{5}{16}$	$\frac{3}{10}$	$2\frac{1}{12}$	6
	$\frac{6}{7}$	$1\frac{5}{6}$	$3\frac{2}{9}$	7	$1\frac{1}{10}$	$3\frac{5}{12}$	$1\frac{1}{4}$
	$+3$	$+\frac{1}{6}$	$+1\frac{1}{9}$	$+4\frac{3}{16}$	$+2\frac{7}{10}$	$+1\frac{11}{12}$	$+2\frac{1}{4}$
	$10\frac{6}{7}$	$5\frac{2}{6}$	$9\frac{3}{9}$	$14\frac{3}{16}$	$4\frac{1}{10}$	$7\frac{5}{12}$	$9\frac{2}{4}$

Extra Practice. For more practice, work Set 45.

Teaching Pages 112 and 113

Pupil's Objectives. (*a*) To review addition and subtraction of mixed numbers without and with carrying and borrowing; (*b*) to write problems using numbers given in examples.

Background. No new concepts are needed for adding and subtracting mixed numbers containing like-fractions. This kind of computation is slightly more difficult, however, since carrying and borrowing may be required. Therefore, pupils first see that it is better to add the fraction column first. If the result of this addition is an improper fraction, it can be changed to a mixed number and the correct number of one's carried to the one's column. Similarly with subtraction, if the subtraction in the fraction column is not possible, one whole must be borrowed and changed to the number of parts shown by the fraction part of the minuend. Of course, whenever fractions occur that can be reduced, they should be written in simplest form.

Note that side work is shown in box B on page 112 in the text. Some children will need to write out this step but others will be able to think it. As

Subtracting Mixed Numbers

Without and with borrowing [O]

1. Ray dug $6\frac{3}{4}$ bu. of potatoes and sold $4\frac{1}{4}$ bu. of them. How many bushels were left?

Explain the work in box A, subtracting first the fractions, then the whole numbers.

Why was the answer changed to $2\frac{1}{2}$? for best form

How could you check the answer? Adding

A

$$6\tfrac{3}{4}$$
$$-4\tfrac{1}{4}$$
$$2\tfrac{2}{4} = 2\tfrac{1}{2}$$

2. Mr. Lee has driven $6\frac{7}{10}$ mi. of the 9 mi. to town. How much farther must he drive?

In $9 - 6\frac{7}{10}$ (box B), there are no $\frac{1}{10}$'s from which to take $\frac{7}{10}$. Why can we borrow 1 whole, changing 9 to $8\frac{10}{10}$? $9 = 8+1, \text{or } 8+\frac{10}{10}$,

Explain the subtracting in box B. or $8\frac{10}{10}$

B

$$9 = 8\tfrac{10}{10}$$
$$-6\tfrac{7}{10} = 6\tfrac{7}{10}$$
$$2\tfrac{3}{10}$$

3. From a cheese weighing $6\frac{1}{8}$ lb., Mr. West, the grocer, has cut $2\frac{7}{8}$ lb. How much is left?

In $6\frac{1}{8} - 2\frac{7}{8}$ (box C), $\frac{7}{8}$ cannot be taken from $\frac{1}{8}$. So we must borrow 1 from 6, change that 1 to $\frac{8}{8}$, and add it to the $\frac{1}{8}$, making $\frac{9}{8}$.

Explain all the subtracting in box C.

C

$$6\tfrac{1}{8} = 5\tfrac{9}{8}$$
$$-2\tfrac{7}{8} = 2\tfrac{7}{8}$$
$$3\tfrac{2}{8} = 3\tfrac{1}{4}$$

4. On the board find the remainder for $6 - \frac{3}{4}$. $5\frac{1}{4}$

[W]

Copy and subtract. Check by adding.

	a	b	c	d	e	f	g
5.	$8\frac{3}{5}$	$4\frac{2}{3}$	6	$3\frac{3}{10}$	2	10	$7\frac{1}{4}$
	$-2\frac{1}{5}$	-1	$-3\frac{1}{6}$	$-1\frac{7}{10}$	$-\frac{4}{7}$	$-\frac{4}{5}$	$-5\frac{3}{4}$
	$6\frac{2}{5}$	$3\frac{2}{3}$	$2\frac{5}{6}$	$1\frac{3}{5}$	$1\frac{3}{7}$	$9\frac{1}{5}$	$1\frac{1}{2}$
6.	$6\frac{1}{3}$	$5\frac{4}{7}$	10	$8\frac{1}{6}$	$2\frac{7}{10}$	$5\frac{3}{16}$	$3\frac{7}{20}$
	$-2\frac{2}{3}$	$-1\frac{5}{7}$	$-5\frac{1}{2}$	$-\frac{5}{6}$	$-\frac{7}{10}$	$-1\frac{7}{16}$	$-\frac{9}{20}$
	$3\frac{2}{3}$	$3\frac{6}{7}$	$4\frac{1}{2}$	$7\frac{1}{3}$	2	$3\frac{3}{4}$	$2\frac{2}{10}$

Extra Practice. For more practice, work Set 46.

Write three problems using the numbers in Ex. 5a to 5c.

with all such aids, encourage the dropping of its use as soon as possible.

Pre-book Lesson. Use fractional pieces and put together mixed numbers in which the sums of the fractions will be as much as, or greater than, a whole. Call attention to the fact that only fractions with the same fractional unit can be added. As pupils write the record for the work with materials, have them use side work for changing improper fractions to mixed numbers and for reducing the fractions. Then see how many of your pupils can work similar examples without actually showing the side work.

Book Lesson (page 112). *Ex. 1–4:* Oral work. *Rows 5 and 6:* Written work.

Pre-book Lesson (page 113). Use fractional pieces to show examples in subtraction in which the fraction in the minuend is smaller than the fraction in the subtrahend, thereby requiring conversion of a whole to fractional parts. Record the work on the board. Show how to check the subtraction with both fractional pieces and numbers.

Differentiations and Extensions

1. Give *all pupils* an opportunity to apply learnings in addition and subtraction of fractions and mixed numbers. Consider the possibility of using problems in which measurements are needed for planning or for design.

2. **Extra Practice Sets 45 and 46**, reproduced below with answers, may be assigned as needed.

Set 45. Addition of mixed numbers with like-fractions; without and with carrying

Add. Reduce fractions in answers to lowest terms.

	a	b	c
1.	$3\frac{1}{2} + 2\frac{1}{2}$ 6	$1\frac{1}{4} + 2\frac{3}{4} + 5\frac{1}{4}$ $9\frac{1}{4}$	$6\frac{3}{8} + 3\frac{5}{8} + 2\frac{3}{8}$ $12\frac{3}{8}$
2.	$1\frac{5}{16} + 2\frac{9}{16} + 5\frac{9}{16}$ $9\frac{1}{16}$	$15\frac{3}{8} + 9\frac{1}{8} + 2\frac{3}{8}$ $27\frac{3}{8}$	$3\frac{5}{6} + 1\frac{1}{6} + 9\frac{5}{6}$ $14\frac{5}{6}$
3.	$5\frac{3}{10} + 8\frac{7}{10}$ 14	$18\frac{2}{3} + 6\frac{2}{3}$ $25\frac{1}{3}$	$2\frac{5}{32} + 8\frac{9}{32} + 4\frac{5}{32}$ $14\frac{19}{32}$
4.	$4\frac{5}{12} + 3\frac{7}{12}$ 8	$5\frac{3}{20} + 9\frac{13}{20}$ $14\frac{4}{5}$	$7\frac{5}{24} + 2\frac{7}{24} + 1\frac{19}{24}$ $11\frac{7}{24}$

Set 46. Subtraction of mixed numbers with like-fractions; without and with borrowing

Subtract. Give fractions in answers in best form.

	a	b	c	d
1.	$4\frac{3}{8} - 3\frac{1}{8}$ $1\frac{1}{4}$	$5\frac{5}{16} - 2\frac{3}{16}$ $3\frac{1}{8}$	$14\frac{7}{8} - 6\frac{3}{8}$ $8\frac{1}{2}$	$19\frac{1}{2} - 9\frac{1}{2}$ 10
2.	$15\frac{7}{16} - 12\frac{9}{16}$ $3\frac{1}{8}$	$10\frac{3}{5} - 5\frac{2}{5}$ $5\frac{1}{5}$	$9\frac{1}{4} - 8\frac{3}{4}$ $\frac{1}{2}$	$7\frac{5}{8} - 7\frac{3}{8}$ $\frac{1}{4}$
3.	$6\frac{9}{32} - 3\frac{3}{32}$ $3\frac{3}{16}$	$11\frac{5}{6} - 5\frac{1}{6}$ $6\frac{2}{3}$	$6\frac{1}{2} - 6\frac{1}{2}$ 0	$3\frac{7}{10} - 2\frac{3}{10}$ $1\frac{2}{5}$
4.	$5\frac{3}{4} - 4\frac{1}{4}$ $1\frac{1}{2}$	$9\frac{7}{12} - 7\frac{5}{12}$ $2\frac{1}{6}$	$4\frac{9}{16} - 2\frac{5}{16}$ $2\frac{1}{4}$	$8\frac{1}{3} - 6\frac{2}{3}$ $1\frac{2}{3}$
5.	$9\frac{3}{10} - 6\frac{7}{10}$ $2\frac{3}{5}$	$17\frac{7}{8} - 6\frac{1}{8}$ $11\frac{3}{4}$	$25\frac{1}{3} - 13\frac{2}{3}$ $11\frac{2}{3}$	$6\frac{9}{16} - 4\frac{3}{16}$ $2\frac{3}{8}$

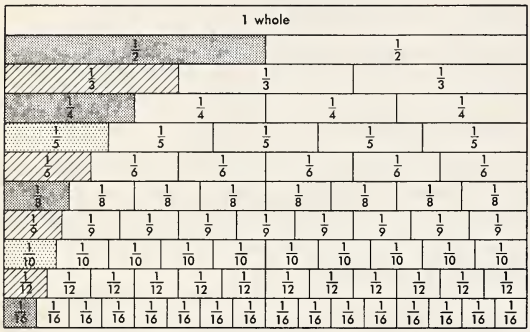
NOTES

What Is a Common Denominator?

Meaning [O]

Judy and Mary are both 12 years old, so we say that 12 is their *common* age, that is, the age they both have.

1. What is the common fractional unit for the fractions $\frac{5}{12}$ and $\frac{7}{12}$? $\frac{1}{12}$ What is their **common denominator**? 12



2. To find a common denominator for $\frac{1}{2}$ and $\frac{3}{8}$, use the chart above. Lay your ruler so that it runs straight down the chart from the $\frac{1}{2}$ point. Then look down to the bar divided into $\frac{1}{8}$'s. How many $\frac{1}{8}$'s does it take to equal exactly $\frac{1}{2}$? 4 $\frac{1}{2} = \frac{4}{8}$.

3. When we write $\frac{1}{2}$ and $\frac{3}{8}$ as $\frac{4}{8}$ and $\frac{3}{8}$, what common fractional unit do we show? $\frac{1}{8}$ What is the common denominator? 8

4. Use the chart to find five other ways in which $\frac{1}{2}$ can be changed. $\frac{1}{2} = \frac{2}{4}$; $\frac{1}{2} = \frac{3}{6}$; and so on.

Fractions which have a common fractional unit have a common denominator.

You do not need the chart if you know the Golden Rule of Fractions. You can use this rule to help you change any set of fractions into fractions with a common denominator.

5. Look again at the fractions $\frac{1}{2}$ and $\frac{3}{8}$. Can you reduce $\frac{3}{8}$ to halves? No Can 2 be a common denominator? No Can $\frac{1}{2}$ be changed to eighths? Yes Can 8 be a common denominator? Yes By what will you multiply both terms of $\frac{1}{2}$ to change it to eighths? 4 $\frac{1}{2} = \frac{4}{8}$.

6. The fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{16}$ can have what number as a common denominator? 16 Explain how you would change $\frac{1}{2}$ and $\frac{3}{4}$ to sixteenths by the Golden Rule of Fractions $\frac{1\times8}{2\times8}=\frac{8}{16}$ $\frac{3\times4}{4\times4}=\frac{12}{16}$

[W]

Use the chart and write these pairs of fractions so that each pair will have a common denominator. (HELPER. Always change to a smaller fractional unit.)

7. $\frac{1}{2}$ and $\frac{3}{4}$ $\frac{2}{4}$
8. $\frac{3}{8}$ and $\frac{5}{16}$ $\frac{6}{16}$
9. $\frac{7}{8}$ and $\frac{11}{16}$ $\frac{14}{16}$
10. $\frac{1}{3}$ and $\frac{5}{12}$ $\frac{4}{12}$

Use the Golden Rule of Fractions and write these pairs of fractions so that each pair will have a common denominator:

11. $\frac{1}{4}$ and $\frac{7}{8}$ $\frac{2}{8}$
12. $\frac{5}{6}$ and $\frac{11}{12}$ $\frac{10}{12}$
13. $\frac{4}{5}$ and $\frac{7}{10}$ $\frac{8}{10}$
14. $\frac{3}{4}$ and $\frac{9}{16}$ $\frac{12}{16}$

To Keep in Practice

Whole numbers [W]

Do what the signs tell you to do. Check your work. In the division examples, write any remainder in a fraction.

	a	b	c	d	e	f
1.	59	60	542	3,641	$91.25	$84.69
	87	97	638	297	3.76	27.93
	92	13	929	78	46.27	6.38
	+64	+48	+628	+5,467	+0.96	+4.94
	302	218	2,737	9,483	$142.24	$723.94
2.	871	964	4,100	2,060	$81.00	$90.00
	−463	−371	−1,628	−782	−36.54	−6.37
	408	593	2,472	1,278	$44.46	$83.63
3.	63	76	290	8,006	$40.60	$32.27
	×59	×47	×604	×354	×298	×68
	3,717	3,572	175,160	2,834,124	$12,098.80	$2,194.36
4.	42)966 23	36)884 24 4/9	68)380 5 10/17	18)1,338 74 3/9	36)$9.36 $0.26	27)$8.37 $0.31

Teaching Pages 114 and 115

Pupil's Objectives. (*a*) To review and practice finding common denominators; (*b*) to maintain skills in working with whole numbers in the four basic processes.

Background. Pupils are first led to realize that the term *common denominator* applies when two or more fractions have a denominator that is *common*, or the same. In most cases, this term is used to refer to a *common fractional unit*, but the two terms are not synonymous, and pupils must be alert to make the distinction when required. Next, pupils use a chart so that they can get a feeling for the idea that if the fractional unit for a smaller fraction will measure a larger fraction a number of times, the denominator for the fractional unit will be the common denominator for the two fractions involved.

The Golden Rule of Fractions, which pupils have just studied, is then introduced at the bottom of page 114 and the top of page 115, and pupils are shown how they may use this rule in finding a common denominator for fractions. For pages 114 and 115, the idea is essentially that the fraction with the larger fractional unit must be changed to have a smaller fractional unit and more parts. This means that the numerator and denominator of this

fraction must be multiplied by a number which will make the new denominator common to the fraction with the smallest fractional unit.

Pre-book Lesson (page 114 and top of page 115)

1. Have children refer to names that are common for groups that are being put together. They might mention, for example, that the thing that is common to a group of 2 pencils and a group of 3 pencils is the fact that both groups are made up of pencils. Then, ask them what is common to 2 inches and 1 foot 3 inches. Lead them to see that the 1 foot 3 inches could be changed to inches so that the measurement unit would be common to both groups being put together. Point out that, in this case, the larger unit was changed to a smaller unit in obtaining the common measurement unit.

2. Show a chart similar to the one on page 114 in the text and ask pupils what they would do if they were asked to add $\frac{1}{4}$ and $\frac{1}{2}$. See if you can lead them to suggest that they would have to change the $\frac{1}{2}$ to $\frac{2}{4}$ in order to have a common fractional unit.

3. Finally, remind pupils that fractions can be changed to other fractions of equal value by applying the Golden Rule of Fractions. See if they can then tell you how they would work with figures in changing $\frac{1}{2}$ to $\frac{2}{4}$.

Book Lesson (page 114 and top of page 115). *Ex. 1–6:* Oral work. *Ex. 7–14:* Written work.

Differentiations and Extensions (page 114 and top of page 115)

1. *Slower learners* may need much work with fraction strips or with a fraction chart in changing fractions so that a common denominator is present. If strips are used, point out each time that the fraction with the largest fractional unit is always changed so that it has more units each of which is smaller in size. As they do this be sure that the number record is kept simultaneously, for it is quite important that *slower learners* eventually learn how to change fractions without using fractional parts of objects or fraction charts.

2. Write a few pairs of fractions on the board and let *more capable children* change these without referring to a fraction chart. Be sure, however, that they can explain just what would be happening if they were using a chart or fractional parts of objects. These children may also be asked to find common denominators for three fractions as in $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{16}$. Be sure, however, that the common denominator is present in one of the three fractions.

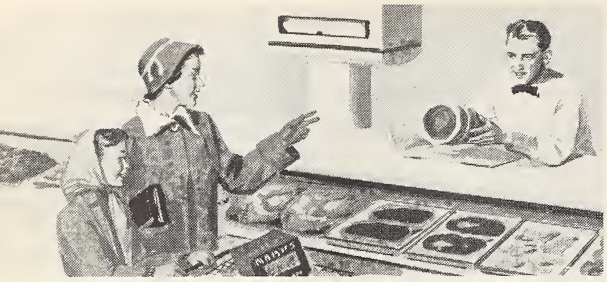
Book Lesson (bottom of page 115). *Rows 1–4:* Written work.

1. Before pupils start the work on this page review the method of checking each kind of example. Sample examples, with checks, may be left on the board for reference.

2. Ask *more capable children* first to give an estimate for each of the examples and to use it as a check of their exact answer.

Differentiations and Extensions (bottom of page 115). In an oral discussion, let *more capable children* tell other ways in which they might add in row 1 of the written work. For Ex. 1**a** somebody might suggest adding the tens first and then the ones. Somebody else might suggest adding the first two addends and then the second two addends and combining the two sums thus obtained. Another person might suggest adding the 59 to the 7 in 87 (66) and then adding to this the 80 (146), continuing in the same way for the rest of the addends. Whatever the case, be sure that the pupils are allowed freedom to discover as many ways as they can think of for adding in these examples. Then, following the same pattern, let them work on rows 2–4.

NOTES



At the Market

A. and S. of unlike-fractions; common denominator present [O]

1. At the market, Ann's mother bought $\frac{1}{2}$ lb. of bacon and $\frac{3}{4}$ lb. of sliced ham. How much did the meat weigh in all?

Since both $\frac{1}{2}$ and $\frac{3}{4}$ are parts of a pound, can we add these fractions? First we must change one of them so that they will have a common fractional unit. In box A, $\frac{1}{2}$ is changed to $\frac{2}{4}$. 2 fourths + 3 fourths = 5 fourths.

Now explain all the work in box A.

A

$\frac{1}{2} = \frac{2}{4}$

$+\frac{3}{4} = \frac{3}{4}$

$\frac{5}{4} = 1\frac{1}{4}$

2. The roast that the butcher showed Ann's mother weighed $7\frac{5}{8}$ lb. This was more than she wanted, so he cut off a piece. Then the roast weighed $5\frac{1}{2}$ lb. How much was cut off?

What common denominator should the fractions have? 8

By what can you multiply both terms of the fraction $\frac{1}{2}$ to change $\frac{1}{2}$ to eighths? 4 $\frac{1}{2} = \frac{4}{8}$

Explain the work in box B.

B

$7\frac{5}{8} = 7\frac{5}{8}$

$-5\frac{1}{2} = 5\frac{4}{8}$

$2\frac{1}{8}$

Unlike-fractions cannot be added or subtracted until some change is made so that they have a common denominator.

Tell the common denominator to be used in each of Ex. 3–6.

3. $\frac{1}{3} + \frac{1}{6} =$ 6 4. $\frac{3}{10} + \frac{1}{5} =$ 10 5. $2\frac{1}{4} - \frac{1}{8} =$ 8 6. $1\frac{5}{6} - \frac{1}{3} =$ 6

Copy and add. First *think*, "What should be the common denominator?" Check by adding upward.

	a	b	c	d	e	f	g
7.	$\frac{4}{5}$	$\frac{7}{8}$	$4\frac{1}{2}$	$\frac{3}{10}$	$\frac{4}{9}$	$7\frac{1}{3}$	$\frac{1}{4}$
	$+\frac{9}{10}$	$+\frac{1}{2}$	$+2\frac{1}{4}$	$+1\frac{2}{5}$	$+\frac{1}{3}$	$+1\frac{1}{3}$	$+\frac{5}{12}$
	$1\frac{7}{10}$	$1\frac{3}{8}$	$6\frac{3}{4}$	$1\frac{7}{10}$	$\frac{7}{9}$	$8\frac{2}{3}$	$\frac{2}{3}$
8.	$3\frac{5}{8}$	$2\frac{1}{10}$	$1\frac{1}{2}$	$\frac{4}{5}$	$12\frac{7}{10}$	$\frac{5}{16}$	$60\frac{2}{9}$
	$+\frac{1}{4}$	$+4\frac{1}{2}$	$+\frac{5}{12}$	$+\frac{7}{20}$	$+17\frac{1}{5}$	$+\frac{7}{8}$	$+7\frac{2}{3}$
	$3\frac{7}{8}$	$6\frac{3}{5}$	$1\frac{11}{12}$	$1\frac{3}{20}$	$29\frac{9}{10}$	$1\frac{3}{16}$	$67\frac{8}{9}$

Copy and subtract. Check your remainders.

9.	$\frac{7}{10}$	$\frac{3}{4}$	$2\frac{3}{4}$	$11\frac{1}{3}$	$3\frac{9}{10}$	$46\frac{2}{8}$	$\frac{5}{6}$
	$-\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{5}{8}$	$-\frac{1}{6}$	$-\frac{2}{5}$	$-4\frac{4}{9}$	$-\frac{2}{3}$
	$\frac{1}{5}$	$\frac{2}{3}$	$2\frac{1}{8}$	$11\frac{1}{6}$	$3\frac{1}{2}$	$42\frac{2}{9}$	$\frac{1}{6}$
10.	$\frac{2}{3}$	$4\frac{7}{8}$	$3\frac{11}{12}$	$6\frac{7}{12}$	$\frac{7}{10}$	$3\frac{7}{10}$	$4\frac{11}{12}$
	$-\frac{5}{12}$	$-2\frac{1}{2}$	$-1\frac{1}{3}$	$-3\frac{1}{4}$	$-\frac{2}{5}$	$-1\frac{1}{5}$	$-2\frac{3}{4}$
	$\frac{7}{12}$	$2\frac{3}{8}$	$2\frac{7}{12}$	$3\frac{3}{4}$	$\frac{3}{10}$	$2\frac{1}{2}$	$2\frac{1}{6}$

Extra Practice. Work Sets 47-49.

Problems Using Fractions

Differentiating A. and S. [W]

First think whether you should add (find the total) or subtract (find a remainder or a difference). Then work the problem.

1. Hilda was $48\frac{1}{8}$ in. tall when summer vacation began. During vacation she grew $\frac{3}{4}$ in. How tall was she then? A., $48\frac{7}{8}$"

2. Mrs. Hunt bought $7\frac{1}{2}$ yd. of cotton dress material. She used $4\frac{1}{8}$ yd. for a dress. How many yards did she have left? S., $3\frac{3}{8}$ yd.

3. Paul jumped $9\frac{9}{10}$ ft. in a running jump and Fred jumped $9\frac{1}{2}$ ft. How much farther did Paul jump than Fred? S., $\frac{2}{5}$ ft.

4. Sam put three boards together for a box cover. They were $3\frac{1}{8}$ in., 4 in., and $3\frac{3}{4}$ in. wide. How wide was the cover? A., $10\frac{7}{8}$"

5. Clara tried to draw a line $5\frac{3}{4}$ in. long. It was only $5\frac{3}{8}$ in. long. How much too short was it? S., $\frac{3}{8}$"

Teaching Pages 116, 117, and 118

Pupil's Objectives. (*a*) To review and practice addition and subtraction of mixed numbers with unlike-fractions but with common denominator present, first without then with carrying and borrowing; (*b*) to solve mixed addition and subtraction problems involving fractions.

Background. More than likely your *more capable children* will have no difficulty in adding and subtracting unlike-fractions when the common denominator is present. They probably will be able to proceed with the written work that starts at the top of page 117 without any of the review on page 116. *Slower learners* may need some slight review of the idea that only like-fractions can be added and that sometimes this entails finding a common denominator by changing one of the fractions.

Pre-book Lesson (page 116 and the top of page 117). Remind pupils that up until now they have added fractions that are like-fractions, but

that they are now prepared to add unlike-fractions. Make a comparison with the addition of whole numbers and ask how they could add 2 dogs and 3 cats. See if they realize that the groups would have to be thought of as *animals*, since this classification would include both dogs and cats. Have them state a common name for each of the following pairs: boys and girls (children); daisies and buttercups (flowers); trout and perch (fish); bees and wasps (insects).

Book Lesson (page 116 and top of page 117). *Ex. 1–6:* Oral work. *Rows 7–10:* Written work.

1. Have pupils notice that $\frac{1}{2}$ in box A is changed to a smaller fractional unit. Ask them to use the fraction chart on page 114 in the text to determine what the fractional units will be when they add in Ex. 3–6 at the bottom of page 116. Then ask them to justify their choices. It would be well to reiterate the fact that the fraction with the largest denominator has the smallest fractional unit.

2. At this time, it might be well to encourage *all pupils* to show side work for changing improper fractions to mixed numbers and for reducing fractions, so that you may follow their thinking and diagnose errors. Occasional use of side work may also help *more capable pupils* consider the orderly arrangement of written work.

Book Lesson (bottom of page 117). *Ex. 1–5:* Written work.

Differentiations and Extensions (pages 116 and 117)

1. See Part IV of this Manual (page 313) for the directions for making a fraction slide rule. *More capable pupils* may be able to make some of these slide rules for themselves and for others. Each *slower learner* may then have as a partner a person who will explain the principle and show him how to operate the slide rule.

2. Assign **Extra Practice Sets 47, 48, and 49.** (These sets appear with answers on Manual page 136.)

Book Lesson (page 118). *Ex. 1 and 2:* Oral work. *Rows 3 and 4:* Written work. Pupils may scan some of the examples in row 3 in the text to determine what the common denominator is and to decide whether or not the sum of the fractions exceeds a whole, making carrying necessary. To follow the same procedure for subtraction, they should scan the examples in row 4, this time determining if the fraction in the subtrahend is greater than the fraction in the minuend, thereby making borrowing necessary.

More Adding and Subtracting of Unlike-Fractions

Carrying and borrowing [O]

1. For the candy sale, Jean made $\frac{3}{4}$ lb. of fudge on Monday, $2\frac{1}{2}$ lb. on Tuesday, and then $1\frac{1}{8}$ lb. on Wednesday. How much candy did she make in the three days?

A

$$\begin{array}{r} \frac{3}{4} = \frac{6}{8} \\ 2\frac{1}{2} = 2\frac{4}{8} \\ +1\frac{1}{8} = 1\frac{1}{8} \\ \hline 3\frac{11}{8} = 4\frac{3}{8} \end{array}$$

Box A. Why is 8 and not 4 the common denominator? *it measures both ½ and ¼*

Is the sum $3\frac{11}{8}$ in best form? *No* Explain. *contains improper fraction*

In best form, $\frac{11}{8}$ is $1\frac{3}{8}$. We can carry this 1 and put it with the 3 that we already have in the sum. Then the sum in best form is $4\frac{3}{8}$.

2. After feeding the chickens from $2\frac{1}{2}$ lb. of feed, Tom found that he had $\frac{3}{4}$ lb. left. How much feed had he used?

B

$$\begin{array}{r} 2\frac{1}{2} = 2\frac{2}{4} = 1\frac{6}{4} \\ -\frac{3}{4} = \frac{3}{4} = \frac{3}{4} \\ \hline 1\frac{3}{4} \end{array}$$

Box B. What is the common denominator? *4* Can we subtract $\frac{3}{4}$ from $\frac{2}{4}$? *No*

Before we can subtract, we must borrow 1 whole, or $\frac{4}{4}$. We put this $\frac{4}{4}$ with the $\frac{2}{4}$ that we already have in the minuend, to make $\frac{6}{4}$ in all. Now can we subtract? *Yes*

Explain the subtraction in box B.

[W]

Copy and add. Check by adding upward.

	a	b	c	d	e	f	g
3.	$2\frac{3}{5}$	$3\frac{2}{3}$	$1\frac{3}{4}$	$16\frac{4}{9}$	$1\frac{5}{8}$	$23\frac{2}{3}$	$11\frac{2}{5}$
	$+\frac{7}{10}$	$+\frac{5}{6}$	$+\frac{7}{8}$	$+2\frac{2}{3}$	$\frac{3}{4}$	$45\frac{5}{12}$	$48\frac{1}{10}$
	$3\frac{3}{10}$	$4\frac{1}{2}$	$2\frac{5}{8}$	$19\frac{7}{9}$	$+2\frac{3}{8}$	$+2\frac{1}{6}$	$+2\frac{3}{5}$
					$4\frac{3}{4}$	$71\frac{1}{4}$	$62\frac{1}{10}$

Copy and subtract. Check your remainders.

4.	$2\frac{1}{3}$	$2\frac{5}{12}$	$14\frac{1}{6}$	$22\frac{1}{4}$	$21\frac{1}{4}$	$43\frac{5}{12}$	$15\frac{1}{2}$
	$-1\frac{5}{6}$	$-\frac{3}{4}$	$-1\frac{1}{2}$	$-\frac{7}{12}$	$-9\frac{3}{8}$	$-1\frac{2}{3}$	$-2\frac{7}{10}$
	$\frac{1}{2}$	$1\frac{2}{3}$	$12\frac{2}{3}$	$21\frac{2}{3}$	$11\frac{7}{8}$	$41\frac{3}{4}$	$12\frac{4}{5}$

Extra Practice. Work Sets 50 and 51.

Differentiations and Extensions (page 118)

1. *All pupils*, or a selected group, may be asked to survey the local stores to discover typical uses of fractions. Include various types of stores: hardware, grocery, drug, furniture, dime stores, and others. Certain pupils may be appointed to examine the labels on canned goods, with the permission of the store managers, to see if the measures are ever expressed as mixed numbers or as measure-

ments that may be converted to mixed numbers (such as 1 lb. 8 oz.). When appropriate, these fractions and mixed numbers may be incorporated into problem situations and used by the pupils for further practice in addition and subtraction of unlike-fractions.

2. Assign **Extra Practice Sets 50 and 51**. (These sets appear with answers on Manual page 137.)

Set 47. Changing fractions to a common denominator

Change to fractions with the denominator asked for.

	a	b
1.	$\frac{1}{2}\ \frac{2}{4},\ \frac{3}{4}$; to 4ths	$\frac{1}{2}\ \frac{4}{8},\ \frac{1}{4}\ \frac{2}{8},\ \frac{5}{8}$; to 8ths
2.	$\frac{1}{2}\ \frac{8}{16},\ \frac{1}{4}\ \frac{4}{16},\ \frac{7}{8}\ \frac{14}{16},\ \frac{5}{16}$; to 16ths	$\frac{1}{2}\ \frac{3}{6},\ \frac{5}{6}$; to 6ths
3.	$\frac{1}{2}\ \frac{6}{12},\ \frac{5}{6}\ \frac{10}{12},\ \frac{7}{12}$; to 12ths	$\frac{1}{2}\ \frac{5}{10},\ \frac{3}{5}\ \frac{6}{10}$; to 10ths
4.	$\frac{1}{2}\ \frac{5}{10},\ \frac{1}{5}\ \frac{2}{10}$; to 10ths	$\frac{1}{2}\ \frac{5}{10},\ \frac{4}{5}\ \frac{8}{10},\ \frac{3}{10}$; to 10ths

Change so fractions have a common fractional unit.

	a	b	c	d
5.	$\frac{1}{2}\ \frac{2}{4},\ \frac{1}{4}$	$\frac{3}{4}\ \frac{6}{8},\ \frac{7}{8}$	$\frac{1}{2}\ \frac{5}{10},\ \frac{3}{10}$	$\frac{1}{8}, \frac{1}{2}\ \frac{4}{8}$
6.	$\frac{3}{4}\ \frac{6}{8},\ \frac{5}{8}, \frac{1}{2}\ \frac{4}{8}$	$\frac{2}{3}\ \frac{4}{6},\ \frac{1}{2}\ \frac{3}{6},\ \frac{1}{6}$	$\frac{1}{2}\ \frac{6}{12},\ \frac{11}{12}, \frac{1}{6}\ \frac{2}{12}$	$\frac{7}{12}, \frac{5}{6}\ \frac{10}{12}$
7.	$\frac{1}{2}\ \frac{5}{10}, \frac{1}{5}\ \frac{2}{10}, \frac{1}{10},$	$\frac{1}{16}\ \frac{2}{32},\ \frac{3}{32}, \frac{1}{4}\ \frac{8}{32}$	$\frac{7}{10}, \frac{2}{5}\ \frac{4}{10}$	$\frac{3}{8}\ \frac{6}{16}, \frac{1}{2}\ \frac{8}{16}, \frac{5}{16}$
8.	$\frac{3}{5}\ \frac{12}{20},\ \frac{7}{20}, \frac{1}{4}\ \frac{5}{20}$	$\frac{5}{8}\ \frac{10}{16},\ \frac{3}{4}\ \frac{12}{16},\ \frac{3}{16}$	$\frac{9}{16}, \frac{7}{8}\ \frac{14}{16}$	$\frac{9}{32}, \frac{7}{16}\ \frac{14}{32}$

Set 48. Addition of unlike-fractions; common denominator present; no carrying

Add. Give answers in best form.

	a	b	c	d	e
1.	$3\frac{1}{2} + \frac{1}{4}\ 3\frac{3}{4}$	$\frac{3}{8} + 9\frac{1}{2}\ 9\frac{7}{8}$	$4\frac{3}{10} + \frac{1}{5}\ 4\frac{1}{2}$	$16\frac{1}{2} + \frac{1}{8}\ 16\frac{5}{8}$	$\frac{9}{16} + 6\frac{3}{8}\ 6\frac{15}{16}$
2.	$2\frac{1}{6} + \frac{1}{2}\ 2\frac{2}{3}$	$8\frac{3}{10} + \frac{1}{2}\ 8\frac{4}{5}$	$\frac{1}{2} + 6\frac{5}{12}\ 6\frac{11}{12}$	$5\frac{3}{10} + \frac{1}{2}\ 5\frac{4}{5}$	$2\frac{1}{2} + \frac{7}{16}\ 2\frac{15}{16}$
3.	$\frac{3}{4} + 4\frac{1}{8}\ 4\frac{7}{8}$	$\frac{5}{16} + 7\frac{1}{2}\ 7\frac{13}{16}$	$9\frac{1}{4} + \frac{9}{16}\ 9\frac{13}{16}$	$\frac{1}{12} + 11\frac{5}{6}\ 11\frac{11}{12}$	$4\frac{3}{5} + \frac{3}{10}\ 4\frac{9}{10}$
4.	$4\frac{1}{6} + \frac{7}{12}\ 4\frac{3}{4}$	$12\frac{5}{8} + \frac{3}{16}\ 12\frac{13}{16}$	$25\frac{1}{2} + \frac{1}{6}\ 25\frac{2}{3}$	$4\frac{5}{8} + \frac{1}{4}\ 4\frac{7}{8}$	$\frac{3}{4} + 1\frac{1}{16}\ 1\frac{13}{16}$

Set 49. Subtraction of unlike-fractions; common denominator present; no borrowing

Subtract. Reduce fractions in answers to lowest terms.

1.	$9\frac{1}{2} - \frac{1}{4}\ 9\frac{1}{4}$	$5\frac{9}{16} - \frac{1}{2}\ 5\frac{1}{16}$	$6\frac{1}{4} - \frac{1}{8}\ 6\frac{1}{8}$	$12\frac{5}{6} - \frac{7}{12}\ 12\frac{1}{4}$	$18\frac{1}{2} - \frac{1}{6}\ 18\frac{1}{3}$
2.	$6\frac{5}{8} - \frac{1}{2}\ 6\frac{1}{8}$	$3\frac{7}{12} - \frac{1}{6}\ 3\frac{5}{12}$	$4\frac{5}{8} - \frac{5}{16}\ 4\frac{5}{16}$	$5\frac{7}{10} - \frac{2}{5}\ 5\frac{3}{10}$	$3\frac{9}{16} - \frac{1}{2}\ 3\frac{1}{16}$
3.	$2\frac{3}{4} - \frac{5}{16}\ 2\frac{7}{16}$	$1\frac{1}{2} - \frac{3}{10}\ 1\frac{1}{5}$	$10\frac{1}{2} - \frac{5}{12}\ 10\frac{1}{12}$	$14\frac{3}{4} - \frac{1}{2}\ 14\frac{1}{4}$	$9\frac{7}{8} - \frac{1}{4}\ 9\frac{5}{8}$
4.	$8\frac{7}{10} - \frac{1}{2}\ 8\frac{1}{5}$	$10\frac{5}{6} - \frac{1}{2}\ 10\frac{1}{3}$	$7\frac{4}{5} - \frac{3}{10}\ 7\frac{1}{2}$	$7\frac{15}{16} - \frac{3}{4}\ 7\frac{3}{16}$	$12\frac{13}{16} - \frac{3}{8}\ 12\frac{7}{16}$

Set 50. Column addition; unlike-fractions; common denominator present; with carrying

Add. Give fractions in answers in best form.

	a	b	c
1.	$3\frac{1}{2} + 4\frac{1}{4} + 5\frac{1}{8}$ $12\frac{7}{8}$	$7\frac{1}{5} + 16\frac{1}{2} + 9\frac{3}{10}$ 33	$2\frac{5}{12} + 13\frac{5}{6} + 7\frac{1}{2}$ $23\frac{3}{4}$
2.	$6\frac{3}{4} + 2\frac{1}{2} + 1\frac{3}{16}$ $10\frac{7}{16}$	$15\frac{1}{6} + 8\frac{7}{12} + 5\frac{1}{2}$ $29\frac{1}{4}$	$32\frac{1}{4} + 2\frac{9}{16} + 10\frac{1}{2}$ $45\frac{5}{16}$
3.	$1\frac{1}{6} + 7\frac{5}{12} + 8\frac{1}{2}$ $17\frac{1}{12}$	$20\frac{1}{2} + 6\frac{7}{8} + 4\frac{3}{4}$ $32\frac{1}{8}$	$9\frac{1}{2} + 3\frac{7}{10} + 2\frac{4}{5}$ 16
4.	$6\frac{1}{8} + 12\frac{1}{2} + 8\frac{3}{4}$ $27\frac{3}{8}$	$2\frac{15}{16} + 18\frac{1}{4} + 3\frac{5}{8}$ $24\frac{13}{16}$	$16\frac{3}{4} + 5\frac{7}{8} + 2\frac{7}{16}$ $25\frac{1}{16}$
5.	$1\frac{1}{4} + 8\frac{3}{8} + 7\frac{5}{16}$ $16\frac{15}{16}$	$8\frac{1}{4} + 5\frac{1}{2} + 10\frac{13}{16}$ $24\frac{9}{16}$	$2\frac{1}{2} + 5\frac{1}{5} + 10\frac{1}{10}$ $17\frac{4}{5}$

Set 51. Subtraction of unlike-fractions; common denominator present; with borrowing

Subtract. Give fractions in answers in lowest terms.

	a	b	c	d
1.	$6\frac{1}{2} - 3\frac{3}{4}$ $2\frac{3}{4}$	$7\frac{3}{8} - 5\frac{1}{2}$ $1\frac{7}{8}$	$10\frac{5}{16} - 9\frac{5}{8}$ $\frac{11}{16}$	$8\frac{3}{4} - 6\frac{15}{16}$ $1\frac{13}{16}$
2.	$5\frac{1}{8} - 2\frac{3}{16}$ $2\frac{15}{16}$	$4\frac{1}{6} - 3\frac{1}{2}$ $\frac{2}{3}$	$17\frac{1}{2} - 12\frac{5}{6}$ $4\frac{2}{3}$	$10\frac{3}{10} - 9\frac{1}{2}$ $\frac{4}{5}$
3.	$15\frac{5}{6} - 9\frac{11}{12}$ $5\frac{11}{12}$	$9\frac{3}{10} - 6\frac{4}{5}$ $2\frac{1}{2}$	$6\frac{1}{8} - 3\frac{1}{4}$ $2\frac{7}{8}$	$23\frac{3}{5} - 8\frac{7}{10}$ $14\frac{9}{10}$
4.	$8\frac{3}{16} - 4\frac{1}{2}$ $3\frac{11}{16}$	$16\frac{1}{2} - 6\frac{7}{12}$ $9\frac{11}{12}$	$7\frac{1}{2} - 4\frac{7}{8}$ $2\frac{5}{8}$	$9\frac{1}{2} - 4\frac{13}{16}$ $4\frac{11}{16}$
5.	$10\frac{5}{12} - 7\frac{5}{6}$ $2\frac{7}{12}$	$5\frac{3}{4} - 1\frac{7}{8}$ $3\frac{7}{8}$	$22\frac{1}{4} - 12\frac{1}{2}$ $9\frac{3}{4}$	$5\frac{3}{16} - 1\frac{3}{4}$ $3\frac{7}{16}$
6.	$7\frac{1}{2} - 6\frac{9}{10}$ $\frac{3}{5}$	$8\frac{5}{12} - 2\frac{1}{2}$ $5\frac{11}{12}$	$3\frac{3}{8} - 1\frac{3}{4}$ $1\frac{5}{8}$	$11\frac{1}{8} - 10\frac{1}{2}$ $\frac{5}{8}$

NOTES

Finding a Common Denominator

Common denominator not present [O]

So far, in adding or subtracting unlike-fractions you have used as common denominator a denominator that was given. For example, for $1\frac{3}{4} + \frac{5}{8}$, the denominator of $\frac{5}{8}$ is the common denominator. For $\frac{2}{3} - \frac{1}{6}$, what is the common denominator? 6

In Ex. 1–7 below, the fractions also are unlike, but none of the given denominators can be used as the common denominator of the fractions.

1. Betty mixed some pancakes. She used $\frac{3}{4}$ c. of milk and $\frac{1}{3}$ c. of water. How much liquid did she use?

A
$\frac{3}{4} = \frac{?}{?}$
$+\frac{1}{3} = \frac{?}{?}$

To find a common denominator for adding, turn back to the fraction chart on page 114. Can $\frac{1}{3}$ be measured exactly by $\frac{1}{4}$? No; so 4 is not the common denominator.

From the chart, the smaller fractions you find which will exactly measure $\frac{1}{3}$ are $\frac{1}{6}, \frac{1}{9}, \frac{1}{12}$. The smaller fractions which will exactly measure $\frac{1}{4}$ are $\frac{1}{8}, \frac{1}{12}, \frac{1}{16}$.

B
$\frac{3}{4} = \frac{?}{12}$
$+\frac{1}{3} = \frac{?}{12}$

Then what fraction measure is common to both $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{1}{12}$ What are the common fractional unit and the common denominator for $\frac{3}{4}$ and $\frac{1}{3}$? 12

In box B, explain how to change $\frac{3}{4}$ and $\frac{1}{3}$ both to $\frac{1}{12}$'s. Multiply $\frac{3}{4}$ by $\frac{3}{3}$; $\frac{1}{3}$ by $\frac{4}{4}$

C
$\frac{3}{4} = \frac{9}{12}$
$+\frac{1}{3} = \frac{4}{12}$
$\frac{13}{12}$, or $1\frac{1}{12}$

Now explain how to finish the addition in box C.

Using the chart on page 114, tell the common fractional unit and the common denominator to be used in adding these:

2. $\frac{2}{3}$ and $\frac{1}{2}$ — 6, 6
3. $\frac{5}{6}$ and $\frac{3}{4}$ — 12, 12
4. $\frac{3}{5}$ and $\frac{1}{2}$ — 10, 10
5. $\frac{1}{4}$ and $\frac{1}{3}$ — 12, 12
6. $\frac{1}{2}$ and $\frac{2}{5}$ — 10, 10
7. $\frac{1}{8}$ and $\frac{1}{4}$ — 12, 12

[W]

Now add the fractions in Ex. 2–7.

Teaching Pages 119, 120, and 121

Pupil's Objectives. (*a*) To find a common denominator when it is not a denominator in a given fraction, first, by use of the fraction chart and, secondly, by determining a denominator that will contain all the given denominators; (*b*) to learn a short way of finding the smallest common denominator; (*c*) to practice addition and subtraction examples in which the common denominator is not present; (*d*) to find mistakes made in a set of problems and to correct them.

Background. Pupils sometimes have difficulty in finding a common denominator when one of the denominators for the fractions involved is not the denominator to be used. On page 119 the pupils are introduced to this problem and referred to the chart on page 114 in the text so that they will see that what must be done is to find a fractional unit that will measure both of the fractions an even number of times. For Ex. 1 on page 119, they discover from the chart that only $\frac{1}{12}$ will measure both $\frac{1}{4}$ and $\frac{1}{3}$. Once the common denominator, or common fractional unit, is discovered, pupils should be able to proceed as in boxes B and C on page 119, changing both fractions to a common fractional unit in preparation for adding.

On page 120 pupils are given a "short cut" for finding the common denominator—multiplying the two given denominators to find a common denominator. However, since this sometimes gives a denominator that is too large (as would occur in box B on page 120) they are led to discover another way for finding the *smallest* common denominator and this way is then stated in the generalization at the bottom of page 120.

It is well to point out to pupils that in all cases of adding and subtracting unlike-fractions we look at the fraction with the smallest fractional unit (the largest common denominator) first. If it will measure the other fractions in the addition or subtraction, it is the smallest common fractional unit. If it will not measure the other fractions, we make the fractional unit twice as small. Or, to put it another way, we make the denominator twice as large by multiplying it by 2. We then check this with the other fractions. If it will measure the other fractions, it is the smallest common fractional unit or the lowest common denominator that will contain all the fractions without being unnecessarily large.

Book Lesson (page 119). *Ex. 1–7:* Oral work. *Bottom of the page:* Written work.

1. As pupils explain box B, reference may be made to the Golden Rule of Fractions, pages 106 and 115 in the text.

2. If pupils need more practice with the fraction chart in order to understand what fractional units measure given fractions, they may work out exercises like the following, and from the answers find fractional units that are common to two or three fractions:

$\frac{1}{2}$ may be measured an even number of times by $\frac{1}{4}$'s, $\frac{1}{6}$'s, _?_, _?_, _?_, _?_ , _?_

A Short Cut

Finding smallest common denominator [O]

Finding a common denominator by making a chart is not always a quick way to work. You need to know a short cut.

1. In the addition of $\frac{3}{4} + \frac{1}{3}$ in box A, you know from your work with the chart that 12 is the common denominator. Could you find the same denominator by multiplying together the two given denominators? Yes $3 \times 4 = 12$

A

$\frac{3}{4} = \frac{9}{12}$

$+\frac{1}{3} = \frac{4}{12}$

$\frac{13}{12}$

By multiplying denominators, find a common denominator for

2. $\frac{2}{3}$ and $\frac{1}{2}$. 6 3. $\frac{1}{2}$ and $\frac{2}{5}$. 10 4. $\frac{2}{3}$ and $\frac{4}{5}$. 15 5. $\frac{2}{3}$ and $\frac{3}{4}$ and $\frac{7}{8}$. 96

You can always work with a common denominator found by multiplying together given denominators but, as in Ex. 5, sometimes this gives a very large denominator. It is better to find the *smallest* number that will exactly contain all given denominators.

6. What is the smallest number that will exactly contain 3, 4, and 8 (box B)?

Will 8, the denominator of $\frac{7}{8}$, contain 4, the denominator of $\frac{3}{4}$? Yes Will it contain 3, the denominator of $\frac{2}{3}$? No

B

$\frac{2}{3} = \frac{24}{24}$

$\frac{3}{4} = \frac{24}{24}$

$\frac{7}{8} = \frac{24}{24}$

Try 2×8, or 16. Will 16 contain 4? Yes Will it contain 3? No

Try 3×8, or 24. Will 24 contain 4? Yes Will it contain 3? Yes

Because 24 is the smallest number that contains all the denominators, we call it the **smallest common denominator.**

Find the smallest common denominator for

7. $\frac{3}{4}$ and $\frac{5}{6}$. 12 8. $\frac{1}{6}$ and $\frac{3}{8}$. 24 9. $\frac{5}{12}$ and $\frac{7}{8}$. 24 10. $\frac{1}{2}$, $\frac{7}{8}$, $\frac{2}{3}$. 24

When no given denominator is a common denominator, we can find the smallest common denominator by multiplying the largest given denominator first by 2, then by 3, and so on, until we get a number that exactly contains all the given denominators.

[W]

Copy and work, using the smallest common denominator.

	a	b	c	d	e	f	g
11.	$\frac{1}{2}$	$\frac{3}{4}$	$3\frac{5}{6}$	$4\frac{3}{5}$	$2\frac{2}{3}$	$8\frac{3}{8}$	$9\frac{1}{6}$
	$+\frac{2}{3}$	$-\frac{2}{5}$	$+1\frac{3}{4}$	$-1\frac{1}{2}$	$-\frac{4}{5}$	$-4\frac{3}{4}$	$+2\frac{7}{8}$
	$\frac{1}{6}$	$\frac{7}{12}$	$5\frac{7}{12}$	$3\frac{1}{10}$	$1\frac{13}{15}$	$3\frac{9}{24}$	$12\frac{1}{24}$
12.	$\frac{5}{8}$	$2\frac{5}{12}$	$3\frac{1}{3}$	$4\frac{2}{3}$	$2\frac{3}{4}$	$3\frac{5}{8}$	$7\frac{1}{3}$
	$\frac{1}{6}$	$\frac{2}{3}$	$7\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{1}{2}$	$\frac{4}{8}$	$3\frac{5}{8}$
	$+\frac{1}{2}$	$+\frac{3}{8}$	$+1\frac{5}{8}$	$+2\frac{7}{8}$	$+1\frac{3}{5}$	$+1\frac{1}{2}$	$+2\frac{2}{3}$
	$\frac{7}{24}$	$\frac{17}{24}$	$12\frac{11}{24}$	$10\frac{17}{24}$	$7\frac{11}{20}$	$5\frac{7}{8}$	$13\frac{5}{24}$

Extra Practice. Work Sets 52 and 53.

What Is Wrong?

Problems; A., S. of unlike-fractions [O]

Tell why these children's solutions are wrong.

1. Marge: Yesterday I spent $\frac{1}{2}$ hr. on my music lesson in the morning and $\frac{1}{4}$ hr. in the afternoon. How long did I spend in all yesterday? $\frac{3}{4}$ hr.

2. Tom: I rode my bicycle $\frac{3}{4}$ mi. from my house to Ed's house, and then $\frac{1}{4}$ mi. farther, on the same road, to the pond. How far is it from my house to the pond? 1 mile

3. Bill: Our house lot contains $\frac{5}{8}$ acre. When my father bought $\frac{7}{8}$ acre next to ours, how much land did he own? $1\frac{1}{2}$ A.

4. Jean: I bought three candy bars weighing $\frac{7}{8}$ oz., $\frac{3}{8}$ oz., and $\frac{1}{4}$ oz. How many ounces of candy was that? $1\frac{1}{2}$ oz.

5. Alec: Dad said that he'd give me either $\frac{3}{8}$ of the money in his pocket or $\frac{2}{3}$ of it, whichever I thought was the larger amount. I chose $\frac{3}{8}$. $\frac{2}{3}$ is more than $\frac{3}{6}$

Why wrong?

Marge $\frac{2}{4} + \frac{1}{4} = \frac{3}{8}$

Tom $\frac{3}{4} + \frac{1}{4} = \frac{4}{8}$, or $\frac{1}{2}$

Bill $\frac{1}{2} = \frac{1}{2}$ $\frac{5}{8} + \frac{7}{8} = 1\frac{12}{16}$, or $\frac{3}{4}$

Jean $\frac{7}{8}$ $\frac{3}{8}$ $+\frac{1}{4} = \frac{2}{8}$ $1\frac{12}{8} = 1\frac{4}{8}$, or $1\frac{1}{2}$ $\frac{1}{2} = \frac{1}{2}$

Alec $\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$ $\frac{3}{8} \quad \frac{2}{3} \quad \frac{1}{8}$

$\frac{1}{3}$ may be measured an even number of times by $\frac{1}{6}$'s, _?_, _?_, _?_

$\frac{1}{4}$ may be measured an even number of times by $\frac{1}{8}$'s, _?_, _?_

$\frac{1}{6}$ may be measured an even number of times by _?_, _?_

$\frac{1}{8}$ may be measured an even number of times by _?_

Pupils may notice themselves the relationships between the denominators and their multiples in the completed chart above. This will naturally lead to the "short way," introduced on the next page, of finding the lowest common denominator.

Book Lesson (page 120 and top of page 121). *Ex. 1–10:* Oral work. *Rows 11 and 12:* Written work.

1. Point out that the "short way" not only saves time, but also the procedure may be done when a chart is not available.

2. For Ex. 7 and 8, have pupils compare the lowest common denominator with the one found by multiplying the given denominators. Bring out the fact that 24 and 48 are both common denominators, as are 12 and 24, for these examples, but that more calculations may be done mentally with the *lowest common denominator* and that it is not, therefore, so cumbersome to use.

3. Call the attention of your pupils to the fact that the "short way" works equally well with subtraction as with addition. They may do a few subtraction examples, using fractional pieces to show the ideas of differences or of "taking away," before starting the written lesson.

Book Lesson (bottom of page 121). *Ex. 1–5:* Oral work. Call particular attention to the kinds of errors made in working the problems. Have the pupils do the problems correctly at the board.

Differentiations and Extensions

1. See Part IV of this Manual for a description of calculating strips (pages 312 and 313) that can be used in measuring fractional pieces. These may be particularly helpful in small-group work for *slower learners* as they try to find the fractional units and the lowest common denominators for addition and subtraction examples.

2. Assign **Extra Practice Sets 52 and 53** as needed.

Set 52. Addition of mixed numbers; common denominator not present

Add. Reduce fractions in answers to lowest terms.

	a	b	c
1.	$4\frac{1}{2} + 3\frac{2}{3}$ $8\frac{1}{6}$	$12\frac{1}{5} + 16\frac{1}{2}$ $28\frac{7}{10}$	$8\frac{2}{3} + 19\frac{3}{5} + 4\frac{7}{10}$ $32\frac{29}{30}$
2.	$6\frac{1}{3} + 5\frac{3}{4}$ $12\frac{1}{12}$	$4\frac{2}{3} + 2\frac{3}{8} + 1\frac{7}{12}$ $8\frac{5}{8}$	$29\frac{2}{3} + 10\frac{4}{5}$ $40\frac{7}{15}$
3.	$2\frac{1}{2} + 3\frac{1}{3} + 4\frac{1}{4}$ $10\frac{1}{12}$	$18\frac{1}{3} + 10\frac{3}{10}$ $28\frac{19}{30}$	$1\frac{3}{5} + 2\frac{1}{4} + 8\frac{9}{10}$ $12\frac{3}{4}$
4.	$8\frac{2}{3} + 4\frac{7}{8}$ $13\frac{13}{24}$	$3\frac{5}{9} + 1\frac{1}{3} + 2\frac{3}{4}$ $7\frac{23}{36}$	$16\frac{2}{3} + 18\frac{3}{4} + 2\frac{5}{16}$ $37\frac{35}{48}$
5.	$9\frac{1}{3} + 3\frac{2}{5} + 7\frac{3}{4}$ $20\frac{3}{4}$	$14\frac{1}{4} + 8\frac{4}{5}$ $23\frac{1}{20}$	$5\frac{1}{3} + 1\frac{5}{6} + 3\frac{2}{9}$ $10\frac{7}{18}$
6.	$10\frac{4}{5} + 19\frac{5}{6}$ $30\frac{19}{30}$	$12\frac{1}{2} + 2\frac{3}{5} + 5\frac{2}{3}$ $20\frac{23}{30}$	$9\frac{1}{4} + 4\frac{1}{3} + 5\frac{1}{6}$ $18\frac{3}{4}$

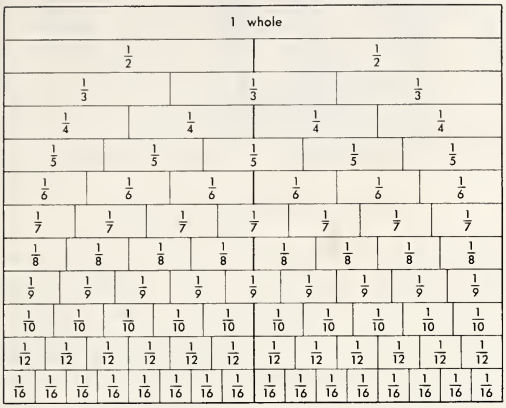
Set 53. Subtraction of mixed numbers; common denominator not present

Subtract. Give fractions in answers in best form.

	a	b	c	d
1.	$5\frac{1}{2} - 3\frac{2}{3}$ $1\frac{5}{6}$	$9\frac{2}{5} - 3\frac{1}{2}$ $5\frac{9}{10}$	$13\frac{1}{6} - 7\frac{1}{4}$ $5\frac{11}{12}$	$8\frac{1}{6} - 7\frac{7}{9}$ $\frac{7}{18}$
2.	$16\frac{1}{3} - 12\frac{3}{4}$ $3\frac{7}{12}$	$18\frac{1}{4} - 15\frac{1}{3}$ $2\frac{11}{12}$	$7\frac{1}{2} - 5\frac{4}{5}$ $1\frac{7}{10}$	$15\frac{2}{9} - 14\frac{3}{4}$ $\frac{17}{36}$
3.	$11\frac{3}{4} - 10\frac{4}{5}$ $\frac{19}{20}$	$6\frac{1}{6} - 1\frac{5}{9}$ $4\frac{11}{18}$	$4\frac{1}{3} - 1\frac{3}{16}$ $3\frac{7}{48}$	$10\frac{5}{8} - 9\frac{2}{3}$ $\frac{23}{24}$
4.	$9\frac{2}{3} - 6\frac{7}{8}$ $2\frac{19}{24}$	$21\frac{3}{8} - 19\frac{7}{12}$ $1\frac{19}{24}$	$16\frac{2}{9} - 10\frac{5}{6}$ $5\frac{7}{18}$	$6\frac{1}{5} - 3\frac{1}{4}$ $2\frac{19}{20}$
5.	$25\frac{3}{8} - 15\frac{2}{3}$ $9\frac{17}{24}$	$12\frac{1}{3} - 8\frac{1}{2}$ $3\frac{5}{6}$	$5\frac{1}{2} - 3\frac{3}{5}$ $1\frac{9}{10}$	$9\frac{3}{4} - 5\frac{5}{6}$ $3\frac{11}{12}$
6.	$8\frac{1}{5} - 4\frac{1}{3}$ $3\frac{13}{15}$	$4\frac{3}{4} - 1\frac{8}{9}$ $2\frac{31}{36}$	$9\frac{1}{4} - 4\frac{2}{3}$ $4\frac{7}{12}$	$3\frac{1}{16} - 1\frac{1}{3}$ $1\frac{35}{48}$
7.	$7\frac{1}{4} - 2\frac{5}{6}$ $4\frac{5}{12}$	$14\frac{1}{3} - 12\frac{3}{5}$ $1\frac{11}{15}$	$10\frac{2}{3} - 8\frac{4}{5}$ $1\frac{13}{15}$	$3\frac{1}{12} - 1\frac{5}{8}$ $1\frac{11}{24}$

Rounding and Estimating

Meaning; fractions and mixed numbers [O]



1. On the chart above, is $\frac{3}{4}$ more or less than $\frac{1}{2}$? Why do we round $\frac{3}{4}$ to 1 instead of to 0? nearer! Would $2\frac{3}{4}$ be rounded to 2 or to 3?

2. To round $6\frac{3}{8}$, first look at $\frac{3}{8}$ on the chart. Is $\frac{3}{8}$ more or less than $\frac{1}{2}$? Would $6\frac{3}{8}$ be rounded to 6 or to 7?

3. Is $\frac{11}{12}$ more or less than $\frac{1}{2}$? Round $5\frac{11}{12}$ to a whole number. 6

4. Is $\frac{7}{16}$ more or less than $\frac{1}{2}$? Round $15\frac{7}{16}$ to a whole number. 15

In rounding a mixed number, when the fraction is equal to $\frac{1}{2}$ or greater than $\frac{1}{2}$, add 1 to the whole number and drop the fraction. When the fraction is less than $\frac{1}{2}$, drop it.

By rounding, estimate answers for these: (When the fraction is exactly $\frac{1}{2}$, round to the next larger whole number.)

5. $3\frac{1}{2} + 8\frac{11}{12}$ 4+9=13

6. $7\frac{2}{5} - 4\frac{9}{10}$ 7−5=2

7. $3\frac{2}{3} + 2\frac{3}{4} + 4\frac{1}{2}$ 4+3+5=12

Fractions in the Kitchen

A. and S. of unlike-fractions [W]

Write all your work for Ex. 1–4.

1. Bill's father, Mr. Oaks, cooked the dinner when his club met at his house. He made his favorite beef stew. To the cooked beef, he added $1\frac{1}{4}$ cups of sliced onions, $7\frac{1}{2}$ cups of diced potatoes, $1\frac{1}{4}$ cups of diced carrots, and $1\frac{2}{3}$ cups of diced turnips. How many cups of vegetables did he use in all? $11\frac{5}{12}$, or $11\frac{2}{3}$ cups

2. His recipe for fudge pie takes 9 oz. of grated chocolate. He used up an opened package of grated chocolate containing $7\frac{1}{2}$ oz. How much more chocolate did he have to take from another package? $1\frac{1}{2}$ oz.

3. When Mr. Oaks started to make mayonnaise he found that there was only $1\frac{1}{8}$ cups of olive oil in the house. The recipe required $2\frac{1}{2}$ cups of oil, so he used peanut oil for the remainder. How much peanut oil did he need to use? $1\frac{3}{8}$ cups

4. Earlier in the day, Mrs. Oaks had made raisin squares from a recipe which calls for $1\frac{1}{2}$ cups of raisins and $2\frac{1}{4}$ cups of shredded coconut. She measured $1\frac{1}{2}$ cups of raisins in a 4-cup measure, and added coconut in the same cup. To what mark on the cup did the coconut come? $3\frac{3}{4}$ cups

5. Bring from home a recipe with unlike-fractions.

Teaching Pages 122 and 123

Pupil's Objectives. (*a*) To learn to round fractions and mixed numbers in order to estimate sums and remainders; (*b*) to solve addition and subtraction problems with unlike-fractions.

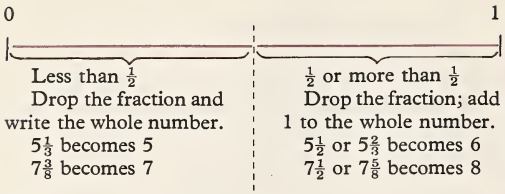
Background. Since it is often necessary to estimate answers for examples containing fractions, it is important for pupils to be able to round fractions and mixed numbers. The principle is the same as in rounding whole numbers. If a fraction is $\frac{1}{2}$ or larger than $\frac{1}{2}$, we round it to 1; if it is smaller than $\frac{1}{2}$, we think of it as 0.

On page 122 in the text pupils use a fraction chart the midpoint of which is marked in red. However, when a chart is not available and pupils are working with figures only, they will have to discover that when the numerator is two or more times the denominator, the fraction is greater than $\frac{1}{2}$ and that in all other cases it is smaller than $\frac{1}{2}$.

Book Lesson (page 122). *Ex. 1–7:* Oral work.

1. A diagram often leaves a stronger impression than words alone. The plan at the top of the next page may be put on the board or on a chart with other illustrative numbers.

Rounding Numbers with Fractions


0
1
Less than $\frac{1}{2}$
Drop the fraction and write the whole number.
$5\frac{1}{3}$ becomes 5
$7\frac{3}{8}$ becomes 7
$\frac{1}{2}$ or more than $\frac{1}{2}$
Drop the fraction; add 1 to the whole number.
$5\frac{1}{2}$ or $5\frac{2}{3}$ becomes 6
$7\frac{1}{2}$ or $7\frac{5}{8}$ becomes 8

2. Pupils may also be assigned an exercise similar to the one below.

Write some fractions that would be *dropped* when rounding mixed numbers. Give:

a. Three fractions with fractional unit of $\frac{1}{8}$.
b. Two fractions with fractional unit of $\frac{1}{6}$.
c. Five fractions with fractional unit of $\frac{1}{12}$.

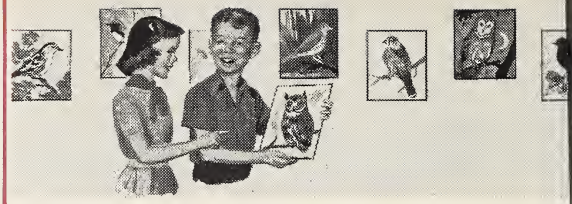
Book Lesson (page 123). *Ex. 1–4:* Written work. For Ex. 5, pupils are asked to bring from home recipes containing unlike-fractions. These may be used as the basis for constructing and solving more problems.

Teaching Pages 124, 125, and 126

Pupil's Objectives. (*a*) To solve problems using several different problem-solving helps; (*b*) to identify problems having too few or too many numbers; (*c*) to supply missing information for problems.

Background. It is generally agreed that ability to solve problems depends upon ability to read and understand the problems and on a thorough understanding of the meanings of the four basic processes. Both of these abilities are probably pretty much directly related to the amount of experience pupils have had in working with objects. It is primarily in this way that meanings are attached to words and that understanding of the basic processes is developed. On pages 124 and 125 in the text these two points are made and a third helper in solving problems is added—that rounding numbers and estimating answers is a good way for deciding how reasonable an answer is in view of the problem situation.

Notice that the meanings of the four basic processes are given at the top of page 125 in the text. These will serve as a summary, but you first ought to be sure that your pupils can state these ideas without reference to the text.



Helps for Solving Problems

[O]

You have been working with whole numbers and fractions, both in practice and in problems. Do you always know how to solve the problems? Here are some ways to help yourself.

What is asked?

One way to help yourself is to tell in your own words what you are to find. Tell what you must find in problems 1–3.

1. Bob collects bird pictures. He has pictures of 4 warblers, 3 owls, 3 hawks, and 2 tanagers. How many pictures has he? (12)

2. A jar filled with hard candies weighs 1 lb. It contains $10\frac{1}{2}$ oz. of candy. How much does the empty jar weigh? ($5\frac{1}{2}$ oz.)

3. Bill found on the beach a wide plank 8 ft. long. His father said that this size of plank is worth 24¢ for each foot of the length. How much was Bill's plank worth? ($1.92)

Do you put groups together or separate groups?

Another help is to ask yourself whether the problem tells about groups that are to be combined or about groups that are to be compared or separated. Do this for problems 4–6.

4. When Sally started piano lessons, her mother paid the teacher $2.50 a week. How much was the cost of lessons for 5 weeks? put together ($12.50)

5. A year ago Linda weighed $98\frac{3}{4}$ lb. Now she weighs $104\frac{1}{2}$ lb. How many pounds has she gained? separated ($5\frac{3}{4}$ lb.)

6. Mrs. Hamilton bought a lamp base for $2.25 and a lampshade for 69¢. How much did she pay for the complete lamp? put together ($2.94)

On page 126 pupils are given practice in deciding whether there are too many or too few numbers in various problem situations. Extraneous or missing information is typical in everyday problem situations and pupils should learn to cope with this difficulty.

Book Lesson (pages 124 and 125). *Ex. 1–12:* Oral work. *Bottom of page 125:* Written work.

1. *Slower learners* may be allowed another helper, that of illustrating the problem situa-

A.? S.? M.? D.?

Even if you know whether to put groups together or to separate or compare groups, you must decide what process to use.

Putting groups together

We add to find the total when we put together unequal groups.
We multiply to find the total when we put together equal groups.

Separating groups

We subtract to find the other part of a group or the difference between two groups.
We divide to find how many equal small groups in a larger group or how many in each of a number of equal groups.

For problems 7–10, tell whether you should add, subtract, multiply, or divide.

7. Ken's pad of paper has 48 sheets. If he uses 3 sheets a day, how long will the pad last? D. (16 days)

8. The hall clock in Ted's house was made in the year 1750. About how old is the clock now? S.

9. At $2.10 a foot, how much will a 24-foot aluminum extension ladder cost? M. ($50.40)

10. A cake recipe directs that $\frac{1}{2}$ cup of milk be added first, and $\frac{3}{4}$ cup of milk later. How much milk is used in all? A. (1¼ c.)

What is the estimated answer?

It is a good plan to use round numbers and estimate the answer before you work a problem. This helps you to decide whether your answer is reasonable. Estimate answers for problems 11–12.

11. Joe is 12 yr. old, and his father is 41. How old was Joe's father when Joe was born? est., 30 (29)

12. Ed watched television $1\frac{1}{4}$ hr. on Friday and $2\frac{1}{2}$ hr. on Saturday. How many hours was that in all? est., 4 (4⅙)

[W]

Now go back and solve problems 1–12.

Too Many Numbers or Too Few?

Help in problem-solving [O]

Sometimes there are more numbers in a problem than you need, and sometimes not enough. For each of problems 1–9, tell whether there are too many numbers or too few.

1. Sue paid $1.47 for 3 yd. of percale. How much change should she get from a $5.00 bill? Not needed: 3 yd. ($3.53)

2. For Christmas presents, Sally pasted colored paper on matchbook covers. If she packs 25 matchbooks in a Christmas box, how many boxes can she fill from 2 store packages of matchbooks? Number of matchbooks in 2 store packages needed.

3. Ted has 8 pieces of straight track for his model railway. How many more pieces does he need in order to lay 9 ft. of straight track? Length of a straight piece of track needed.

4. If a bushel of potatoes weighs 60 lb., what does a peck of potatoes weigh? Must know 1 bu. = 4 pk. (15 lb.)

5. At the diner where Joe's father eats lunch, coffee with cream costs 10¢ but black coffee costs only 7¢. How much would Joe's father have to pay for 2 cups of black coffee? Not needed: 10¢ (14¢)

6. Mrs. Alger bought 4 lamb chops priced at 79¢ a pound. How much did they cost? Total weight, not the number of chops needed.

7. Ted bought a box of candy and gave the clerk 50¢. How much change did he get? Cost of candy needed.

8. Mary had 4 yd. of cotton material. She used $\frac{1}{2}$ yd. for one apron and $\frac{7}{8}$ yd. for another apron. Was there enough material left for a dress? Material required for one dress needed.

9. Mr. Farwell said that the piece of molding was just $\frac{3}{4}$ inch too short. How long a piece of molding did he need? Length of Mr. Farwell's moulding needed.

[W]

Now go back and solve problems 1–9. When you come to a problem that has too few numbers, first supply the necessary information. Then work the problem.

tions with sketches, with number lines, or with bundles representing groups. These pupils may develop a page of such pictures for their arithmetic notebooks.

2. Although pupils may be able to distinguish between putting groups together or separating groups, they may have difficulty in differentiating between addition and multiplication and between subtraction and division. Stress the underlying principle that multiplication and division are the efficient ways of dealing with *equal* groups.

Book Lesson (page 126). *Ex. 1–9:* Oral work. *Bottom of the page:* Written work.

1. For each of these problems, ask pupils to tell what information is required for solving. When extra information is given, they should identify this information and disregard it. When insufficient data are given, have the class select reasonable figures to be used by all in the written lesson.

2. *More capable pupils* may omit the oral work. If they do, check their work carefully for the reasonableness of the data they supplied.

Teaching Pages 127, 128, and 129

Pupil's Objective. To take the four regular end-of-chapter tests.

Pre-book Lesson

1. Make certain that pupils assume the proper attitude toward the tests and are not frightened by them. Assure them that speed is not as important as accuracy.

2. Remind pupils again that the function of a test is to help them locate their weaknesses so that they may be corrected as soon as possible.

Book Lesson (page 127). *Ex. 1–14:* Written work. Test of Information and Meaning 3.

Book Lesson (top of page 128). *Rows 1–8:* Written work. Diagnostic Test 3. If needed, demonstrate the use of study pages and practice sets. Practice sets with answers are reproduced on previous pages in the Manual. For their location, refer to the heading *Practice* in the index for this Manual.

Book Lesson (bottom of page 128 and top of page 129). *Ex. 1–8:* Written work. Problem Test 3.

Book Lesson (bottom of page 129). *Ex. 1–14:* Written work. Computation Test 3. If necessary this test may be used for still further practice as there is an alternate computation test on page 302 of this Manual.

Differentiations and Extensions. Use the table below as an aid in entering scores on the test record cards.

TABLE OF PER CENTS FOR CHAPTER 3 SCORES

Score	Per Cents for Problem Test 3
1	13
2	25
3	38
4	50
5	63
6	75
7	88
8	100

Score	Per Cents for Computation Test 3	Score	Per Cents for Computation Test 3
1	4	15	54
2	7	16	58
3	11	17	61
4	14	18	64
5	18	19	68
6	21	20	71
7	25	21	75
8	29	22	79
9	32	23	82
10	36	24	86
11	39	25	89
12	43	26	93
13	46	27	96
14	50	28	100

Do You Understand?

Test of Information and Meaning 3

On your paper write the words or numbers that will complete Ex. 1–5 correctly.

1. The numerator and the denominator of a fraction are called its terms

2. In best form, $5\frac{12}{8} = 6\frac{1}{2}$. 3. $1 = \frac{7}{7}$.

4. To subtract $4\frac{3}{4}$ from 30, we must change 30 to $29\frac{4}{4}$

5. To subtract $5\frac{9}{10}$ from $8\frac{3}{10}$, we must change $8\frac{3}{10}$ to $7\frac{13}{10}$

6. Write an improper fraction; a mixed number; a proper fraction.

7. Does $11\frac{3}{4}$ mean $\boxed{11 + \frac{3}{4}}$ or $11 \times \frac{3}{4}$?

8. Which of the following improper fractions is equal to $9\frac{5}{7}$?

$\frac{14}{7}$, $\left(\frac{68}{7}\right)$, $\frac{45}{7}$

9. Which is larger:

a. $\frac{1}{2}$ or $\left(\frac{2}{3}\right)$? b. $\frac{2}{3}$ or $\left(\frac{3}{4}\right)$? c. $\frac{5}{6}$ or $\left(\frac{7}{8}\right)$? d. $\left(\frac{3}{10}\right)$ or $\frac{3}{18}$?

10. In which of the following sets are the fractions like-fractions?

a. $\frac{1}{6}$, $\frac{2}{8}$, $\frac{5}{12}$ b. $\frac{5}{16}$, $\frac{5}{8}$, $\frac{5}{6}$ c. $\boxed{\frac{3}{10}, \frac{7}{10}, \frac{1}{10}}$

11. Use the Golden Rule of Fractions to change

a. $\frac{2}{3}$ to ninths; $\frac{6}{9}$ b. $\frac{5}{8}$ to twenty-fourths; $\frac{15}{24}$ c. $\frac{15}{18}$ to lowest terms. $\frac{5}{6}$

12. Write the smallest common denominator for each of the following examples:

a. $\frac{2}{5} + \frac{1}{3}$ 15 b. $\frac{5}{6} - \frac{3}{8}$ 24 c. $\frac{1}{6} + \frac{5}{9}$ 18 d. $\frac{13}{16} - \frac{3}{4}$ 16

13. Round each of these to the nearest whole number:

a. $4\frac{2}{5}$ 5 b. $18\frac{1}{4}$ 18 c. $16\frac{5}{12}$ 16 d. $12\frac{3}{8}$ 12

14. Write estimated answers for the following examples:

a. $4\frac{1}{3} + 8\frac{5}{6} + 2\frac{7}{8} + 1\frac{3}{10}$ 17 c. $13\frac{3}{5} - 7\frac{1}{2}$ 6

b. $89\frac{15}{16} - 21\frac{1}{8}$ 69 d. $6\frac{1}{2} + 2\frac{2}{5} + 4\frac{3}{10} + 7\frac{2}{3}$ 21

127

NOTES

Do You Make Mistakes?

Diagnostic Test 3

		Study Pages	Practice Sets
1.	Write the fractional unit for each. $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{12}$ $\frac{1}{9}$ $\frac{1}{16}$ $\frac{1}{3}$ $\frac{1}{4}$	99	
2.	Which is larger? $\frac{1}{6}$ or $\frac{1}{5}$ $\frac{1}{8}$ or $\frac{1}{10}$ $\frac{1}{4}$ or $\frac{1}{2}$ $\frac{1}{3}$ or $\frac{1}{4}$	102	
3.	Change to equal fractions. $\frac{3}{4} = \frac{6}{8}$ $\frac{4}{6} = \frac{2}{3}$ $\frac{9}{12} = \frac{3}{4}$ $\frac{12}{16} = \frac{3}{4}$	106-107	32, 41
4.	Change to best form. $\frac{17}{8} 2\frac{1}{8}$ $\frac{21}{6} 3\frac{1}{2}$ $\frac{8}{8} 1$ $\frac{10}{4} 2\frac{1}{2}$ $\frac{11}{3} 3\frac{2}{3}$	108-109	42
5.	Add or subtract. a $\frac{1}{5} + \frac{4}{5} 1$ b $\frac{7}{8} - \frac{3}{4} \frac{1}{8}$ c $\frac{1}{6} + \frac{5}{6} 1$ d $\frac{9}{10} - \frac{3}{10} \frac{3}{5}$	110-111	43, 44
6.	Change to fractions with smallest common denominator. $\frac{3}{8}$ and $\frac{5}{6}$ $\frac{4}{6}$ $\frac{9}{12}$ $\frac{3}{4}$ and $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{8}$ and $\frac{1}{4}$ $\frac{2}{8}$ $\frac{1}{3}$ and $\frac{5}{6}$ $\frac{2}{6}$ $\frac{5}{6}$	114-115, 119, 120	47
7.	Add or subtract. $6\frac{1}{4} + 2\frac{1}{2} 8\frac{3}{4}$ $7\frac{5}{8} + 2\frac{2}{8} 10\frac{1}{2}$ $4\frac{5}{8} - 1\frac{3}{8} 3\frac{2}{8}$ $5\frac{1}{8} - 3\frac{2}{8} 1\frac{4}{8}$	116-117, 118	45, 46 48-51
8.	Add or subtract. $4\frac{1}{5} - 2\frac{2}{5} 1\frac{4}{5}$ $5\frac{2}{3} + 7\frac{1}{2} 12\frac{7}{6}$ $9\frac{3}{4} - 7\frac{5}{12} 2\frac{1}{2}$ $3\frac{1}{2} + 8\frac{2}{3} 12\frac{1}{6}$	119-120	52, 53

Can You Solve Problems?

Problem Test 3

First write your (estimate) of the answer before solving each problem.

1. At the cheese counter, Mother bought $8\frac{1}{2}$ oz. of Danish cheese and $2\frac{2}{3}$ oz. of Swiss cheese in packages. How much more did the Danish cheese weigh than the Swiss cheese? (6), $5\frac{5}{6}$ oz.

2. Jim said that he needed $4\frac{1}{2}$ yd. of cord to rig his toy sailboat. His mother gave him $6\frac{1}{3}$ yd. How much more did she give him than he said he needed? (1), $1\frac{5}{6}$ yd.

3. Mother bought spices in boxes—cinnamon, $1\frac{1}{2}$ oz.; cloves, $1\frac{1}{8}$ oz.; nutmeg, $1\frac{1}{4}$ oz. What was the total weight? (4 oz.), $3\frac{7}{8}$ oz.

4. On her way to Grandma's, Lucy rode her bicycle $2\frac{7}{8}$ mi. and pushed it $\frac{1}{4}$ mi. How far was it to Grandma's? (3), $3\frac{1}{8}$ mi.

5. After filling the empty sugar can from a 5-pound bag of sugar, Mrs. Lee had $2\frac{3}{8}$ lb. left. How much did the can hold? (2 lb.), $2\frac{5}{8}$ lb.

6. One house lot is $85\frac{3}{4}$ ft. wide and one next to it is $98\frac{7}{8}$ ft. What is the total width of the two house lots? (185), $184\frac{5}{8}$ ft.

7. Linda wrote $\frac{7}{16}$ of the package of cards and Dale wrote $\frac{1}{2}$ of the package. Which wrote more? Dale How much more? $\frac{1}{16}$

8. Mrs. Douglas had $5\frac{1}{2}$ yd. of curtain material and bought $3\frac{3}{4}$ yd. more of the same kind. How much curtain material did she have then? (10 yd.), $9\frac{1}{4}$ yd.

How Well Can You Figure?

Computation Test 3

1. Change these numbers to best form:

$8\frac{6}{8} 8\frac{3}{4}$ $\frac{12}{16} \frac{3}{4}$ $\frac{35}{7} 5$ $5\frac{12}{8} 6\frac{1}{2}$ $\frac{39}{4} 9\frac{3}{4}$ $1\frac{13}{10} 2\frac{3}{10}$ $6\frac{14}{16} 6\frac{7}{8}$ $2\frac{9}{6} 3\frac{1}{2}$

2. Subtract each of the following fractions from 1:

$\frac{2}{3} \frac{1}{3}$ $\frac{7}{10} \frac{3}{10}$ $\frac{5}{9} \frac{4}{9}$ $\frac{5}{8} \frac{3}{8}$ $\frac{3}{7} \frac{4}{7}$ $\frac{8}{11} \frac{3}{11}$ $\frac{5}{6} \frac{1}{6}$ $\frac{3}{4} \frac{1}{4}$

Copy and work the following:

3. $4\frac{2}{3} + 1\frac{5}{6} = 6\frac{1}{2}$

4. $9\frac{3}{4} - 2\frac{1}{8} = 7\frac{5}{8}$

5. $8\frac{1}{2} + 4\frac{7}{8} = 13\frac{3}{8}$

6. $24\frac{5}{6} - 9\frac{1}{2} = 15\frac{2}{3}$

7. $8\frac{1}{2} + 2\frac{3}{4} = 11\frac{1}{4}$

8. $7\frac{1}{8} - 2\frac{1}{2} = 4\frac{5}{8}$

9. $24\frac{1}{12} - 8\frac{5}{6} = 15\frac{1}{4}$

10. $36\frac{1}{3} + 7\frac{5}{9} = 43\frac{8}{9}$

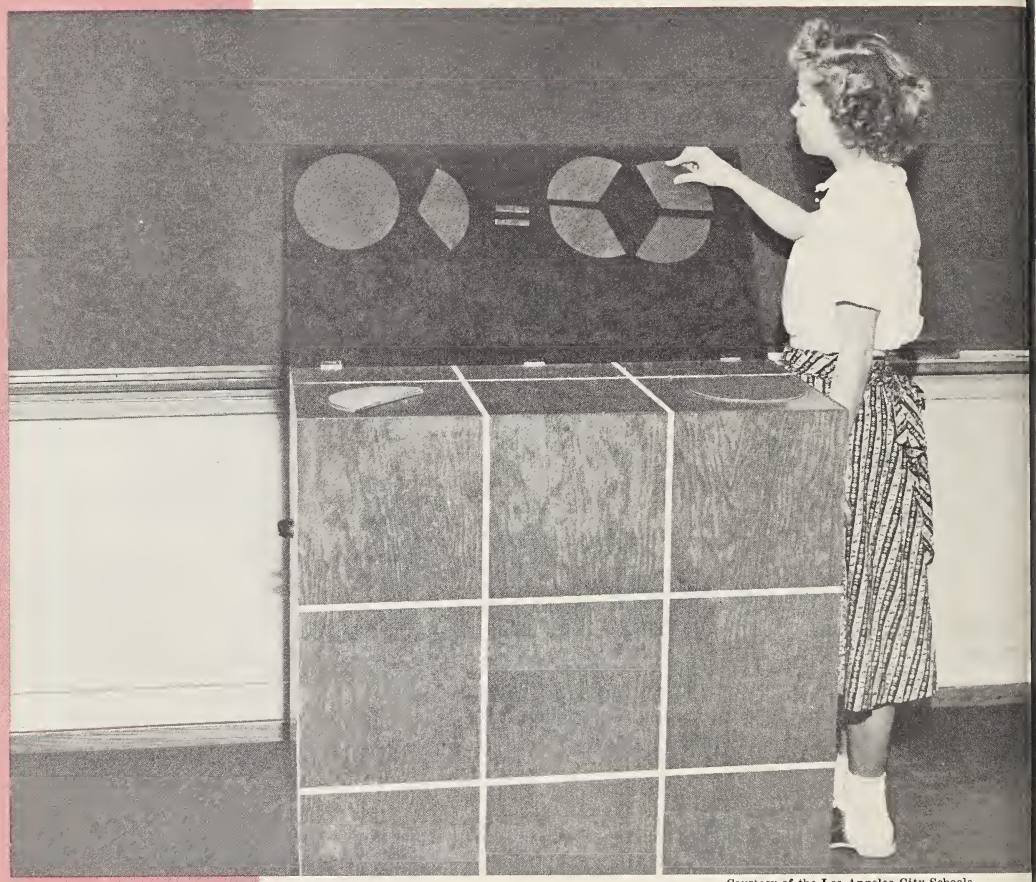
11. $4\frac{1}{3} + 8\frac{1}{4} = 12\frac{7}{12}$

12. $16\frac{4}{5} + 8\frac{7}{10} = 25\frac{1}{2}$

13. $18\frac{1}{2} - 5\frac{1}{3} = 13\frac{1}{6}$

14. $16\frac{1}{5} - 8\frac{2}{3} = 7\frac{13}{15}$

NOTES



Courtesy of the Los Angeles City Schools

This pupil finds that the flannel board attached to the top of the "cubic yard" is useful in showing why $1\frac{1}{3} = \frac{4}{3}$.

Teaching Chapter 4 of Grade 6

Introduction

Some reteaching occurs in Chapter 4 (scales, tables, graphs, and the like), but the major portion of this chapter is devoted to multiplication involving decimals which is new material for Grade 6.

I. Learning Outcomes in Chapter 4

1. Understanding and ability in multiplication involving fractions
2. Ability to estimate products in multiplying fractions
3. Understanding of, and ability to use, scales, tables, tallies, picture graphs, and bar graphs
4. Ability to solve problems
5. Understanding of various specific concepts, generalizations, relationships, and skills
6. Desirable emotionalized responses (attitudes, appreciations, values)

Understanding and Ability in Multiplication Involving Fractions (pages 131, 132, 135–142, 146–148, 150–153, 156, 164, 165)

The program for learning to multiply fractions is very carefully organized so as to make the best use of previous understandings and skills developed in relation to work with whole numbers and fractions. The sequence of topics in this chapter is as follows:

a. Two factors in multiplying fractions

- Whole number times a fraction
- Whole number times a mixed number (both the horizontal and vertical form)
- Fraction times a whole number
- Fraction times a fraction
- Fraction times a mixed number
- Mixed number times a whole number (both the horizontal and vertical form)
- Mixed number times a fraction
- Mixed number times a mixed number

b. Cancellation

c. More than two factors in multiplying fractions

Pupils have already discovered in working with whole numbers that multiplication involves combining several equal groups of objects or numbers. This idea is most easily carried over to multiplying fractions through the use of examples having whole-number multipliers and fraction or mixed-number multiplicands. In either type of example some of the techniques pupils have learned to use in multiplying whole numbers may be used.

The work may be shown (*a*) concretely (on a flannel board or with parts cut from equal wholes); (*b*) using pictures of objects; (*c*) on a number line; (*d*) as a process of adding several equal groups; (*e*) as multiplication. All of these techniques are used in leading pupils to realize that they may multiply the whole number times the numerator of the fraction to discover the number of equal parts in the total, and that they may write this product over the denominator of the fraction to show the size as well as the number of parts. Multiplying a whole number times a mixed number involves only one new step, the changing of the mixed number to an improper fraction. This simple step is presented on page 134, and pupils then use what they know about multiplying a whole number times a fraction to multiply a whole number times a mixed number in horizontal form. This is immediately followed by a page devoted to multiplying a whole number times a mixed number in vertical form, since computation involving large mixed-number multiplicands is frequently more easily done in this way.

In developing the fractional-part meaning of division in early grades, pupils learned that finding $\frac{1}{4}$ of 8 is simply a matter of dividing 8 by 4. In Grade 5, preparatory work with multiple fractions occurred (as $\frac{2}{5}$ of 15) but the word "of" was always used in place of the times sign ($\times$) that will be used in Grade 6. The first step, then, is for pupils to realize that an example like $\frac{1}{4}$ of 8 is really a multiplication example. This is not an easy concept, for pupils have grown used to thinking of the product as being larger than the multiplicand, whereas in this situation the product will be less than the multiplicand. A lesson designed to help pupils understand the meaning of multiplication when the multiplier is a fraction is presented on page 138 and is followed by work that leads to the computational procedure for multiplying a fraction times a whole number.

As the work in multiplying fractions continues, pictures are used so that pupils will *see* the meaning of multiplying a fraction times a fraction. In this way, pupils are helped to develop the idea that multiplying the denominators gives the size of the

new fractional unit and that multiplying the numerators gives the number of equal parts in the new fraction. Multiplying a fraction times a mixed number involves no new ideas, but some pupils may find it necessary to use side work in changing the mixed number to an improper fraction before multiplying.

After a summary of the eight different kinds of examples in multiplying fractions, pupils go on to develop skill in multiplying by mixed numbers. Pupils are shown that they may multiply by each part of the mixed-number multiplier (as with whole numbers), first by the fraction part of the mixed number, then by the whole number part. At the same time, they are shown how the mixed number could be changed to an improper fraction and the multiplication accomplished as in multiplying a fraction times a whole number. As pupils learn to multiply a mixed number times a fraction or a mixed number, they are shown the short way of multiplying (i.e., they learn to multiply the numerators and denominators directly without first rewriting the example, as shown in box A on page 148).

Once all the different kinds of multiplication examples involving fractions are presented, pupils are introduced to the idea of cancellation. They learn that cancellation is nothing more than dividing both terms of a fraction by the same number *before* multiplying, thereby avoiding the necessity of reducing the product after the multiplication. This is followed by a page of practice with all the different kinds of multiplication examples in which pupils are asked to use cancellation when they can.

If pupils really understand multiplying fractions, they will have no difficulty in multiplying when there are more than two factors. Essentially the procedure is to change all the factors to fractions, using cancellation when possible, so that the numerators and denominators may be multiplied.

It should be noted that what has been said here does not complete the program for multiplying fractions. A very important aspect of the program, rounding and estimating answers, is discussed under the special heading which follows.

Ability to Estimate Products in Multiplying Fractions (pages 154, 155, 157, 158)

The pages listed after this heading are only those that are specifically related to rounding and estimating products in multiplying fractions. There are, however, many opportunities to use these skills in the work in the text that is devoted to presenting the computational procedure and these are mentioned in the Manual whenever they occur, along with suggestions for teaching these skills.

Ability to estimate products in multiplying fractions is at least as important as ability to estimate products in work with whole numbers. First, it is an exceedingly useful skill in practical affairs. Second, practice in estimating products very often leads pupils to a better understanding of the meaning of multiplying fractions, especially when the multiplier is a fraction. Third, as pupils study the relationships in multiplication examples in order to estimate products, it very often happens that they discover ways of working without using pencil and paper.

Some of the most useful ideas for estimating products are presented on page 154 in the text. All of these ideas involve the size of the product in relation to the multiplicand when the multiplier is less than, equal to, or more than 1. Another important skill in estimating products is ability to find the range within which a product will fall. This frequently involves ability to round fractions. For examples of lessons that will help pupils develop these abilities in estimating products see pages 154, 158, 161, 163, and 168 in this Manual.

Understanding of, and Ability to Use, Scales, Tables, and Graphs (pages 159–164, 166–168)

Most of the work with graphs in Chapter 4 is part of the reteaching program. The meaning and use of a scale is reintroduced and then used by pupils much as they would use it in their everyday affairs. That is, they are asked to find the dimensions of objects drawn to scale in encyclopedias and in reading distances on maps.

Pupils then see how a tally sheet is used in gathering information and how this data is recorded in a table for easier reading. Except for learning the meaning of a key in the graph, the transition from work with tables to work with picture graphs is slight.

Pupils have had experience with horizontal- and vertical-bar graphs in Grade 5. Here in Grade 6 they are asked to make graphs of their own on a very simple level.

Ability to Solve Problems (pages 133, 137, 143, 145, 149, 157, 158)

The problem-solving program in Grade 6 is extended to include problems involving fractions. Pupils differentiate addition and subtraction in problems containing fractions. They work with problems that require the multiplying of fractions, and they solve problems involving measures written as fractions. They also receive practice in testing answers by estimating and in choosing the best estimate for problems. Because many other aspects of solving problems occur in the development devoted to computational procedures, it is wise for you to think of the problem-solving program as being the *reason* for learning arithmetic, not a special skill to be taught in isolation.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

bar graph
cancel, cancellation
graph
horizontal-bar graph
key (in a graph)
picture graph
scale
scale drawing
table (of numbers)
tally
vertical-bar graph

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 4 will be found on the following pages in the text, and on the corresponding reproduced pages in the Manual: pages 132, 135, 136, 140, 141, 142, 147, 150, 154, 155.

3. *Skills beyond those already listed:*

a. Ability to change mixed numbers to improper fractions (page 134)
b. Skill in using fractions in a game (page 144)
c. Ingenuity in finding other ways to multiply fractions (page 169)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

The end of Chapter 4 is a good place to test your pupils' development of understanding, skills, and abilities. A term test is provided in the Manual for this purpose and is referred to at the end of this chapter. However, an evaluation of attitudes, appreciations, and values in relation to arithmetic is the responsibility of the teacher. Make plans now which will allow you to observe in an objective way your pupils' emotional response to the arithmetic program. A check list of desirable responses may be helpful in this appraisal, and an anecdotal record beside each pupil's name will preserve your observations for later use.

An All-Day Hike

M. of a fraction by a whole number; meaning [O]

1. Six boys who were going on an all-day hike put up their own lunches. For each boy they allowed $\frac{1}{4}$ can of meat. How many cans of meat did they use in all?

To find this answer, you can use the drawing at the right. Count the $\frac{1}{4}$'s that you see. Which is the best way to write the number of cans used: $\frac{6}{4}$ or $1\frac{2}{4}$ or $1\frac{1}{2}$? Explain. Fractions in lowest terms

You can also add to find the answer:

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{6}{4}$, or $1\frac{2}{4}$, or $1\frac{1}{2}$.

These addends are equal, so you can multiply instead of adding. For 6 boys, how many $\frac{1}{4}$'s were used? Six $\frac{1}{4}$'s = $\frac{6}{4}$. From box A, tell how to give the answer in best form. Change improper fraction to mixed number.

A
Six $\frac{1}{4}$'s = $\frac{6}{4}$
$\frac{6}{4} = 1\frac{2}{4}$, or $1\frac{1}{2}$

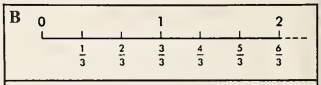
2. They allowed $\frac{1}{3}$ of a package of ginger snaps for each of the 6 hikers. How many packages did they need? 2

Box B shows how to find the answer with a number line. On line B, six $\frac{1}{3}$'s = $\frac{6}{3}$, or 2.

Explain box C.

The work in box D is like multiplying any whole number. Explain. 6 X 1 = 6; 6 X 1 ft. = 6 ft.; etc.

Box E shows how to multiply the fraction. $6 \times \frac{1}{3}$ is six $\frac{1}{3}$'s, or 2.

B


C
$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{6}{3}$, or 2

D
6 × 1 third = 6 thirds, or $\frac{6}{3}$, or 2

E
$6 \times \frac{1}{3} = \frac{6}{3} = 2$

3. As in box D, say these multiplications: **a.** $4 \times \frac{1}{4}$; **b.** $5 \times \frac{1}{3}$; **c.** $6 \times \frac{1}{2}$; **d.** $10 \times \frac{1}{5}$; **e.** $9 \times \frac{1}{6}$; **f.** $12 \times \frac{1}{8}$. (1) (1/3) (3) (2) (1/2) (1/4) [W]

4. Now, as in box E, write the multiplications in Ex. 3.

Teaching Pages 130, 131, and 132

Pupil's Objective. To develop the meaning of multiplying a whole number by a fraction and to practice multiplication of this kind.

Background. Chapter 4 introduces for the first time a complete development of multiplication involving fractions. The first three pages deal with the multiplication of a fraction by a whole number. If the development of this concept is initiated through the use of objects, pupils will have no trouble in understanding the meaning of the process. Just as with whole-number multiplication, pupils may count parts of objects, count

intervals on a number line, add equal groups, or, finally, multiply using both words and figures in working with fractions. All of these points are made on page 131 in the text, and then on page 132 pupils receive more detailed work in multiplying using figures.

The generalization on page 132 touches on an important point that ought to be stressed—since it is the numerator that shows the number of equal parts, it is the numerator that we work with in multiplying a fraction by a whole number (or in adding to check). The work in boxes A and B shows the written form that pupils may adopt.

Teacher's Preparation. Plan materials which may be used in presenting multiplication of fractions in concrete and semi-concrete form. The following devices and objects have many applications: fractional pieces (possibly prepared for use with a flannel board); calculating strips for fractions (with corresponding number lines); and paper, scissors, paste, and crayons for pupils to prepare their own representations.

Pre-book Lesson (pages 130 and 131)

1. Allow pupils an opportunity to experiment with concrete and semi-concrete materials so that they will discover that the number of parts, as represented by the numerator, is to be multiplied, while the size of the parts, as represented by the denominator, remains the same.

2. Present a problem such as the following:

Plans are being made for a party where ice cream is to be served to 8 people. Find how many quarts would be used if each quart of ice cream makes 6 servings; if each quart makes 8 servings; if each quart makes 10 servings.

Illustrate with pictures or with oak tag rectangles measured to show the portions described, as below. Have pupils determine the fractional unit and the way of expressing the multiplication.

Fractional unit = $\frac{1}{6}$

$8 \times \frac{1}{6} = ?$

First have them determine the answer by counting and by adding; then let them explain why the answer may be found by multiplication. The work may be completed on the board as below.

Eight $\frac{1}{6}$'s $= \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$ (quarts of ice cream)

In like manner, pupils may determine how much ice cream will be used if the choice is either that of 8 portions or 10 portions to a quart.

3. Present examples so that some pupils may manipulate fractional pieces and calculating strips, while other pupils draw simple pictures or representative forms.

4. Have the children state in their own words a rule for multiplying a fraction by a whole number. Lead them to include the understandings that are stated in the generalization on page 132 if you can.

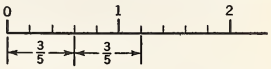
How Many in All?

Whole number × fraction [O]

1. Ted's school is $\frac{3}{5}$ mile from his home. How far does he walk in going to school and home again?

You can add to find the answer. $2 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5}$, or $\frac{6}{5}$. In best form, $\frac{6}{5} = 1\frac{1}{5}$.

You can use a number line. $2 \times \frac{3}{5} = \frac{6}{5}$, or $1\frac{1}{5}$.



Remember that the 3 in $\frac{3}{5}$ means that there are three $\frac{1}{5}$'s. Then for $2 \times \frac{3}{5}$ there would be twice as many $\frac{1}{5}$'s, or six $\frac{1}{5}$'s. Explain. 2 x 3 fifths = 6 fifths

Box A shows you how to find $2 \times \frac{3}{5}$ with figures. What is the multiplier? 2 the multiplicand? $\frac{3}{5}$ the product? $\frac{6}{5}$, or $1\frac{1}{5}$

A
$$2 \times \frac{3}{5} = \frac{2 \times 3}{5}, \text{ or } \frac{6}{5}, \text{ or } 1\frac{1}{5}$$

2. Explain the multiplication in box B. In best form, $\frac{45}{4} = 11\frac{1}{4}$

B
$$15 \times \frac{3}{4} = \frac{15 \times 3}{4}, \text{ or } \frac{45}{4}, \text{ or } 11\frac{1}{4}$$

To multiply a fraction by a whole number, multiply the numerator of the fraction by the whole number to get the total number of equal parts. Then show the size of the parts by writing the same denominator as in the given fraction.

[W]

Multiply as in box B. Give your answers in best form.

	a	b	c	d	e	f
3.	$6 \times \frac{1}{2} 3$	$6 \times \frac{2}{3} 4$	$3 \times 1\frac{3}{8}$	$5 \times \frac{3}{4} 3\frac{3}{4}$	$4 \times 2\frac{3}{5}$	$7 \times \frac{1}{2} 3\frac{1}{2}$
4.	$4 \times \frac{2}{3} 2\frac{2}{3}$	$8 \times \frac{2}{3} 5\frac{1}{3}$	$4 \times \frac{3}{8} 1\frac{1}{2}$	$3 \times \frac{2}{5} 2$	$6 \times \frac{7}{8} 5\frac{1}{4}$	$3 \times \frac{2}{5} 1\frac{1}{5}$
5.	$6 \times \frac{3}{4} 4\frac{1}{2}$	$7 \times \frac{3}{8} 2\frac{5}{8}$	$10 \times \frac{3}{4} 7\frac{1}{2}$	$8 \times \frac{4}{5} 6\frac{2}{5}$	$9 \times \frac{2}{3} 6$	$3 \times \frac{7}{8} 2\frac{5}{8}$
6.	$12 \times \frac{5}{6} 10$	$14 \times \frac{2}{3} 9\frac{1}{3}$	$9 \times \frac{3}{5} 5\frac{2}{5}$	$5 \times \frac{3}{8} 1\frac{7}{8}$	$7 \times \frac{2}{3} 4\frac{2}{3}$	$7 \times \frac{5}{8} 4\frac{3}{8}$

Check your answers for row 3 by adding.

Extra Practice. Work Set 54.

Book Lesson (pages 130 and 131). *Ex. 1–3:* Oral work. *Ex. 4:* Written work. Remind pupils to give all final answers in best form.

Pre-book Lesson (page 132). Because numerators larger than 1 are introduced on this page, you may wish to extend the kinds of practice with concrete and representative objects that were suggested for page 131. For example, you may suggest that each child at the party was allowed two portions of ice cream, thus introducing the examples $8 \times \frac{2}{6}$, $8 \times \frac{2}{8}$, $8 \times \frac{2}{10}$.

Book Lesson (page 132). *Ex. 1 and 2:* Oral work. *Rows 3–6:* Written work. Spend considerable time helping pupils understand the form for multiplying the numerator by the whole number as shown in boxes A and B in the text.

Differentiations and Extensions

1. In all work involving multiplication of fractions be sure you allow pupils an opportunity to develop quick ways to work without pencil and paper. For example, on page 132 some pupils could be asked to try to give as many of the multiplications as possible as an oral lesson. For the benefit of the other members of the group, be sure each child explains his thinking process.

For Ex. 3**c**, pupils should see immediately that $3 \times \frac{1}{8}$ is three $\frac{1}{8}$'s ($\frac{3}{8}$). It would be too bad for *all pupils* to use pencil and paper for an easy example such as this. For Ex. 3**f**, the same procedure may be used, but pupils will obtain an improper fraction for an answer. They should be able to change this to a mixed number quickly. For Ex. 3**a**, some pupils will probably think, "Since two $\frac{1}{2}$'s make a whole, six $\frac{1}{2}$'s will make three wholes." Some pupils may be able to do all of rows 3–6 without pencil and paper. They may surprise you by the understanding they show in suggesting various ways to find products.

2. Assign **Extra Practice Set 54**, reproduced below, as needed.

Set 54. Multiplying a fraction by a whole number

	a	b	c	d	e
1.	$4 \times \frac{1}{2}$ 2	$10 \times \frac{2}{5}$ 4	$1 \times \frac{1}{7}$ $\frac{1}{7}$	$3 \times \frac{8}{9}$ $2\frac{2}{3}$	$11 \times \frac{3}{5}$ $6\frac{3}{5}$
2.	$9 \times \frac{5}{3}$ 15	$12 \times \frac{3}{8}$ $4\frac{1}{2}$	$3 \times \frac{14}{5}$ $8\frac{2}{5}$	$8 \times \frac{7}{8}$ 7	$2 \times \frac{2}{7}$ $\frac{4}{7}$
3.	$5 \times \frac{1}{4}$ $1\frac{1}{4}$	$8 \times \frac{7}{4}$ 14	$16 \times \frac{5}{8}$ 10	$7 \times \frac{4}{5}$ $5\frac{3}{5}$	$15 \times \frac{3}{4}$ $11\frac{1}{4}$
4.	$6 \times \frac{2}{3}$ 4	$9 \times \frac{5}{6}$ $7\frac{1}{2}$	$14 \times \frac{1}{5}$ $1\frac{4}{5}$	$5 \times \frac{11}{8}$ $6\frac{7}{8}$	$7 \times \frac{13}{2}$ $45\frac{1}{2}$

Fraction Problems

Differentiating A. and M.; using a number line [O]

When Alex worked problems 1 to 5 on this page, he first wrote the examples which follow the problems. For which problem did Alex write the wrong example? *Problem 4*

1. If it takes $\frac{3}{4}$ yd. of nylon to make a scarf, how much nylon will be needed for 3 such scarves? $3 \times \frac{3}{4} = 2\frac{1}{4}$ yd.

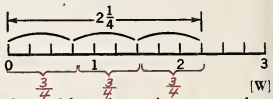
2. Mr. Lusk raises Angora rabbits. He has just sheared 3 of them. One yielded $\frac{3}{8}$ lb. of hair, another $\frac{1}{2}$ lb., and the third $\frac{1}{4}$ lb. How many pounds of hair did Mr. Lusk get from the three rabbits together? $\frac{3}{8} + \frac{1}{2} + \frac{1}{4} = 1\frac{1}{8}$ lb.

3. Each day Mr. Lusk feeds his rabbits $\frac{1}{4}$ pk. of oats. How many pecks does he feed them in 7 days? $7 \times \frac{1}{4} = 1\frac{3}{4}$ pk.

4. If $\frac{2}{3}$ cup of sugar is needed for one apple pie, how many cups are needed for 4 apple pies? $4 \times \frac{2}{3} = 2\frac{2}{3}$ c.

5. Tom's notebook weighs $\frac{3}{8}$ pound. How much would five such notebooks weigh? $5 \times \frac{3}{8} = 1\frac{7}{8}$ lb.

Explain how to work problem 1 on this number line.



[W]

Now find the answer for each problem by using a number line. Then write each example with its answer in best form.

Teaching Page 133

Pupil's Objective. To solve addition and multiplication problems involving fractions by using a number line.

Background. Differentiating addition and multiplication as shown on this page is new only in that fractions are involved. Of course, pupils already know that multiplication is just a quick way of adding equal groups, and so all of the problems on page 133 in the text could be solved by addition. However, you will want to help your children

realize that when equal groups are put together, multiplication is the more direct process. This may also be a good time to point out again that fractions can only be combined when they have the same fractional unit (are like-fractions).

Book Lesson. *Ex. 1–5:* Oral work. *Bottom of the page:* Written work.

1. In the oral discussion, have pupils tell the difference in the meaning of the addition and multiplication processes.

2. The plus sign that Alex wrote for Example 4 will give you a chance to stress the importance of understanding basic meanings as well as the importance of being careful in writing process signs.

3. For all but Example 4, pupils may use their ruler as a number line, provided that it shows eighths.

Differentiations and Extensions

1. If pupils draw number lines for each of problems 1–5, you may want to have some of them selected for display on the bulletin board. An explanation of what is shown on each number line probably ought to accompany each illustration.

2. Again let *more capable children* tell ways that the abstract work might be done without pencil and paper.

Teaching Pages 134 and 135

Pupil's Objectives. (*a*) To learn to change a mixed number to an improper fraction; (*b*) to multiply a mixed number by a whole number by changing the mixed number to an improper fraction.

Background. Now that pupils know how to multiply a fraction by a whole number, they should have very little difficulty in multiplying a mixed number by a whole number, since mixed numbers may be changed to improper fractions.

The work on page 134 will help pupils realize that they can change a mixed number so that the resultant improper fraction will have the same fractional unit as the fraction of the mixed number. As pictured at the top of page 134 in the text, have pupils do this work using objects so that the mechanical changing will make sense.

You will notice in the box on page 135 that the only new step is changing the mixed number to a fraction before multiplying. Some pupils may have to do this as side work, but with time and practice they should learn to make the change without writing the work.

Teacher's Preparation. Colored construction paper, cut so that mixed numbers may be represented, helps to vary practice materials in a way that appeals to children.

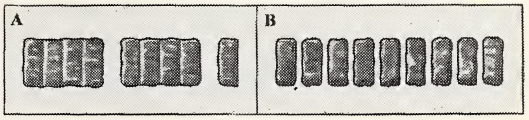
Pre-book Lesson (page 134)

1. Pupils may explore the possibilities of changing mixed numbers or whole numbers to improper fractions by manipulating the colored parts cut from construction paper.

Group work may be organized by letting each group be responsible for demonstrating to the class

Making a Fraction from a Mixed Number

Changing to improper fraction [O]



1. In box A there are $2\frac{1}{4}$ candy bars. In box B the $2\frac{1}{4}$ bars have been cut up into $\frac{1}{4}$'s. How many $\frac{1}{4}$'s are there? 9

2. From boxes A and B, you know that the mixed number $2\frac{1}{4}$ is equal to the improper fraction $\frac{9}{4}$.

3. On the board draw candy bars to find out what improper fraction is equal to $3\frac{1}{3}$. $\frac{10}{3}$ to $4\frac{1}{2}$. $\frac{9}{2}$

In changing 1 whole to $\frac{1}{4}$'s, you know that there are four $\frac{1}{4}$'s. In changing to $\frac{1}{3}$'s, you know that there are three $\frac{1}{3}$'s, and so on.

4. You can change 6 to $\frac{1}{4}$'s without making a drawing.

In 1 whole there are four $\frac{1}{4}$'s, so in 6 wholes there are 6 times as many $\frac{1}{4}$'s, or 24 of them. $6 = \frac{24}{4}$

In this same way tell how many

5. $\frac{1}{3}$'s in 5. $\frac{15}{3}$ 6. $\frac{1}{2}$'s in 7. $\frac{14}{2}$ 7. $\frac{1}{5}$'s in 8. $\frac{40}{5}$

You can change a mixed number to an improper fraction in much the same way. The denominator of the fraction shows you what the fractional unit of the improper fraction is to be.

8. Change $3\frac{1}{4}$ to an improper fraction.

HELPER. $3 = \frac{12}{4}$. $\frac{12}{4}$ plus the other $\frac{1}{4}$ equals $\frac{13}{4}$.

Change these mixed numbers to improper fractions without making a drawing:

9. $5\frac{1}{2}$ $\frac{11}{2}$ 10. $8\frac{1}{3}$ $\frac{25}{3}$ 11. $4\frac{2}{3}$ $\frac{14}{3}$ 12. $6\frac{3}{5}$ $\frac{33}{5}$ 13. $3\frac{1}{8}$ $\frac{25}{8}$ 14. $7\frac{3}{4}$ $\frac{31}{4}$

Extra Practice. Work Set 55.

134

the changing of a mixed number (such as $1\frac{7}{8}$, $2\frac{1}{2}$, or $4\frac{1}{4}$) or a whole number to an improper fraction. Whereas some pupils within each group may be responsible for providing demonstrations with concrete or semi-concrete materials, other pupils within the same group may contribute by drawing diagrams or writing explanations of the steps taken. With the approval of the other members of the class, or on the basis of suggestions offered by them, the materials displayed by each group may be made into permanent charts.

2. Before they turn to the work in the text, see if *more capable pupils* can suggest a way of changing mixed numbers to improper fractions using figures only. To change $1\frac{7}{8}$, they might follow these steps:

$$1\frac{7}{8} = 1 + \frac{7}{8};\ 1 = \frac{8}{8};\ \frac{8}{8} + \frac{7}{8} = \frac{15}{8};\ 1\frac{7}{8} = \frac{15}{8}$$

Book Lesson (page 134). *Ex. 1–14:* Oral work.

1. For Ex. 9–14, *slower learners* may need to tell to what fractional unit they will change each mixed number. For example, Ex. 9 requires that $5\frac{1}{2}$ be changed to halves while Ex. 10 requires that $8\frac{1}{3}$ be changed to thirds.

2. You may assign for practice rows 2, 3, and 4 on page 135 at this time if you so wish.

Book Lesson (page 135). *Ex. 1:* Oral work. *Rows 2–10:* Written work. Most pupils will probably apply with ease the procedure just studied, that of changing mixed numbers to improper fractions, to the work presented here in multiplication. It may help *slower learners* to draw diagrams for Ex. 1, and a few other examples, to assist them in the process.

Multiplying a Mixed Number

Whole number × mixed number [O]

1. Mrs. Fox bought 6 wedges of cheese each marked $1\frac{1}{3}$ oz. How many ounces of cheese did she buy? $6 \times 1\frac{1}{3} = ?$

The box shows one way to find $6 \times 1\frac{1}{3}$. The mixed number $1\frac{1}{3}$ is changed to $\frac{4}{3}$ before it is multiplied. Explain each step in finding the product, 8.

$$6 \times 1\tfrac{1}{3} = 6 \times \tfrac{4}{3}$$

$$6 \times \tfrac{4}{3} = \tfrac{6 \times 4}{3}, \text{ or } \tfrac{24}{3}, \text{ or } 8$$

Mrs. Fox bought 8 ounces of cheese.

To multiply a mixed number by a whole number, change the mixed number to an improper fraction. Then multiply this fraction by the whole number.

[W]

Change these mixed numbers to improper fractions:

	a	b	c	d	e	f
2.	$1\frac{5}{6}$ $\frac{11}{6}$	$8\frac{1}{2}$ $\frac{17}{2}$	$1\frac{3}{4}$ $\frac{7}{4}$	$7\frac{1}{3}$ $\frac{22}{3}$	$3\frac{5}{8}$ $\frac{29}{8}$	$1\frac{2}{3}$ $\frac{5}{3}$
3.	$3\frac{1}{3}$ $\frac{10}{3}$	$2\frac{1}{2}$ $\frac{5}{2}$	$5\frac{1}{4}$ $\frac{21}{4}$	$3\frac{1}{2}$ $\frac{7}{2}$	$2\frac{3}{4}$ $\frac{11}{4}$	$3\frac{1}{6}$ $\frac{19}{6}$
4.	$2\frac{1}{4}$ $\frac{9}{4}$	$3\frac{1}{7}$ $\frac{22}{7}$	$2\frac{3}{10}$ $\frac{23}{10}$	$2\frac{5}{12}$ $\frac{29}{12}$	$3\frac{3}{8}$ $\frac{27}{8}$	$4\frac{1}{5}$ $\frac{21}{5}$

As in the box above, multiply in rows 5–7.

	a	b	c	d	e
5.	$8 \times 6\frac{3}{4}$ 54	$6 \times 2\frac{7}{8}$ $17\frac{1}{4}$	$9 \times 1\frac{3}{4}$ $15\frac{3}{4}$	$5 \times 4\frac{2}{3}$ $23\frac{1}{3}$	$4 \times 2\frac{5}{8}$ $11\frac{1}{2}$
6.	$7 \times 3\frac{2}{5}$ $23\frac{4}{5}$	$5 \times 2\frac{4}{15}$ $11\frac{1}{3}$	$6 \times 2\frac{5}{9}$ $15\frac{1}{3}$	$4 \times 3\frac{7}{12}$ $14\frac{1}{3}$	$8 \times 1\frac{3}{16}$ $9\frac{1}{2}$
7.	$2 \times 2\frac{4}{5}$ $5\frac{3}{5}$	$7 \times 2\frac{4}{7}$ 18	$8 \times 5\frac{1}{6}$ $41\frac{1}{3}$	$3 \times 2\frac{1}{8}$ $6\frac{3}{8}$	$6 \times 4\frac{2}{9}$ 25

Find products for the examples in rows 8–10. Check your answers for row 8 by adding.

8.	$5 \times \frac{1}{8}$ $\frac{5}{8}$	$4 \times \frac{5}{6}$ $3\frac{1}{3}$	$8 \times 2\frac{4}{9}$ $19\frac{5}{9}$	$5 \times 1\frac{9}{10}$ $9\frac{1}{2}$	$12 \times \frac{3}{4}$ 9
9.	$18 \times \frac{1}{6}$ 3	$3 \times 2\frac{11}{12}$ $8\frac{3}{4}$	$4 \times \frac{5}{9}$ $2\frac{2}{9}$	$2 \times 3\frac{5}{16}$ $6\frac{5}{8}$	$6 \times 7\frac{2}{3}$ 46
10.	$2 \times \frac{5}{32}$ $\frac{5}{16}$	$4 \times 1\frac{7}{20}$ $5\frac{2}{5}$	$5 \times \frac{2}{15}$ $\frac{2}{3}$	$3 \times \frac{7}{24}$ $\frac{7}{8}$	$6 \times 1\frac{1}{12}$ 6

Extra Practice. Work Set 56.

Differentiations and Extensions

1. For Ex. 9 to 14 at the bottom of page 134 and rows 2–4 on page 135 in the text, see if *more capable pupils* can tell you how they think in changing the mixed numbers to improper fractions. Some may be able to discover for themselves that they only need to multiply the whole number times the denominator and add the numerator in order to discover the numerator of the improper fraction.

2. For rows 5–7 on page 135, be sure pupils receive practice in estimating products. They should be able to change mixed numbers to the nearest whole number and multiply mentally. *More capable pupils* frequently discover ways of deriving exact answers using the estimating procedure. For example, in Ex. 5**a**, some of these pupils may see that they can think of $6\frac{3}{4}$ as 7. They then multiply 8×7 obtaining a product 56. However, since they increased the multiplicand by $\frac{1}{4}$ in rounding, the product will be $8 \times \frac{1}{4}$, or 2 too large, the exact answer being 54.

3. Rows 8–10 at the bottom of page 135 contain examples with multiplicands that are fractions or mixed numbers. Let pupils attempt to do some of these examples orally. They should be able to do examples 8**a**, **b**, and **e** orally and they should give estimates for examples containing mixed numbers as multiplicands.

4. Pupils may not realize that the quantity cooking done in places where food is served to large

groups requires larger units of measure. *More capable pupils* may complete the following recipes, writing their answers in best form.

Vegetable Chowder

	25 portions	*50 portions*
chopped celery	1 c.	2 c.
diced potatoes	$\frac{1}{2}$ gal.	1 gal.
chopped carrots	$1\frac{1}{2}$ qt.	3 qt.
liquid (cooking liquid and milk)	$\frac{3}{4}$ gal.	$1\frac{1}{2}$ gal.
salt pork, chopped	$\frac{1}{2}$ lb.	1 lb.
sliced onion	1 pt.	1 qt.
sifted flour	$\frac{1}{2}$ c.	1 c.

Muffins

	25 portions	*50 portions*
dried white of egg	$\frac{1}{4}$ c.	$\frac{1}{2}$ c.
sifted flour	1 qt.	2 qt.
baking powder	$\frac{1}{4}$ c.	$\frac{1}{2}$ c.
salt	2 tsp.	4 tsp.
sugar	$\frac{1}{2}$ c.	1 c.
milk	$1\frac{3}{4}$ c.	$3\frac{1}{2}$ c.
melted fat	$\frac{1}{2}$ c.	1 c.

5. **Extra Practice Sets 55 and 56** give additional practice in changing mixed numbers to improper fractions and in multiplying mixed by whole numbers.

Set 55. Changing mixed numbers to improper fractions

Change these mixed numbers to improper fractions.

	a	b	c	d	e	f	g
1.	$4\frac{1}{2}$ $\frac{9}{2}$	$5\frac{3}{4}$ $\frac{23}{4}$	$12\frac{1}{3}$ $\frac{37}{3}$	$9\frac{1}{4}$ $\frac{37}{4}$	$2\frac{5}{8}$ $\frac{21}{8}$	$11\frac{1}{10}$ $\frac{111}{10}$	$25\frac{1}{2}$ $\frac{51}{2}$
2.	$2\frac{3}{16}$ $\frac{35}{16}$	$16\frac{2}{3}$ $\frac{50}{3}$	$3\frac{1}{7}$ $\frac{22}{7}$	$16\frac{1}{2}$ $\frac{33}{2}$	$8\frac{3}{10}$ $\frac{83}{10}$	$4\frac{5}{12}$ $\frac{53}{12}$	$32\frac{3}{8}$ $\frac{259}{8}$
3.	$1\frac{1}{8}$ $\frac{9}{8}$	$6\frac{2}{5}$ $\frac{32}{5}$	$9\frac{7}{8}$ $\frac{79}{8}$	$7\frac{3}{32}$ $\frac{227}{32}$	$3\frac{3}{20}$ $\frac{63}{20}$	$10\frac{2}{3}$ $\frac{32}{3}$	$7\frac{5}{16}$ $\frac{117}{16}$
4.	$9\frac{2}{3}$ $\frac{29}{3}$	$3\frac{7}{12}$ $\frac{43}{12}$	$1\frac{1}{6}$ $\frac{7}{6}$	$8\frac{7}{16}$ $\frac{135}{16}$	$15\frac{5}{6}$ $\frac{95}{6}$	$5\frac{3}{5}$ $\frac{28}{5}$	$30\frac{3}{4}$ $\frac{123}{4}$

Set 56. Multiplying a mixed number by a whole number

	a	b	c	d	e
1.	$6 \times 9\frac{7}{10}$ $58\frac{1}{5}$	$9 \times 3\frac{1}{7}$ $28\frac{2}{7}$	$7 \times 13\frac{2}{3}$ $95\frac{2}{3}$	$6 \times 1\frac{7}{12}$ $9\frac{1}{2}$	$8 \times 16\frac{1}{4}$ 130
2.	$4 \times 10\frac{3}{8}$ $41\frac{1}{2}$	$3 \times 14\frac{3}{4}$ $44\frac{1}{4}$	$2 \times 19\frac{5}{9}$ $39\frac{1}{9}$	$4 \times 18\frac{3}{5}$ $74\frac{2}{5}$	$6 \times 13\frac{4}{5}$ $82\frac{4}{5}$
3.	$2 \times 8\frac{5}{16}$ $16\frac{5}{8}$	$4 \times 11\frac{5}{6}$ $47\frac{1}{3}$	$8 \times 4\frac{7}{16}$ $35\frac{1}{2}$	$5 \times 17\frac{7}{8}$ $89\frac{3}{8}$	$2 \times 15\frac{1}{3}$ $30\frac{2}{3}$
4.	$3 \times 12\frac{1}{2}$ $37\frac{1}{2}$	$9 \times 9\frac{2}{5}$ $84\frac{3}{5}$	$5 \times 15\frac{5}{8}$ $78\frac{1}{8}$	$7 \times 10\frac{6}{7}$ 76	$5 \times 2\frac{7}{15}$ $12\frac{1}{3}$

NOTES

Multiplying a Mixed Number Another Way

A whole number × a mixed number [O]

1. The length of Mr. Kent's steps is $2\frac{1}{2}$ ft. From the picture, find the length of a row in his garden. 7 × $2\frac{1}{2}$ = ?

You can change $2\frac{1}{2}$ to an improper fraction as on page 134. Another way is to separate the mixed number into parts and multiply the parts, as in box A.

$2\frac{1}{2} = 2 + \frac{1}{2}$. In box A, explain how we multiply parts.

Name the partial products.

A

$2\frac{1}{2}$
$\times 7$
$3\frac{1}{2}$
14
$17\frac{1}{2}$

Side work: $7 \times \frac{1}{2} = \frac{7}{2}$, or $3\frac{1}{2}$

B

$2\frac{5}{6}$
$\times 8$
$6\frac{2}{3}$
16
$22\frac{2}{3}$

Side work: $8 \times \frac{5}{6} = \frac{?}{6}$, or _?_

2. Box B. Tell the parts of $2\frac{5}{6}$. $2\frac{5}{6}$

Explain the side work Ralph did to get the partial product $6\frac{2}{3}$.

Explain the rest of the work.

To multiply a mixed number by a whole number, you can multiply the parts separately, first the fraction and then the whole number, and then add the partial products.

[W]

Copy and multiply as in the boxes above.

	a	b	c	d	e	f
3.	$4 \times 1\frac{1}{2}$ 6	$5 \times 3\frac{3}{4}$ $18\frac{3}{4}$	$6 \times 4\frac{2}{3}$ 28	$4 \times 1\frac{3}{4}$ 7	$3 \times 2\frac{1}{3}$ 7	$4 \times 3\frac{2}{5}$ $13\frac{3}{5}$
4.	$3 \times 4\frac{1}{6}$ $12\frac{1}{2}$	$7 \times 5\frac{2}{3}$ $39\frac{2}{3}$	$4 \times 2\frac{1}{8}$ $8\frac{1}{2}$	$2 \times 1\frac{3}{8}$ $2\frac{3}{4}$	$8 \times 2\frac{1}{2}$ 20	$4 \times 8\frac{2}{3}$ $34\frac{2}{3}$

Check each answer for row 3 by multiplying as on page 135.

Using Fractions in Problems

M. of fractions and mixed numbers by whole numbers [W]

Write your work to find the answers. You can multiply as on page 135 or as on page 136, whichever way is easier for you.

1. The advertisement for a mountain climber's pack sack gave the weight as $12\frac{1}{2}$ lb. What will 2 of these packs weigh? 25 lb.

2. Tony planted 5 rows of bush beans $1\frac{1}{2}$ feet apart. How many feet was it from the first row to the fifth? 6 ft.

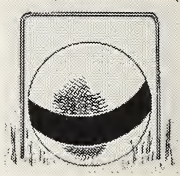
HELPER. Draw a diagram to see whether you multiply $1\frac{1}{2}$ by 4 or by 5.

3. Sally's recipe for cookies calls for $2\frac{1}{4}$ cups of flour. If she doubles the recipe, how much flour should she use? $4\frac{1}{2}$ cups

4. If she allows $1\frac{3}{8}$ yards for each apron, how much material will Mrs. Hooper need for 3 aprons? $4\frac{1}{8}$ yd.

5. To get ready for a weight-lifting contest, Mr. Henry added $\frac{3}{4}$ lb. each day to the bar he lifted for practice. If the bar weighed 75 lb. at first, how much did it weigh after he added this much weight on each of 28 days? 96 lb.

6. Elinor's sister works in a candy store. If she packs $1\frac{1}{2}$ lb. of candy in each box, how much candy will she use in filling 12 boxes? 18 lb.



7. According to the official rules for croquet, the width of the wickets should be $3\frac{3}{8}$ inches,—which leaves a $\frac{1}{8}$-inch margin on each side of the ball as it passes through the wicket. How much room does the croquet ball take? $3\frac{1}{8}$"

8. Kenny's father wants to make two rope swings for the backyard playground. Each swing will take $18\frac{1}{2}$ feet of rope. How much will the rope cost at $8\frac{1}{2}$¢ a foot? (Count any fraction of a cent in the answer as another cent.) $3.1

Teaching Pages 136 and 137

Pupil's Objectives. (*a*) To learn how to multiply a mixed number by a whole number in vertical form; (*b*) to discover that the two ways of multiplying a mixed number by a whole number may be used as mutual checks; (*c*) to solve problems involving the kinds of multiplication recently studied.

Background. Right after the development of the horizontal method for multiplying a whole number times a mixed number comes the vertical method shown in boxes A and B on page 136. This is a very useful technique, especially when the multiplicand is a large mixed number and the multiplier is a small whole number. Pupils who have previously been estimating products may already have discovered this technique of multiplying by parts and adding the partial products. Some pupils will need to write out that part of the work involving multiplying the fraction. This should be shown as side work as in boxes A and B. Note at the bottom of page 136 that the horizontal way of multiplying may be used as a simple check for the vertical way

of multiplying a whole number times a mixed number.

Book Lesson (page 136). *Ex. 1 and 2:* Oral work. *Rows 3 and 4:* Written work. Before pupils start the written assignment, have them work out with you a guide to be written on the board, somewhat as below.

Multiplying a Mixed Number by a Whole Number

$7\frac{3}{4}$ ← Multiply the fraction first.
$\times\ 6$ $(6 \times \frac{3}{4} = \frac{18}{4} = 4\frac{1}{2})$ ← Do side work to be sure that the partial product is in the best form.
$4\frac{1}{2}$ ←
42 ← Write the partial product.
$46\frac{1}{2}$ ← Multiply the whole numbers, writing the partial product in the correct place.
Add the partial products to get the final product.

Multiplying Horizontally as a Check

$6 \times 7\frac{3}{4}$ $\qquad$ $6 \times \frac{31}{4} = \frac{186}{4} = 46\frac{1}{2}$

Book Lesson (page 137). *Ex. 1–8:* Written work. Inspect the problems on this page with your pupils. Stress one or two areas which you know need strengthening. For example, you may ask pupils how many two-step problems they can find. You may wish to give specific help for weaknesses that you have noticed in multiplication of this kind, particularly for your *slower learners*.

Differentiations and Extensions

1. Let pupils estimate answers for rows 3 and 4 at the bottom of page 136 in the text. Then, to be sure pupils are maintaining previously-learned skills, have them turn to page 132 and do again rows 5 and 6 on that page.

2. *More capable pupils* may use the vertical method of multiplying mixed numbers in writing equivalents for measures, as suggested in the exercise below.

If you know that $5\frac{1}{2}$ yd., or $16\frac{1}{2}$ ft., equal 1 rod, you should be able to complete the statements below. Check your answers by finding if the number of feet in each case is three times the number of yards.

4 rods = $5\frac{1}{2}$ yd., or $16\frac{1}{2}$ ft.

6 rods = 33 yd., or 99 ft.

7 rods = $38\frac{1}{2}$ yd., or $115\frac{1}{2}$ ft.

Can We Multiply by a Fraction?

Meaning: fraction × whole number [O]

1. In box A, the red multiplier is one half as large as the black multiplier. The red product is also $\frac{1}{2}$ as large as the black product.

2. Finish: In box B, the red multiplier is one half as large as the black multiplier; so the red product is also one half as large as the black product.

3. If making a multiplier one half as large also makes the product one half as large, what product would you get for the red question mark in box C? 2

4. You are probably surprised to find that a multiplier can be a fraction. Find the product of $\frac{1}{3} \times 6$ by explaining the work in boxes D and E. In each box the missing product is $\frac{1}{3}$ the preceding product.

For box E you found that $\frac{1}{3} \times 6 = 2$. Would you get the same answer for $\frac{1}{3}$ of 6? Yes Instead of saying "$\frac{1}{3}$ times 6," we usually think and say, "$\frac{1}{3}$ of 6."

A
$4 \times 4 = 16$
$2 \times 4 = 8$

B
$2 \times 4 = 8$
$1 \times 4 = 4$

C
$1 \times 4 = 4$
$\frac{1}{2} \times 4 = ?$

D
$3 \times 6 = 18$
$1 \times 6 = 6$

E
$1 \times 6 = 6$
$\frac{1}{3} \times 6 = 2$

Read the following, saying "of" each time you see ×.

	a	b	c	d	e
5.	$\frac{1}{8} \times 16$ 2	$\frac{1}{5} \times 20$ 4	$\frac{1}{2} \times 24$ 12	$\frac{1}{7} \times 28$ 4	$\frac{1}{6} \times 42$ 7
6.	$\frac{1}{4} \times 8$ 2	$\frac{1}{3} \times 12$ 4	$\frac{1}{9} \times 27$ 3	$\frac{1}{6} \times 18$ 3	$\frac{1}{8} \times 32$ 4
7.	$\frac{1}{2} \times 12$ 6	$\frac{1}{8} \times 24$ 3	$\frac{1}{4} \times 28$ 7	$\frac{1}{5} \times 15$ 3	$\frac{1}{7} \times 21$ 3
8.	$\frac{1}{3} \times 18$ 6	$\frac{1}{6} \times 30$ 5	$\frac{1}{9} \times 36$ 4	$\frac{1}{4} \times 20$ 5	$\frac{1}{8} \times 40$ 5
9.	$\frac{1}{5} \times 25$ 5	$\frac{1}{4} \times 16$ 4	$\frac{1}{6} \times 48$ 8	$\frac{1}{3} \times 15$ 5	$\frac{1}{9} \times 45$ 5

Thinking "of" for ×, say the products for rows 5–9. For Ex. 5a, say, "$\frac{1}{8}$ times 16 = $\frac{1}{8}$ of 16, or 2."

Teaching Pages 138 and 139

Pupil's Objectives. (*a*) To learn the meaning of multiplying a whole number by a fraction; (*b*) to practice reading this kind of multiplication example; (*c*) to practice multiplying whole numbers by unit and multiple fractions.

Background. Multiplying fractions and mixed numbers by whole numbers should not have caused any difficulty, since the idea of combining equal groups to find a total by multiplying is not new. However, the idea of multiplying by a fraction is a more difficult concept and so boxes A–E on page 138 are used to show why a fraction may be used as a multiplier. Then the idea is extended

(under rows 5–9 on page 138) so that pupils may think the word "of" for the times sign, thereby realizing that multiplying by $\frac{1}{2}$ is the same as dividing by 2 (fractional-part division).

The pupil is then ready for the next step—multiplication by a multiple fraction with a choice of two ways in which to work (boxes F and G on page 139 in the text). He has been prepared for finding a unit fraction times a number by finding one of the equal parts of a number. He should see, therefore, that two, three, or more parts will be two, three, or more times one of the equal parts. The quicker way to work, shown in box G, is the one that should be adopted.

Book Lesson. *Ex. 1–14:* Oral work. *Rows 15–19:* Written work.

1. In discussing boxes A, B, and C, use diagrams on the board, as below, to help pupils understand how a fraction can be a multiplier and how the product can be smaller than the multiplicand.

$4 \times 4 = 16$ $2 \times 4 = 8$ $1 \times 4 = 4$ $\frac{1}{2} \times 4 = 2$

X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X

2. When pupils have completed the work on page 138, dictate the examples below and ask them to write each as a multiplication example.

a. $\frac{1}{4}$ of 16 *b.* $\frac{1}{8}$ of 32 *c.* $\frac{1}{5}$ of 30 *d.* $\frac{1}{2}$ of 14

3. The multiplication shown in box F should proceed smoothly as it is an extension of previous learnings. Do not dwell too long on this method, for the one in box G will be used more often.

Differentiations and Extensions

1. After all of the work on page 138 in the text has been completed, have the examples in row 5, with the products, written on the board along with the examples and products for row 5 on page 135. Let pupils compare the products for these two types of examples to see if they can derive the generalizations given below (and also at the bottom of page 154 in the text).

a. If the multiplier is less than 1, the product will be less than the multiplicand.

b. If the multiplier is 1, the product will be equal to the multiplicand.

c. If the multiplier is more than 1, the product will be more than the multiplicand.

10. $\frac{2}{3} \times 12 = ?$ *Think,* "$\frac{2}{3}$ of 12 = ?"

Without any trouble, you can tell the product for $\frac{1}{3}$ of 12.

$\frac{2}{3}$ is 2 times as much as $\frac{1}{3}$, so $\frac{2}{3}$ of 12 is 2 times as much as $\frac{1}{3}$ of 12. Explain the work in box F.

F

$\frac{2}{3} \times 12 = ?$

$\frac{1}{3}$ of 12 = 4, so

$\frac{2}{3}$ of 12 = 2×4, or 8

Box G shows a quicker way to find $\frac{2}{3} \times 12$. In box F, we found $\frac{1}{3}$ of 12 and then made that answer twice as large. In box G, we double the multiplicand and then take $\frac{1}{3}$ of this product. See if you can explain why both boxes show the same final product. $\frac{1}{3}$ of 2 times a number is the same as 2 times $\frac{1}{3}$ of the number.

G

$\frac{2}{3} \times 12 = ?$

$\frac{2 \times 12}{3} = \frac{24}{3}$, or 8

11–14. Explain the work in boxes H–K.

H

$\frac{3}{4} \times 8 = \frac{3 \times 8}{4}$, or $\frac{24}{4}$, or 6

I

$\frac{2}{5} \times 10 = \frac{2 \times 10}{5}$, or $\frac{20}{5}$, or 4

J

$\frac{3}{8} \times 10 = \frac{3 \times 10}{8}$, or $\frac{30}{8}$, or $3\frac{6}{8}$, or $3\frac{3}{4}$

K

$\frac{5}{6} \times 8 = \frac{5 \times 8}{6}$, or $\frac{40}{6}$, or $6\frac{4}{6}$, or $6\frac{2}{3}$

[W]

As in boxes H–K, find **n**, the missing product.

	a	b	c	d
15.	$\frac{3}{4} \times 12 = n$ (9)	$\frac{5}{6} \times 54 = n$ (45)	$\frac{6}{7} \times 35 = n$ (30)	$\frac{7}{8} \times 560 = n$ (490)
16.	$\frac{3}{5} \times 15 = n$ (9)	$\frac{3}{4} \times 36 = n$ (27)	$\frac{8}{3} \times 24 = n$ (16)	$\frac{3}{4} \times 248 = n$ (186)
17.	$\frac{2}{3} \times 19 = n$ ($12\frac{2}{3}$)	$\frac{5}{8} \times 52 = n$ ($32\frac{1}{2}$)	$\frac{5}{9} \times 75 = n$ ($41\frac{2}{3}$)	$\frac{2}{3} \times 129 = n$ (86)
18.	$\frac{3}{8} \times 27 = n$ ($10\frac{1}{8}$)	$\frac{3}{5} \times 21 = n$ ($12\frac{3}{5}$)	$\frac{3}{8} \times 59 = n$ ($22\frac{1}{8}$)	$\frac{5}{6} \times 365 = n$ (304)
19.	$\frac{7}{8} \times 18 = n$ ($15\frac{3}{4}$)	$\frac{2}{3} \times 27 = n$ (18)	$\frac{3}{4} \times 43 = n$ ($32\frac{1}{4}$)	$\frac{3}{8} \times 496 = n$ (186)

Extra Practice. For more practice, work Sets 57 and 58.

Then have them tell, before the written work at the bottom of page 139, whether the products in rows 15–19 will be greater or less than the multiplicands.

2. Let pupils estimate products for rows 15–19 at the bottom of page 139. With this as a basis they should attempt to devise ways for finding exact products without paper and pencil.

Some pupils will be able to look at Ex. 15**a** ($\frac{3}{4} \times 12 = n$) and think "$\frac{1}{4}$ of 12 is 3, so $\frac{3}{4}$ will be 3×3, or 9."

For the same example others may think, "In rounding $\frac{3}{4}$ to 1, we increased the multiplier by $\frac{1}{4}$. We must then subtract $\frac{1}{4}$ of 12 from our estimate. $\frac{1}{4}$ of 12 = 3. $12 - 3 = 9$, the exact answer."

Still others may think to themselves, "We can change the factors and multiply $12 \times \frac{3}{4}$. $12 \times 1 = 12$. $12 \times \frac{1}{4} = 3$. $12 - 3 = 9$, the exact answer."

3. Have *all pupils* answer the following questions using the results of the written work for row 15:

15**a**. How many inches long is a string that is $\frac{3}{4}$ of a foot long?

15**b**. How wide is a piece of cloth that is $\frac{5}{6}$ of 54 inches wide?

15**c**. How many days are there in $\frac{6}{7}$ of 35 days?

15**d**. If $\frac{7}{8}$ of \$560 is spent, how much is left?

While some pupils may treat the last problem above as a two-step problem, *more capable pupils* may see immediately that they can find the answer by taking $\frac{1}{8}$ of \$560.

4. Assign **Extra Practice Sets 57 and 58** as needed. These sets, reproduced below with answers, provide further practice in multiplying with fractions.

Set 57. Multiplying a fraction times a whole number

	a	b	c	d	e
1.	$\frac{1}{2} \times 19$ $9\frac{1}{2}$	$\frac{2}{3} \times 8$ $5\frac{1}{3}$	$\frac{8}{9} \times 5$ $4\frac{4}{9}$	$\frac{4}{5} \times 30$ 24	$\frac{5}{8} \times 18$ $11\frac{1}{4}$
2.	$\frac{3}{5} \times 20$ 12	$\frac{1}{8} \times 12$ $1\frac{1}{2}$	$\frac{5}{6} \times 32$ $26\frac{2}{3}$	$\frac{1}{4} \times 13$ $3\frac{1}{4}$	$\frac{1}{5} \times 16$ $3\frac{1}{5}$
3.	$\frac{1}{7} \times 15$ $2\frac{1}{7}$	$\frac{3}{4} \times 14$ $10\frac{1}{2}$	$\frac{1}{3} \times 25$ $8\frac{1}{3}$	$\frac{5}{7} \times 9$ $6\frac{3}{7}$	$\frac{3}{8} \times 22$ $8\frac{1}{4}$
4.	$\frac{7}{8} \times 6$ $5\frac{1}{4}$	$\frac{2}{5} \times 10$ 4	$\frac{3}{7} \times 35$ 15	$\frac{1}{6} \times 52$ $8\frac{2}{3}$	$\frac{5}{9} \times 4$ $2\frac{2}{9}$

Set 58. Multiplying a fraction times a whole number

1.	$\frac{2}{3} \times 18$ 12	$\frac{3}{8} \times 30$ $11\frac{1}{4}$	$\frac{2}{5} \times 16$ $6\frac{2}{5}$	$\frac{1}{3} \times 231$ 77	$\frac{8}{9} \times 45$ 40
2.	$\frac{1}{4} \times 136$ 34	$\frac{4}{7} \times 35$ 20	$\frac{5}{8} \times 24$ 15	$\frac{4}{5} \times 12$ $9\frac{3}{5}$	$\frac{5}{6} \times 128$ $106\frac{2}{3}$
3.	$\frac{7}{8} \times 14$ $12\frac{1}{4}$	$\frac{3}{4} \times 120$ 90	$\frac{3}{5} \times 19$ $11\frac{2}{5}$	$\frac{5}{7} \times 21$ 15	$\frac{1}{2} \times 315$ $157\frac{1}{2}$

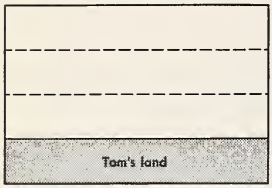
NOTES

Fraction × Fraction

Meaning [O]

1. Tom's father gave him $\frac{1}{4}$ acre of land for a garden of his own. Tom decided to raise potatoes in $\frac{1}{2}$ of his garden. What part of the whole acre did Tom plant with potatoes?

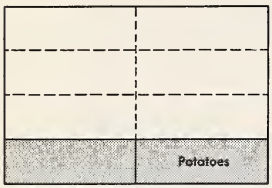
A



The whole acre is shown in picture A. Into how many equal parts is the acre divided? 4 What is the fractional unit? $\frac{1}{4}$

Tom's piece of land is what part of the acre? $\frac{1}{4}$

B



Tom planted potatoes in $\frac{1}{2}$ of $\frac{1}{4}$ acre. Then the part in which he planted potatoes is what fractional part of the whole acre? $\frac{1}{8}$ (Use picture B to help you find the answer.)

From picture B, we see that $\frac{1}{2}$ of $\frac{1}{4}$ (or $\frac{1}{2} \times \frac{1}{4}$) = $\frac{1}{8}$.

We can write, "$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$." The work is done in box C. With figures, how did we find the size of the new fractional unit, $\frac{1}{8}$? by multiplying denominators

C

$$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4}, \text{ or } \frac{1}{8}$$

[W]

2. Draw pictures to find

a. $\frac{1}{3} \times \frac{1}{5}$. $\frac{1}{15}$ b. $\frac{1}{2} \times \frac{1}{8}$. $\frac{1}{16}$ c. $\frac{1}{4} \times \frac{1}{6}$. $\frac{1}{24}$ d. $\frac{1}{5} \times \frac{1}{2}$. $\frac{1}{10}$

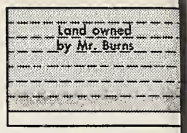
3. Now write with figures the work for finding the products for Ex. 2.

In multiplying a fraction by a fraction, we find the size of the new fractional unit by multiplying the denominators of the given fractions.

[O]

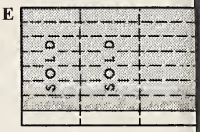
4. Mr. Burns owned $\frac{7}{8}$ acre of land. He sold $\frac{2}{3}$ of it for a house lot. What part of an acre did he sell? $\frac{2}{3} \times \frac{7}{8} = ?$

D



Picture D shows the whole acre and, in blue, the part Mr. Burns owned. What is the fractional unit? $\frac{1}{8}$

E



Now think of each $\frac{1}{8}$ acre as divided into 3 equal parts, as in E. How many of these parts in the whole acre? 24 The new fractional unit is $\frac{1}{24}$. Mr. Burns sold two $\frac{1}{24}$'s from each of his $\frac{1}{8}$'s. He sold $7 \times \frac{2}{24}$, or $\frac{14}{24}$, of an acre in all. Then $\frac{2}{3}$ of $\frac{7}{8}$ must equal $\frac{14}{24}$, or $\frac{7}{12}$.

Study box F. With figures, how do we get the new fractional unit? How do we find the total number of equal parts in the new fraction? Explain the work in box F.

multiplying denominators

multiplying numerators

F

$$\frac{2}{3} \times \frac{7}{8} = \frac{2 \times 7}{3 \times 8}, \text{ or } \frac{14}{24}, \text{ or } \frac{7}{12}$$

[W]

Find the products, as in box F. Give answers in best form.

	a	b	c	d	e	f
5.	$\frac{2}{3} \times \frac{3}{4} \frac{1}{2}$	$\frac{5}{6} \times \frac{1}{2} \frac{5}{12}$	$\frac{2}{3} \times \frac{3}{8} \frac{1}{4}$	$\frac{1}{8} \times \frac{1}{6} \frac{1}{48}$	$\frac{1}{2} \times \frac{2}{3} \frac{1}{3}$	$\frac{3}{5} \times \frac{1}{8} \frac{3}{40}$
6.	$\frac{1}{6} \times \frac{2}{9} \frac{1}{9}$	$\frac{3}{4} \times \frac{7}{8} \frac{21}{32}$	$\frac{2}{3} \times \frac{3}{5} \frac{2}{5}$	$\frac{3}{8} \times \frac{1}{4} \frac{3}{32}$	$\frac{4}{5} \times \frac{1}{2} \frac{2}{5}$	$\frac{1}{2} \times \frac{5}{8} \frac{5}{16}$
7.	$\frac{3}{4} \times \frac{2}{5} \frac{3}{10}$	$\frac{5}{8} \times \frac{3}{10} \frac{3}{16}$	$\frac{1}{6} \times \frac{9}{10} \frac{3}{20}$	$\frac{3}{4} \times \frac{3}{4} \frac{9}{16}$	$\frac{3}{4} \times \frac{2}{3} \frac{1}{2}$	$\frac{3}{8} \times \frac{4}{5} \frac{3}{10}$
8.	$\frac{1}{2} \times \frac{1}{2} \frac{1}{4}$	$\frac{5}{6} \times \frac{2}{3} \frac{5}{9}$	$\frac{4}{5} \times \frac{5}{12} \frac{1}{3}$	$\frac{2}{3} \times \frac{15}{16} \frac{5}{8}$	$\frac{5}{7} \times \frac{7}{20} \frac{1}{4}$	$\frac{5}{18} \times \frac{9}{10} \frac{1}{4}$
9.	$\frac{3}{4} \times \frac{8}{9} \frac{2}{3}$	$\frac{5}{8} \times \frac{5}{6} \frac{25}{48}$	$\frac{1}{2} \times \frac{7}{8} \frac{7}{16}$	$\frac{8}{15} \times \frac{5}{12} \frac{2}{9}$	$\frac{7}{12} \times \frac{4}{5} \frac{7}{15}$	$\frac{7}{10} \times \frac{5}{8} \frac{7}{16}$

Prove your work for row 5 by making drawings like E.

Extra Practice. For more practice, work Sets 59 and 60.

To find the product of two fractions, we multiply the denominators to get our new fractional unit. We multiply the numerators to find the number of equal parts in the new fraction.

Teaching Pages 140 and 141

Pupil's Objectives. (*a*) To discover the meaning of multiplying a fraction by a fraction; (*b*) to practice multiplying a fraction by a fraction.

Background. Once children have learned how to multiply a whole number by a fraction they should have no difficulty in multiplying any number by a fraction. The basic meaning of multiplying a fraction by a unit fraction and by a multiple fraction may be shown with objects or with pictures, as illustrated in A, B, D, and E on pages 140 and 141. The computation in boxes C and F is similar to the work done on pages 138 and 139 in the text.

Pre-book Lesson

1. Have pupils use fractional pieces, calculating strips, and diagrams so that pupils will be convinced that multiplying a fraction by a fraction yields a smaller fractional unit.

$\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{2} \times \frac{1}{2}$

2. Let pupils also discover that decreasing the multiplier decreases the product through working a series of examples such as the one below.

$4 \times \frac{1}{2} = 2$ $2 \times \frac{1}{2} = 1$ $1 \times \frac{1}{2} = \frac{1}{2}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

3. Comparing the answer obtained by using pictures or objects with the answer obtained by using figures only should help pupils realize that $\frac{1}{2}$ of $\frac{1}{2}$ is equal to $\frac{1}{2} \times \frac{1}{2}$, or $\frac{1}{4}$.

Book Lesson. *Ex. 1:* Oral work. *Ex. 2 and 3:* Written work. *Ex. 4:* Oral work. *Rows 5–9:* Written work.

1. Notice that the work on page 140 leads to the generalization stated at the bottom of the page. The emphasis here is in finding the size of the fractional unit obtained; therefore, attention is directed only toward the denominators.

2. On page 141 multipliers that are multiple fractions are introduced for multiplying a fraction by a fraction.

3. Study of box F should help pupils state in their own words a generalization similar to the one found at the bottom of the page.

Differentiations and Extensions

1. Have *all children* examine their schoolroom to find situations which suggest taking a fractional part of a fractional part. It is possible that windows, bulletin boards, or bookcases will reveal opportunities to apply multiplication of a fraction by either a unit fraction or a multiple fraction. Have pupils compare the results obtained by observation with results obtained by the process of multiplication.

2. It is more difficult to estimate products for examples such as those in rows 5–9 on page 141. However, pupils should at least realize that the product will be less than the multiplicand, since in every case the multiplier is less than 1. For examples like **5e**, they may extend this idea a little further, thinking to themselves that the product is going to be just half as big as the multiplicand. In this case some pupils may realize that $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$ and others that $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$, or $\frac{1}{3}$. For examples like **6f**, some pupils might even realize that $\frac{1}{2}$ of $\frac{5}{8} = \frac{2\frac{1}{2}}{8}$ ($2\frac{1}{2}$ eighths). Although this is an unusual answer, it is very often quite convenient, as in taking $\frac{1}{2}$ of $\frac{5}{8}$ in. when using a ruler that does not show sixteenths of an inch.

Keeping in mind the meaning of multiplying by a fraction, let pupils examine some of the other examples in rows 5–9 to see if they can think of ways of arriving at exact answers when the multiplier is a multiple fraction. Some pupils may look at example **5a** and think, "$\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$; therefore, $\frac{2}{3}$ is $\frac{2}{4}$, or $\frac{1}{2}$." Others might look at Ex. **6a** and think "I can reverse the factors, making the example $\frac{2}{3} \times \frac{1}{6}$. $\frac{1}{3}$ of $\frac{1}{6}$ has got to be $\frac{1}{18}$, so $\frac{2}{3}$ of $\frac{1}{6}$ has to be $\frac{2}{18}$, or $\frac{1}{9}$."

3. After the written work on page 141, you may want to assign parts of Extra Practice Sets 54 to 60 in order to make sure that children can work all the different kinds of multiplication examples involving fractions so far introduced. Even better, you might scramble the examples taken from these various sets and write them on the board or mimeograph them so that pupils may obtain mixed practice in the various kinds of multiplication.

4. **Extra Practice Sets 59 and 60** may be assigned to pupils who need more work in multiplying a fraction times a fraction.

Set 59. Multiplying a fraction times a fraction

	a	b	c	d	e	f
1.	$\frac{1}{3} \times \frac{3}{4}$ $\frac{1}{4}$	$\frac{5}{6} \times \frac{9}{15}$ $\frac{1}{2}$	$\frac{5}{16} \times \frac{1}{5}$ $\frac{1}{16}$	$\frac{7}{9} \times \frac{4}{7}$ $\frac{4}{9}$	$\frac{4}{5} \times \frac{7}{8}$ $\frac{7}{10}$	$\frac{1}{8} \times \frac{1}{2}$ $\frac{1}{16}$
2.	$\frac{5}{9} \times \frac{3}{5}$ $\frac{1}{3}$	$\frac{1}{2} \times \frac{2}{5}$ $\frac{1}{5}$	$\frac{1}{7} \times \frac{5}{8}$ $\frac{5}{56}$	$\frac{7}{8} \times \frac{2}{5}$ $\frac{7}{20}$	$\frac{2}{3} \times \frac{7}{10}$ $\frac{7}{15}$	$\frac{5}{8} \times \frac{2}{15}$ $\frac{1}{12}$
3.	$\frac{3}{16} \times \frac{2}{3}$ $\frac{1}{8}$	$\frac{5}{12} \times \frac{6}{7}$ $\frac{5}{14}$	$\frac{3}{4} \times \frac{2}{9}$ $\frac{1}{6}$	$\frac{4}{15} \times \frac{3}{8}$ $\frac{1}{10}$	$\frac{3}{5} \times \frac{5}{18}$ $\frac{1}{6}$	$\frac{3}{10} \times \frac{1}{9}$ $\frac{1}{30}$

Set 60. Multiplying a fraction times a fraction

1.	$\frac{2}{3} \times \frac{1}{2}$ $\frac{1}{3}$	$\frac{5}{12} \times \frac{3}{8}$ $\frac{5}{32}$	$\frac{3}{16} \times \frac{2}{15}$ $\frac{1}{40}$	$\frac{4}{7} \times \frac{7}{9}$ $\frac{4}{9}$	$\frac{11}{16} \times \frac{2}{3}$ $\frac{11}{24}$	$\frac{7}{8} \times \frac{5}{14}$ $\frac{5}{16}$
2.	$\frac{7}{16} \times \frac{2}{7}$ $\frac{1}{8}$	$\frac{2}{5} \times \frac{3}{10}$ $\frac{3}{25}$	$\frac{7}{15} \times \frac{3}{4}$ $\frac{7}{20}$	$\frac{9}{16} \times \frac{5}{9}$ $\frac{5}{16}$	$\frac{1}{10} \times \frac{5}{16}$ $\frac{1}{32}$	$\frac{2}{15} \times \frac{3}{10}$ $\frac{1}{25}$
3.	$\frac{7}{10} \times \frac{5}{6}$ $\frac{7}{12}$	$\frac{8}{15} \times \frac{5}{16}$ $\frac{1}{6}$	$\frac{3}{5} \times \frac{3}{10}$ $\frac{9}{50}$	$\frac{5}{6} \times \frac{7}{8}$ $\frac{35}{48}$	$\frac{3}{16} \times \frac{5}{12}$ $\frac{5}{64}$	$\frac{9}{10} \times \frac{3}{4}$ $\frac{27}{40}$

Fraction × Mixed Number

[O]

1. Last year Mr. Knight used $3\frac{3}{4}$ acres of land for potatoes. This year he plans to use only $\frac{2}{3}$ as much, or how many acres?

We can write $3\frac{3}{4}$ as the improper fraction, $\frac{15}{4}$. So this year Mr. Knight plans to use $\frac{2}{3}$ of $\frac{15}{4}$ acres for potatoes. Explain the work in the box. In best form, the answer is $2\frac{1}{2}$.

$\frac{2}{3} \times 3\frac{3}{4} = \frac{2}{3} \times \frac{15}{4}$, or $\frac{2 \times 15}{3 \times 4}$, or $\frac{30}{12}$, or $2\frac{1}{2}$

Side work
$3\frac{3}{4} = \frac{15}{4}$

[W]

Find products for the following:

2. $\frac{2}{3} \times 2\frac{3}{4}$ $1\frac{5}{6}$
3. $\frac{1}{2} \times 1\frac{1}{2}$ $\frac{3}{4}$
4. $\frac{4}{5} \times 2\frac{2}{3}$ $2\frac{2}{15}$
5. $\frac{3}{8} \times 1\frac{1}{3}$ $\frac{1}{2}$
6. $\frac{5}{8} \times 5\frac{1}{3}$ $3\frac{1}{3}$
7. $\frac{7}{10} \times 1\frac{1}{3}$ $\frac{14}{15}$
8. $\frac{3}{4} \times 2\frac{4}{9}$ $1\frac{5}{6}$
9. $\frac{1}{2} \times 4\frac{4}{5}$ $2\frac{2}{5}$
10. $\frac{2}{3} \times 5\frac{3}{5}$ $3\frac{11}{15}$
11. $\frac{5}{6} \times 4\frac{4}{5}$ 4
12. $\frac{1}{3} \times 2\frac{5}{8}$ $\frac{7}{8}$
13. $\frac{1}{4} \times 2\frac{2}{5}$ $\frac{3}{5}$
14. $\frac{1}{2} \times 3\frac{5}{6}$ $1\frac{11}{12}$
15. $\frac{1}{3} \times 10\frac{2}{3}$ $3\frac{5}{9}$
16. $\frac{4}{5} \times 3\frac{3}{4}$ 3
17. $\frac{1}{7} \times 5\frac{3}{5}$ $\frac{4}{5}$

Extra Practice. For more practice, work Sets 61 and 62.

You multiply a mixed number by a fraction just as you multiply a fraction by a fraction, first changing the mixed number to an improper fraction.

Multiplying with Fractions

[O]

Tell what you must do to solve each problem, but do not solve it yet.

1. Butter is 60¢ a pound. How much will Mrs. Brown pay for $\frac{3}{4}$ lb.? $\frac{3}{4} \times 60¢ = 45¢$, or $0.45

HELPER. When Mrs. Brown buys 3 lb., the cost is $3 \times $0.60. When she buys $\frac{3}{4}$ lb., the cost is $\frac{3}{4} \times $0.60.

2. Sam had 35 papers to sell. He sold $\frac{2}{5}$ of them before lunch. How many papers did he sell before lunch? $\frac{2}{5} \times 35 = 14$

3. After Christmas a card store marked its 15¢ cards down to $12\frac{1}{2}$¢. How much did Mary pay for 7 of these cards? $7 \times 12\frac{1}{2}¢ = 87\frac{1}{2}¢$, or $0.88

HELPER. In figuring a selling price, we round any fraction of a cent in the answer to the next larger whole cent.

4. Miss Pike said that $\frac{7}{8}$ of the class had perfect attendance for the first two months. The total number enrolled in the class was 40. How many pupils had perfect attendance during these two months? $\frac{7}{8} \times 40 = 35$

5. Between 8 o'clock and 2 o'clock Jack had ridden on his bicycle 3 times the distance from his house to Sam's. The boys once measured the distance and found that it was $2\frac{1}{2}$ mi. How far did Jack ride between 8 o'clock and 2 o'clock? $3 \times 2\frac{1}{2} = 7\frac{1}{2}$ mi.

6. Betty and Sue earned money by shelling $2\frac{1}{4}$ lb. of nuts each day for 5 days. How many pounds of nuts did they shell in that time? $5 \times 2\frac{1}{4} = 11\frac{1}{4}$ lb.

7. Barbara worked on her music lesson $\frac{3}{4}$ hr. each day. In 6 days, how much time did she put on her music? $6 \times \frac{3}{4} = 4\frac{1}{2}$ hr.

8. Bob earned 60¢ an hour working after school. In $\frac{2}{3}$ hr. after school, how much did he earn? $\frac{2}{3} \times 60¢ = 40¢$

[W]

Now write out your work to find the answers.

Teaching Pages 142 and 143

Pupil's Objectives. (*a*) To learn how to multiply a mixed number by a fraction and to practice the same; (*b*) to solve problems that involve multiplication with fractions.

Background. The work on page 142 is a simple extension of that on the previous two pages in the text, since all that is involved is the changing of mixed numbers to improper fractions before multiplying by a fraction.

The work on page 143 is, in a way, a test of the work your pupils have been doing. Can they put it into use? This page also serves as one facet in the problem-solving program which spreads throughout the year's work. As an oral lesson, have children tell what must be done to solve each problem.

Pre-book Lesson (page 142). Have pupils consider a situation in which the necessity of finding a fractional part of a mixed number arises. For example, two classes may need to share $3\frac{1}{2}$ jars of

paint equally. Accept any answers which pupils may suggest that are satisfactory methods for finding how much paint each class should receive. Later, results obtained by these methods may be compared with results obtained by computation. Because pupils have already learned to change mixed numbers to improper fractions and to multiply a fraction by a fraction, lead them toward the form of writing the example shown below by encouraging them to state a generalization similar to the one at the bottom of the page in the text.

$\frac{1}{2} \times 3\frac{1}{2}$ $\frac{1}{2} \times \frac{7}{2} = \frac{7}{4} = 1\frac{3}{4}$ (jars)

Book Lesson (page 142). *Ex. 1:* Oral work. *Ex. 2–17:* Written work.

Book Lesson (page 143). *Ex. 1–8:* Oral work. *Bottom of the page:* Written work. Have pupils study each problem to determine the situation described, the answer to be sought, and the method of writing the example. The examples, but not the solutions, may be written on the board so that when they have all been recorded, pupils may use them to summarize the types of examples that will be encountered (e.g., a fraction times a whole number, a whole number times a mixed number). Examples of each type may be found on preceding pages, or some may be written on the board in completed form. Erase the examples that correspond to the problems on the page before the children begin the written work.

Differentiations and Extensions

1. Ex. 2–17 on page 142 offer excellent opportunities for pupils to obtain practice in estimating and in discovering ways of arriving at exact answers without paper and pencil. By now pupils ought to be able to tell that each product will be less than its multiplicand. In estimating for Ex. 2, pupils should be able to round $2\frac{3}{4}$ to 3, realizing that $\frac{1}{3}$ of 3 is 1 and that, therefore, $\frac{2}{3}$ of 3 would be 2. For Ex. 3, some pupils may round $1\frac{1}{2}$ to 2, obtaining an estimate of 1. Other pupils may realize that $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ and that, therefore, $\frac{1}{2}$ of $1\frac{1}{2} = \frac{3}{4}$.

2. Notice in Ex. 3, on page 143, that the helper gives the conventional system for rounding when fractional parts of a cent are involved in buying and selling situations. Pupils should realize that this does not correspond to the usual system for rounding whole numbers for estimating, since *any* fractional part of a cent is rounded upward.

3. Assign **Extra Practice Sets 61 and 62** as needed.

Set 61. Multiplying a mixed number by a fraction

	a	b	c	d	e	f
1.	$\frac{1}{2} \times 2\frac{1}{3}$ $1\frac{1}{6}$	$\frac{3}{8} \times 4\frac{8}{9}$ $1\frac{5}{6}$	$\frac{2}{5} \times 7\frac{5}{9}$ $3\frac{1}{45}$	$\frac{3}{4} \times 1\frac{3}{4}$ $1\frac{5}{16}$	$\frac{5}{7} \times 4\frac{7}{8}$ $3\frac{27}{56}$	$\frac{2}{3} \times 6\frac{1}{2}$ $4\frac{1}{3}$
2.	$\frac{3}{4} \times 1\frac{1}{5}$ $\frac{9}{10}$	$\frac{6}{7} \times 5\frac{1}{6}$ $4\frac{3}{7}$	$\frac{1}{4} \times 8\frac{3}{4}$ $2\frac{3}{16}$	$\frac{5}{8} \times 9\frac{1}{4}$ $5\frac{25}{32}$	$\frac{3}{5} \times 5\frac{1}{2}$ $3\frac{3}{10}$	$\frac{1}{9} \times 1\frac{4}{5}$ $\frac{1}{5}$
3.	$\frac{2}{3} \times 3\frac{2}{3}$ $2\frac{4}{9}$	$\frac{3}{5} \times 6\frac{5}{8}$ $3\frac{39}{40}$	$\frac{7}{8} \times 3\frac{1}{7}$ $2\frac{3}{4}$	$\frac{1}{5} \times 8\frac{5}{6}$ $1\frac{23}{30}$	$\frac{4}{7} \times 4\frac{2}{3}$ $2\frac{2}{3}$	$\frac{3}{8} \times 3\frac{1}{5}$ $1\frac{1}{5}$

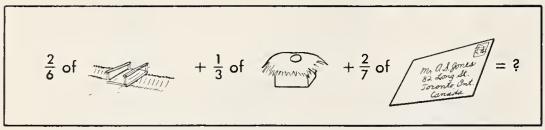
Set 62. Multiplying a mixed number by a fraction

	a	b	c	d	e
1.	$\frac{3}{5} \times 1\frac{2}{3}$ 1	$\frac{1}{2} \times 2\frac{3}{16}$ $1\frac{3}{32}$	$\frac{5}{12} \times 4\frac{1}{2}$ $1\frac{7}{8}$	$\frac{3}{4} \times 9\frac{1}{7}$ $6\frac{6}{7}$	$\frac{6}{7} \times 10\frac{3}{4}$ $9\frac{3}{14}$
2.	$\frac{1}{4} \times 12\frac{1}{4}$ $3\frac{1}{16}$	$\frac{5}{6} \times 4\frac{4}{5}$ 4	$\frac{7}{8} \times 5\frac{1}{7}$ $4\frac{1}{2}$	$\frac{2}{9} \times 12\frac{3}{8}$ $2\frac{3}{4}$	$\frac{1}{5} \times 11\frac{1}{2}$ $2\frac{3}{10}$

NOTES

Mystery Chef

A game with fractions [W]



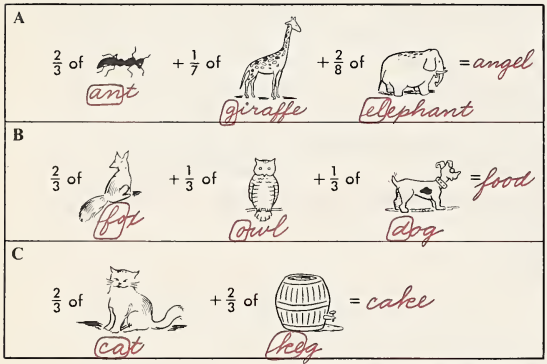
A cook uses some unusual materials to make a special dessert. To learn what the dessert is, use boxes A, B, and C, below. First look at the box above and study these directions for finding the word that is represented there:

The denominator of each fraction gives the number of letters in the name of the thing pictured. The numerator tells the number of letters used from the beginning of the word.

For example, from the box above,

$\frac{2}{6}$ of bridge + $\frac{1}{3}$ of egg + $\frac{2}{7}$ of address = br e ad.

The name of the special dessert contains three words. The first word can be found in box A; the second, in box B; and the third, in box C.



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Problems in Measurement

[W]

Write your work for these problems:

1. A bushel of peas will fill how many 2-quart bags? 16

2. The weights of 5 boys were $79\frac{1}{2}$ lb., 86 lb., $91\frac{1}{4}$ lb., 83 lb., and $95\frac{1}{4}$ lb. Find their average weight. 87 lb.

3. Change 145 minutes to hours and minutes. 2 hr. 25 min

4. Joe ran $\frac{3}{5}$ of a mile. How many feet was that? 3,168

5. A living room is 26 feet long and 17 feet wide. How many square yards of rug material will be needed to cover the entire floor? $49\frac{1}{9}$

Mixed Practice

[W]

Copy and do as the signs say. Check your answers.

	a	b	c	d	e	f
1.	273	8,963	755	3,806	$95.57	$63.07
	5,786	3,826	4,513	7,712	84.26	5.73
	958	859	1,668	59	0.75	98.95
	+675	+407	+268	+942	+4.65	+2.61
	7,692	14,055	7,204	12,519	$185.23	$170.36
2.	768	4,059	20,061	70,350	$73.42	$40.06
	−397	−2,673	−2,581	−3,876	−50.79	−2.79
	371	1,386	17,480	66,474	$22.63	$37.27
3.	630	9,841	2,308	8,002	$6.37	$8.05
	×27	×76	×706	×410	×28	×27
	17,010	747,916	1,629,448	3,280,820	$178.36	$217.35

Write any remainder in a fraction.

4.	81	509	782	$1,814\frac{4}{5}$	$1.56	$2.03
	8)648	7)3,563	4)3,128	5)9,074	6)$9.36	9)$18.27
5.	$600\frac{1}{2}$	$800\frac{5}{7}$	$612\frac{6}{7}$	560	$5.20	$1.89
	6)3,603	9)7,205	7)4,289	8)4,480	4)$20.80	26)$49.14

Be sure that you can say the M. and D. facts on pages 293 and 294.

Teaching Pages 144 and 145

Pupil's Objectives. (*a*) To practice multiplying a fraction times a whole number by means of a game; (*b*) to solve problems involving measures; (*c*) to obtain mixed practice in work with whole numbers.

Background. The game on page 144 should lighten for a moment the work your pupils have been doing in multiplying fractions. Some children will be able to work out this type of puzzle with ease, but those who cannot should not be required to do so.

The work at the top of page 145 is designed to help pupils maintain previously-learned skills and abilities in work with measures, while the work at the bottom of the page accomplishes the same purpose by providing a review of whole-number computations.

Book Lesson (page 144). Written work. Allow *all children* to enjoy the work on this page. Withdraw the *slower learners* who cannot proceed alone, and help them determine the number of letters in each word and to do the multiplication. *More capable children* may be allowed to do much of the multiplication mentally.

Book Lesson (page 145). *Ex. 1–5:* Written work. *Rows 1–5:* Written work.

1. Examine the page with your pupils. For the problems with measures at the top of the page, have pupils state the kind of information that is necessary for each problem. Help them to find sources in which they can find whatever information they have forgotten.

2. Select one or two examples in each row of computation for pupils to use in estimating answers. You may also wish to review briefly the method of checking each process.

Differentiations and Extensions. Show *more capable pupils* how they may adapt the game on page 144 in the text for the purpose of writing original problems. Do a sample puzzle with them, possibly pointing out how classifications other than food could be used. Have the completed puzzles presented to the class for discussion.

Teaching Pages 146 and 147

Pupil's Objectives. (*a*) To see summarized the types of multiplication with fractions that have been studied; (*b*) to learn two ways of multiplying when the multiplier is a mixed number and the multiplicand is a whole number.

Background. The work on page 146 makes a good introduction to the next unit of work in multiplication, for it summarizes the work so far introduced, including those cases in which the multiplier is a whole number or a fraction. It also indicates that there are three more cases which involve mixed-number multipliers.

On page 147 pupils see that, in the same way as when the multiplier was a whole number and the multiplicand a mixed number, there are two choices of method for working when the multiplier is a mixed number. The first method involves multiplying by parts and using partial products. Now that pupils can multiply by a fraction, this method is available to them and is often quite useful. The second method involves changing the mixed number to an improper fraction and multiplying horizontally.

Book Lesson (page 146). *Ex. 1–6:* Oral work.

1. For any type of multiplication example which causes much difficulty, refer pupils to the page on which that type was presented.

Progress in Multiplying with Fractions

Summary of the 8 kinds of example [O]

In multiplying with fractions, you have been using two kinds of multipliers. In one kind of example, the multiplier was a whole number; in the other, it was a fraction.

1. $5 \times \frac{3}{10} = ?$ is an example in which the multiplier is a whole number and the multiplicand is a fraction. Give three more examples of this kind. $5\times\frac{3}{10} = \frac{5\times3}{10} = \frac{15}{10} = 1\frac{5}{10} = 1\frac{1}{2}$

2. Use $5 \times \frac{3}{10} = ?$ to tell how you work such an example.

3. Give an example in which the multiplier is a whole number and the multiplicand is a mixed number.

4. $6 \times 2\frac{1}{4} = 13\frac{1}{2}$ Tell how to work this example by $6\times\frac{1}{4}=1\frac{1}{2}$; $6\times2=12$; $12+1\frac{1}{2}=13\frac{1}{2}$
a. changing $2\frac{1}{4}$ to an improper fraction; b. multiplying parts. $6\times2\frac{1}{4}=6\times\frac{9}{4}=\frac{54}{4}=13\frac{1}{2}$

5. Now write on the board and tell how to work an example with a fraction multiplier and a multiplicand which is
a. a whole number; b. a fraction; c. a mixed number.

There is another kind of example in multiplying with fractions. In this kind the multiplier is a mixed number.

6. Study the boxes at the right. They list all the kinds of multiplication examples with fractions. Find the box which names the multiplier as a mixed number. The box at its right shows how many kinds of multiplicands you can have with a mixed-number multiplier.

Multiplier	Multiplicand
I. Whole Number	a. Fraction b. Mixed Number
II. Fraction	a. Whole Number b. Fraction c. Mixed Number
III. Mixed Number	a. Whole Number b. Fraction c. Mixed Number

The next pages will show you how to work examples when the multiplier is a mixed number.

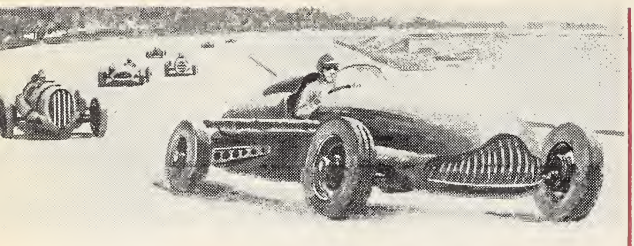
2. Pupils may write a sample example of each kind of multiplication in their notebooks.

Book Lesson (page 147). *Ex. 1:* Oral work. *Rows 2–4:* Written work.

Differentiations and Extensions

1. There will almost certainly arise many opportunities which will allow pupils to use the skill of multiplying a whole number by a mixed number. Suggest some of the possibilities below and then let pupils add ideas of their own to the list.

a. Finding the cost of meat. $3\frac{1}{2}$ pounds at 50¢ a pound.



Multiplying by a Mixed Number

Multiplicand a whole number [O]

1. At the end of an automobile race, car 33 was $3\frac{1}{2}$ laps ahead of car 52. The length of one lap was 5 miles. Car 33 was how many miles ahead of car 52?

It is easy to multiply a whole number by a mixed number if you think of the mixed number in parts. Explain box A.

Also you can find the answer in the fraction way. To what fraction can you change $3\frac{1}{2}$? $\frac{7}{2}$ Explain the work in box B.

A

$$\begin{array}{r} 5 \\ \times 3\frac{1}{2} \\ \hline 2\frac{1}{2} \quad (\frac{1}{2} \times 5) \\ 15 \quad (3 \times 5) \\ \hline 17\frac{1}{2} \end{array}$$

B

$3\frac{1}{2} \times 5 = \frac{7}{2} \times 5$, or $\frac{7 \times 5}{2}$, or $\frac{35}{2}$, or $17\frac{1}{2}$

Side work $3\frac{1}{2} = \frac{7}{2}$

[W]

Work these examples both ways, as in boxes A and B:

	a	b	c	d	e
2.	$4\frac{1}{3} \times 7$ 30 3	$5\frac{2}{3} \times 6$ 34	$4\frac{1}{5} \times 16$ 67 5	$6\frac{1}{3} \times 14$ 88 2/3	$7\frac{1}{2} \times 23$ 172 2
3.	$2\frac{1}{2} \times 4$ 10	$7\frac{3}{4} \times 9$ 69 3/4	$3\frac{1}{8} \times 25$ 78 1/8	$5\frac{1}{2} \times 28$ 154	$8\frac{1}{3} \times 15$ 125
4.	$3\frac{1}{4} \times 8$ 26	$2\frac{1}{4} \times 17$ 38 1/4	$2\frac{3}{5} \times 7$ 18 1/5	$4\frac{2}{3} \times 9$ 42	$4\frac{1}{2} \times 45$ 202 1/2

Extra Practice. For more practice, work Sets 63 and 64.

To multiply a whole number by a mixed number, either multiply by the parts of the mixed number; or change the mixed number to an improper fraction and multiply by the fraction.

b. Finding the number of inches of ribbon in a piece $2\frac{1}{2}$ yards long.

c. Finding about how far one can travel in $1\frac{3}{4}$ hours averaging 30 miles per hour.

2. Be sure that *all pupils* have an opportunity to estimate products for the examples in rows 2–4 on page 147 in the text. This is a simple matter involving rounding the mixed numbers to the nearest whole number and multiplying whole numbers. Sometimes, too, the multiplicand should be rounded. In Ex. **2e**, 23 should be rounded to 20 and multiplied by 8 in obtaining an estimate of 160.

3. Be sure, also, that pupils are given an opportunity to discover for rows 2–4 ways of arriving at exact answers without using pencil and paper. For Ex. **2b**, pupils might think, "If we round $5\frac{2}{3}$ to 6, our answer will be too big by $\frac{1}{3}$ of 6. 6×6, 36. $\frac{1}{3}$ of $6 = 2$, $36 - 2 = 34$, the exact answer." For Ex. **2c**, pupils might think, "$4 \times 16 = 64$. $\frac{1}{5}$ of $16 = 3\frac{1}{5}$. $64 + 3\frac{1}{5} = 67\frac{1}{5}$."

4. More work in multiplying whole numbers by mixed numbers is provided in **Extra Practice Sets 63 and 64.**

Set 63. Vertical multiplication of a whole number by a mixed number

	a	b	c	d	e	f	g	h
1.	4	6	10	5	15	9	24	3
	$\times 2\frac{1}{3}$	$\times 3\frac{1}{8}$	$\times 4\frac{2}{5}$	$\times 9\frac{2}{3}$	$\times 2\frac{5}{6}$	$\times 7\frac{3}{8}$	$\times 3\frac{7}{8}$	$\times 8\frac{5}{9}$
	$9\frac{1}{3}$	$18\frac{3}{4}$	44	$48\frac{1}{3}$	$42\frac{1}{2}$	$66\frac{3}{8}$	93	$25\frac{2}{3}$
2.	12	7	14	16	8	19	52	2
	$\times 5\frac{1}{2}$	$\times 1\frac{3}{5}$	$\times 3\frac{1}{7}$	$\times 6\frac{5}{8}$	$\times 8\frac{3}{4}$	$\times 1\frac{7}{9}$	$\times 3\frac{1}{4}$	$\times 9\frac{1}{6}$
	66	$11\frac{1}{5}$	44	106	70	$33\frac{7}{9}$	169	$18\frac{1}{3}$

Set 64. Horizontal multiplication of a whole number by a mixed number

	a	b	c	d	e
1.	$3\frac{1}{2} \times 5$ $17\frac{1}{2}$	$1\frac{7}{8} \times 15$ $28\frac{1}{8}$	$5\frac{2}{5} \times 18$ $97\frac{1}{5}$	$7\frac{6}{7} \times 7$ 55	$9\frac{3}{8} \times 15$ $140\frac{5}{8}$
2.	$2\frac{1}{8} \times 9$ $19\frac{1}{8}$	$3\frac{1}{7} \times 17$ $53\frac{3}{7}$	$8\frac{1}{3} \times 7$ $58\frac{1}{3}$	$4\frac{7}{9} \times 27$ 129	$8\frac{3}{4} \times 3$ $26\frac{1}{4}$

NOTES

Multiplying by a Mixed Number

Multiplicand a fraction [O]

1. John spent $\frac{3}{5}$ hr. on his homework. Alan spent $2\frac{1}{2}$ times many hours on his, or _?_ hours. $2\frac{1}{2} \times \frac{3}{5} = ?$

We can change $2\frac{1}{2}$, the multiplier, to $\frac{5}{2}$ (box A). Now the example is like which kind of example you have had? fraction × fraction

In box A study the short way to multiply $\frac{5}{2} \times \frac{3}{5}$. What step has been left out? Explain all the work in box A.

$2\frac{1}{2} \times \frac{3}{5} = \frac{5}{2} \times \frac{3}{5}$, or $\frac{5 \times 3}{2 \times 5}$, or $\frac{15}{10}$, or $1\frac{5}{10}$, or $1\frac{1}{2}$
Short way: $\frac{5}{2} \times \frac{3}{5} = \frac{15}{10}$, or $1\frac{5}{10}$, or $1\frac{1}{2}$

$4\frac{1}{2} \times 1\frac{1}{2} = \frac{9}{2} \times \frac{3}{2}$, or $\frac{27}{4}$, or $6\frac{3}{4}$	Side work $4\frac{1}{2} = \frac{9}{2}$ $1\frac{1}{2} = \frac{3}{2}$

Multiplicand a mixed number

2. A tractor uses $1\frac{1}{2}$ gal. of gasoline an hour. How many gallons will it use in $4\frac{1}{2}$ hr.? $4\frac{1}{2} \times 1\frac{1}{2} = ?$ change to improper fractions.

What must we do before we can multiply? Explain the work in box B. Then, to check, work the example on the board, reversing the factors. Do you get the same answer as in box B? Yes

[W]

Multiply in rows 3–7. Write your work the short way. In row 7, check by reversing the factors and multiplying again.

a	b	c	d	e
$2\frac{5}{8} \times \frac{1}{3}$ 7/8	$4\frac{2}{3} \times \frac{1}{4}$ 1/6	$4\frac{5}{6} \times \frac{2}{5}$ 1 4/5	$1\frac{2}{3} \times \frac{3}{4}$ 1/4	$4\frac{1}{2} \times \frac{5}{8}$ 2 1/16
$6\frac{2}{3} \times \frac{3}{5}$ 4	$3\frac{1}{3} \times \frac{7}{10}$ 2 1/3	$2\frac{2}{5} \times \frac{11}{4}$ 6 3/5	$2\frac{3}{4} \times \frac{1}{2}$ 1/8	$4\frac{1}{3} \times \frac{2}{3}$ 2 2/9
$6\frac{1}{4} \times \frac{4}{5}$ 5	$3\frac{2}{3} \times \frac{3}{5}$ 2 1/5	$1\frac{4}{5} \times \frac{5}{8}$ 1/8	$3\frac{1}{4} \times 1\frac{3}{5}$ 5/12	$2\frac{1}{2} \times 1\frac{3}{4}$ 4 3/8
$2\frac{1}{10} \times 2\frac{1}{4}$ 4 29/40	$3\frac{1}{3} \times 5\frac{2}{5}$ 18 2/5	$1\frac{2}{3} \times 2\frac{5}{8}$ 4 3/8	$4\frac{1}{8} \times 2\frac{1}{2}$ 10 5/16	$8\frac{1}{5} \times 7\frac{1}{2}$ 66
$1\frac{3}{8} \times 6\frac{2}{9}$ 8 5/9	$2\frac{1}{3} \times 2\frac{1}{3}$ 5 4/9	$3\frac{1}{3} \times 2\frac{1}{4}$ 7 1/2	$1\frac{1}{5} \times 1\frac{3}{8}$ 1 3/20	$5\frac{1}{5} \times 4\frac{3}{8}$ 22 3/4

Extra Practice. For more practice, work Sets 65–67.

Some Multiplication Problems

[W]

Write and work the multiplication example for each problem.

1. On the average, Mr. Beal's sheep yield $6\frac{1}{2}$ lb. of wool each. How much wool will 10 sheep yield? 65 lb.

2. Each of 144 packages in a carton contains $\frac{3}{4}$ lb. of soap flakes. How many pounds of soap flakes does the carton contain? 108 lb.

3. The lead from old storage batteries is melted and made into $2\frac{1}{2}$-pound chunks. One day a salvage company made 1,387 chunks. That was how many pounds of lead? 3,467½ lb.

4. Bill sold a 3-pound chicken at $27\frac{1}{2}$¢ a pound. How much money should he receive? 82½¢, or 83¢

5. It is $\frac{6}{10}$ mi. from Bob's house to school. When he has walked $\frac{2}{3}$ of the way, how far has he walked? $\frac{2}{5}$ miles

6. A $2\frac{1}{2}$-gallon can contains how much water when $\frac{2}{3}$ full? $1\frac{2}{3}$ gal.

7. The radio announcer said that the stadium was $\frac{3}{4}$ full. If it seats 56,000, about how many people were in the seats? 42,000

8. Mrs. Dean bought $2\frac{3}{4}$ yd. of lace for trimming at 20¢ a yard. How much did she pay for it? 55¢

9. At the market, cube steaks weigh about $\frac{3}{8}$ lb. Mrs. West bought 4 of them. How much did they weigh in all? 1½ lb.

10. How many square feet of glass are there in a picture window $3\frac{1}{2}$ ft. long and $1\frac{5}{6}$ ft. high? 6 5/12 sq. ft.

Teaching Pages 148 and 149

Pupil's Objectives. (*a*) To learn how to multiply by a mixed number when the multiplicand is either a fraction or a mixed number; (*b*) to solve multiplication problems involving fractions.

Background. Most pupils should be able, without a detailed formal introduction, to multiply when the multiplier is a mixed number and the multiplicand a fraction or a mixed number.

Notice that under Ex. 2 on page 148 pupils are led to realize that they can check multiplication involving fractions by reversing factors and multiplying again. Now that all the different types of multiplication involving fractions have been introduced, this check can always be used.

Teacher's Preparation. Obtain or construct fractional parts that can be used with a flannel board and with a fractional chart.

Pre-book Lesson (page 148)

1. Present, using figures which will produce an answer in simplest form, a problem such as the one at the top of the first column on the next page.

On field day the boys will run around the playground $3\frac{1}{2}$ times. The distance around the playground is $\frac{1}{8}$ of a mile. What part of a mile will each boy run?

A class discussion concerning the solving of the problem might proceed along the following lines:

a. Estimate the answer.
b. Illustrate with a drawing.
c. Decide how the answer could be found.
d. Find the answer at the board.
e. Ask questions about the procedure used for solving the problem. (These questions will help to stimulate thinking and to develop clearer understanding.)
f. Discuss the algorism and list the steps taken in working the example.
g. Discuss shorter ways of performing the algorisms.

2. Follow the procedure above for another problem, such as the following:

A horse ran around a track $5\frac{1}{2}$ times without stopping. The track is $1\frac{1}{4}$ miles long. How far did the horse run?

Book Lesson (page 148). *Ex. 1 and 2:* Oral work. *Rows 3–7:* Written work.

Pre-book Lesson (page 149). Put several problems such as the following on the board:

a. The theater was $\frac{2}{3}$ full. If it seated 1,200 people, how many people were in the theater?
b. Mrs. Smith bought 5 porterhouse steaks for dinner. Each steak weighed $1\frac{1}{4}$ lb. How many pounds of steak did Mrs. Smith buy?

Have volunteer pupils go to the board to solve the problems. If some pupils seem to have difficulty, withdraw them, letting the others turn to the written work in the text. Let the children who have been withdrawn discuss and solve on the board some of the problems in the text. If you can, discover at this time whether pupils are having difficulty with the computation or the meaning of multiplication, and reteach as necessary.

Book Lesson (page 149). *Ex. 1–10:* Written work.

Differentiations and Extensions

1. For the examples in rows 3–7 on page 148, pupils should be able to tell that all products will be larger than the multiplicands because the multipliers are larger than 1. Pupils should see, therefore, that they may round the multiplier to the nearest whole number and multiply to obtain an estimate of the product. For Ex. 3**a** pupils would round $2\frac{5}{8}$ to 3 and then see that $3 \times \frac{1}{3} = 1$. For Ex. 3**b**, $4\frac{2}{3}$ rounds to 5. $5 \times \frac{1}{4} = \frac{5}{4}$, or $1\frac{1}{4}$. From Ex. 5**d** to the end of the exercise, both multiplier and multiplicand may be rounded to the nearest whole number in estimating products. Of course, when one of the factors rounds to 1, the other factor will serve as an estimate. In Ex. 7**a**, $1\frac{3}{8}$ rounds to 1, so a good estimate for this example is $6\frac{2}{9}$.

2. Continue your program designed to help children discover ways of working without pencil and paper. For the examples in rows 3–7, it may be helpful for pupils to realize that they can reverse the factors before trying to work the example. For Ex. 4**d** pupils might think "Reverse the factors so that the example becomes $\frac{1}{2} \times 2\frac{3}{4}$. Round $2\frac{3}{4}$ to 3. The product will be $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{8}$, too large. $\frac{1}{2}$ of $3 = 1\frac{1}{2}$, $1\frac{1}{2} - \frac{1}{8} = 1\frac{3}{8}$, the exact answer."

Working without pencil and paper for examples where both the multiplier and the multiplicand is a mixed number is quite difficult and only the *more capable children* should attempt this.

3. *More capable children* may write and solve original problems using the following figures:

$5 \times \frac{1}{3}$ c.	$\frac{4}{5} \times 7$ mi.	$1\frac{1}{2} \times \frac{1}{4}$ lb.
$2\frac{2}{3} \times 9$ gal.	$6 \times 1\frac{3}{4}$ qt.	$\frac{7}{8} \times \frac{3}{4}$ doz.

If the writing of original problems seems too difficult for the group, let them solve the following problems:

a. We made 5 cakes. Each required $\frac{1}{3}$ cup of shortening. How much shortening did we use?
b. Each batch of cookies required $\frac{3}{4}$ cup of flour. To make $1\frac{1}{2}$ times the batch, how much flour will be needed?
c. A jeep used 9 gallons of gas to go around the lake. If it traveled around the lake $2\frac{2}{3}$ times, how much gas would it use?
d. Each boy at camp will drink $1\frac{3}{4}$ quarts of milk each day on the average. How many quarts should be ordered for 6 boys?

4. **Extra Practice Sets 65, 66, and 67** may be assigned as needed.

Set 65. Multiplying a fraction by a mixed number

1. a. $1\frac{1}{2} \times \frac{2}{3}$ **1** **b.** $5\frac{5}{8} \times \frac{5}{9}$ $3\frac{1}{8}$ **c.** $4\frac{1}{6} \times \frac{3}{5}$ $2\frac{1}{2}$

d. $2\frac{1}{3} \times \frac{7}{8}$ $2\frac{1}{24}$ **e.** $4\frac{3}{5} \times \frac{3}{4}$ $3\frac{9}{20}$

2. a. $6\frac{2}{3} \times \frac{4}{5}$ $5\frac{1}{3}$ **b.** $2\frac{5}{6} \times \frac{1}{3}$ $\frac{17}{18}$ **c.** $8\frac{1}{3} \times \frac{5}{7}$ $5\frac{20}{21}$

d. $9\frac{1}{8} \times \frac{2}{5}$ $3\frac{13}{20}$ **e.** $3\frac{7}{9} \times \frac{5}{6}$ $3\frac{4}{27}$

Set 66. Multiplying a fraction by a mixed number

1. a. $3\frac{1}{3} \times \frac{8}{9}$ $2\frac{26}{27}$ **b.** $5\frac{5}{6} \times \frac{6}{7}$ 5 **c.** $3\frac{3}{5} \times \frac{5}{16}$ $1\frac{1}{8}$

d. $4\frac{1}{2} \times \frac{7}{8}$ $3\frac{15}{16}$ **e.** $2\frac{5}{8} \times \frac{14}{3}$ $12\frac{1}{4}$

2. a. $6\frac{2}{5} \times \frac{9}{16}$ $3\frac{3}{5}$ **b.** $3\frac{1}{8} \times \frac{13}{5}$ $8\frac{1}{8}$ **c.** $9\frac{3}{8} \times \frac{4}{15}$ $2\frac{1}{2}$

d. $8\frac{4}{5} \times \frac{5}{16}$ $2\frac{3}{4}$ **e.** $1\frac{1}{2} \times \frac{3}{4}$ $1\frac{1}{8}$

Set 67. Multiplying a mixed number by a mixed number

1. a. $2\frac{1}{3} \times 2\frac{1}{2}$ $5\frac{5}{6}$ **b.** $8\frac{3}{4} \times 2\frac{3}{5}$ $22\frac{3}{4}$ **c.** $5\frac{1}{4} \times 3\frac{2}{3}$ $19\frac{1}{4}$

d. $1\frac{7}{8} \times 1\frac{2}{5}$ $2\frac{5}{8}$ **e.** $4\frac{1}{4} \times 4\frac{1}{4}$ $18\frac{1}{16}$

2. a. $3\frac{1}{7} \times 1\frac{2}{9}$ $3\frac{53}{63}$ **b.** $3\frac{1}{5} \times 2\frac{1}{4}$ $7\frac{1}{5}$ **c.** $2\frac{6}{7} \times 1\frac{3}{4}$ 5

d. $4\frac{1}{6} \times 7\frac{1}{2}$ $31\frac{1}{4}$ **e.** $5\frac{1}{2} \times 2\frac{7}{8}$ $15\frac{13}{16}$

3. a. $6\frac{3}{8} \times 1\frac{5}{6}$ $11\frac{11}{16}$ **b.** $9\frac{5}{9} \times 2\frac{5}{8}$ $25\frac{1}{12}$ **c.** $3\frac{3}{5} \times 4\frac{3}{8}$ $15\frac{3}{4}$

d. $2\frac{5}{8} \times 1\frac{1}{7}$ 3 **e.** $6\frac{1}{3} \times 4\frac{1}{2}$ $28\frac{1}{2}$

Teaching Pages 150 and 151

Pupil's Objective. To discover that cancellation is a way of reducing fractions that shortens the process of multiplying fractions.

Background. Pupils should now be ready to learn about cancellation in multiplying fractions. For most pupils this will be easy. However, don't let your pupils work so fast in picking up cancellation that they fail to realize that it is nothing more than reducing fractions before multiplying numerators and denominators instead of afterwards. For some pupils, you may be able to show how an example like $\frac{1}{10} \times \frac{2}{3}$ could be rewritten as $\frac{1 \times 2}{10 \times 3}$; then as $\frac{2 \times 1}{10 \times 3}$; and finally as $\frac{2}{10} \times \frac{1}{3}$. In the latter form it is easier for pupils to see that cancellation involves reducing fractions ($\frac{2}{10}$ to $\frac{1}{5}$ in this case) before multiplying numerators and denominators.

The work on page 151 gives pupils an opportunity to review the different kinds of multiplication involving fractions while giving further practice in cancellation. Notice that the examples in rows 1 to 11 are separated according to whether the multiplier is a whole number, a fraction, or a mixed number. This should be useful to you in deciding what kinds of multiplication need further reteaching.

A Short Cut—Cancellation

Meaning [O]

1. $\frac{5}{9} \times \frac{2}{5} = \frac{5 \times 2}{9 \times 5} = \frac{10}{45}$. By what number will you divide both terms of $\frac{10}{45}$ to change the answer to best form? 5 $\frac{10}{45} = \frac{2}{9}$

2. Box A shows an easier way to work Ex. 1. When we look at the fractions to be multiplied, we see right away that their product will have 5 as a factor of both the numerator and the denominator. Then we can divide both terms by 5 *before* we multiply. To show that we are dividing by 5, we cross out, or cancel, the 5's. Then we multiply as usual the numbers that are left, 9×1 for the denominator and 1×2 for the numerator. What is the product? $\frac{2}{9}$ Is the answer in best form? Yes

A

$\frac{\overset{1}{\cancel{5}}}{9} \times \frac{2}{\underset{1}{\cancel{5}}} = \frac{2}{9}$

B

$\frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{7}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{12}}} = \frac{1}{2}$

C

$\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{5}{\cancel{15}}} = \frac{21}{40}$

both terms divided by 7

3. Box B. Explain what the cancellation of the 7's means. By what other number have we divided both a numerator and a denominator? 6 Explain all the work. The product is $\frac{1}{2}$.

4. Explain the cancellation in box C. How do we use the Golden Rule of Fractions when we cancel? We divide both numerator and denominator by the same number.

Cancellation is a way of reducing fractions before they are multiplied.

Tell whether you can cancel (✓ = cancellation possible) before multiplying in these:

	a	b	c	d	e	f
5.	$\frac{1}{3} \times \frac{4}{5}$ $\frac{4}{15}$	$\frac{1}{10} \times \frac{\cancel{3}}{\cancel{3}}$ $\frac{1}{15}$	$\frac{1}{5} \times \frac{\cancel{15}}{16}$ $\frac{3}{16}$	$\frac{7}{8} \times \frac{\cancel{3}}{\cancel{14}}$ $\frac{3}{16}$	$\frac{4}{5} \times \frac{\cancel{7}}{8}$ $\frac{7}{10}$	$\frac{1}{9} \times \frac{\cancel{9}}{\cancel{10}}$ $\frac{1}{30}$
6.	$\frac{1}{4} \times \frac{2}{7}$ $\frac{1}{14}$	$\frac{2}{5} \times \frac{7}{9}$ $\frac{14}{45}$	$\frac{1}{8} \times \frac{\cancel{4}}{15}$ $\frac{1}{30}$	$\frac{3}{5} \times \frac{7}{8}$ $\frac{21}{40}$	$\frac{1}{6} \times \frac{\cancel{8}}{15}$ $\frac{4}{45}$	$\frac{2}{3} \times \frac{\cancel{9}}{14}$ $\frac{3}{7}$
7.	$\frac{1}{2} \times \frac{7}{8}$ $\frac{7}{16}$	$\frac{2}{3} \times \frac{\cancel{9}}{10}$ $\frac{3}{5}$	$\frac{2}{7} \times \frac{\cancel{14}}{15}$ $\frac{4}{15}$	$\frac{9}{10} \times \frac{3}{4}$ $\frac{27}{40}$	$\frac{5}{6} \times \frac{\cancel{9}}{10}$ $\frac{3}{4}$	$\frac{3}{5} \times \frac{\cancel{5}}{12}$ $\frac{1}{4}$

[W]

Now multiply in rows 5–7, using cancellation.

150

Pre-book Lesson

1. Present a problem such as the following:

Tom took $\frac{3}{4}$ lb. of hamburg on a hiking trip. He used $\frac{2}{3}$ of it. How much did he have left?

Ask pupils to volunteer to show the solution on the board. Guide subsequent discussion with questions such as these:

a. How should the answer be expressed? (in simplest form)

b. What was done to express the answer in simplest form? (it was reduced, or changed, to lowest terms)

c. How was this done? (by dividing both terms of the fraction by the same number)

d. How could the example have been done a shorter way? (by reducing or cancelling before multiplying)

Using Cancellation

(✓ = cancellation possible) [W]

Multiply. Use cancellation when you can.

	a	b	c	d	e
			A whole number as multiplier		
1.	$35 \times \frac{1}{2} = 17\frac{1}{2}$	$27 \times \frac{1}{6} = 4\frac{1}{2}$	$16 \times \frac{1}{3} = 5\frac{1}{3}$	$21 \times \frac{1}{3} = 49$	$14 \times \frac{3}{5} = 8\frac{2}{5}$
2.	$60 \times \frac{7}{10} = 42$	$30 \times \frac{5}{9} = 16\frac{2}{3}$	$15 \times \frac{1}{5} = 27$	$12 \times \frac{5}{16} = 24$	$8 \times \frac{7}{12} = 4\frac{2}{3}$
3.	$2 \times \frac{3}{8} = \frac{3}{4}$	$6 \times \frac{1}{12} = \frac{1}{2}$	$4 \times \frac{7}{16} = \frac{7}{4}$	$3 \times \frac{5}{12} = \frac{1}{4}$	$4 \times \frac{2}{9} = \frac{8}{9}$
			A fraction as multiplier		
4.	$\frac{3}{10} \times \frac{5}{5} = \frac{3}{25}$	$\frac{2}{3} \times \frac{1}{4} = \frac{1}{10}$	$\frac{3}{16} \times \frac{1}{9} = \frac{1}{12}$	$\frac{8}{3} \times \frac{3}{2} = 4$	$\frac{5}{4} \times \frac{8}{5} = 3\frac{1}{3}$
5.	$\frac{2}{9} \times \frac{3}{5} = \frac{2}{15}$	$\frac{5}{16} \times \frac{8}{15} = \frac{1}{6}$	$\frac{7}{10} \times 2 = 1\frac{2}{5}$	$\frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$	$\frac{2}{3} \times 1\frac{7}{10} = 1\frac{2}{15}$
6.	$\frac{7}{8} \times \frac{4}{5} = \frac{7}{10}$	$\frac{3}{7} \times \frac{5}{6} = \frac{5}{14}$	$\frac{6}{7} \times 4\frac{2}{3} = 4$	$\frac{3}{16} \times 2\frac{2}{5} = \frac{9}{20}$	$\frac{7}{16} \times 3\frac{1}{5} = 1\frac{2}{5}$
7.	$\frac{2}{15} \times \frac{5}{6} = \frac{1}{9}$	$\frac{5}{12} \times 7\frac{1}{2} = 3\frac{1}{2}$	$\frac{9}{4} \times 2 = 4\frac{1}{2}$	$\frac{5}{3} \times 2\frac{7}{10} = 4\frac{1}{2}$	$\frac{15}{2} \times 3 = 22\frac{1}{2}$
8.	$\frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$	$\frac{5}{16} \times 2\frac{2}{5} = \frac{3}{4}$	$\frac{7}{6} \times 4 = 4\frac{2}{3}$	$\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$	$\frac{7}{9} \times \frac{12}{7} = 1\frac{1}{3}$
			A mixed number as multiplier		
9.	$2\frac{1}{2} \times \frac{1}{5} = 2$	$2\frac{1}{6} \times \frac{3}{8} = \frac{13}{16}$	$3\frac{1}{7} \times \frac{1}{2} = 1\frac{4}{7}$	$1\frac{1}{5} \times 2\frac{1}{12} = 2\frac{1}{4}$	$2\frac{2}{5} \times \frac{1}{4} = \frac{3}{5}$
10.	$1\frac{7}{8} \times \frac{3}{10} = \frac{3}{8}$	$1\frac{1}{3} \times \frac{3}{16} = \frac{1}{4}$	$2\frac{2}{9} \times 2 = 4\frac{4}{9}$	$4\frac{1}{2} \times \frac{1}{6} = \frac{3}{4}$	$1\frac{1}{6} \times \frac{3}{10} = \frac{7}{20}$
11.	$1\frac{4}{5} \times 3\frac{1}{3} = 6$	$1\frac{1}{5} \times \frac{7}{12} = \frac{7}{10}$	$4\frac{1}{6} \times \frac{2}{5} = 1\frac{2}{3}$	$1\frac{1}{2} \times 4 = 6$	$3\frac{1}{3} \times 3 = 10$

12. A package of crackers marked "Contents, $\frac{3}{4}$ lb." is half full. What is the weight of the crackers in it now? $\frac{3}{8}$ lb.

13. What will $\frac{3}{4}$ yd. of ribbon cost at 40¢ a yard? 30¢

14. How much will $\frac{2}{3}$ of a $10\frac{1}{2}$-pound cheese weigh? 7 lb.

15. Since 1 rod (rd.) = $5\frac{1}{2}$ yd., how many yards are there in 12 rd.? 66 yd.

16. How many ounces of candy are there in a $5\frac{1}{2}$-pound chocolate egg? 88 oz.

17. At $7\frac{1}{2}$¢ a yard, how much would Mrs. Wallace pay for 24 yd. of lace? $1.80

Extra Practice. For more practice, work Sets 68–70.

151

2. Have a group of *more capable children* go to the board to do these examples:

$\frac{3}{4} \times \frac{6}{9}$ $\quad$ $\frac{2}{3} \times \frac{12}{15}$ $\quad$ $\frac{5}{6} \times \frac{3}{10}$

After the examples have been done, engage in a discussion, the purpose of which is to bring out the principle of reduction and cancellation.

3. Have a group of *slower learners* go to the board to do these examples:

$\frac{5}{6} \times \frac{8}{10}$ $\quad$ $\frac{2}{3} \times \frac{6}{8}$ $\quad$ $\frac{1}{2} \times \frac{4}{8}$

Book Lesson (page 150). *Ex. 1–7:* Oral work. *Bottom of page:* Written work.

Book Lesson (page 151). *Ex. 1–17:* Written work.

Differentiations and Extensions

1. Let pupils multiply in some of the examples in rows 5–7 on page 150 and rows 1–11 on page 151 without first cancelling so that they will realize the efficiency of cancelling.

2. Continue to give children practice in estimating answers and finding products mentally.

3. Assign **Extra Practice Sets 68, 69, and 70** as needed.

Set 68. Cancellation in multiplying fractions

Cancel if you can before you multiply.

1.	a. $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$	b. $\frac{1}{2} \times \frac{8}{9} = \frac{4}{9}$	c. $\frac{5}{8} \times \frac{7}{15} = \frac{7}{24}$
	d. $\frac{2}{3} \times \frac{15}{16} = \frac{5}{8}$	e. $\frac{7}{8} \times \frac{4}{21} = \frac{1}{6}$	f. $\frac{3}{5} \times \frac{5}{12} = \frac{1}{4}$
2.	a. $\frac{5}{9} \times \frac{3}{10} = \frac{1}{6}$	b. $\frac{4}{5} \times \frac{7}{12} = \frac{7}{15}$	c. $\frac{3}{8} \times \frac{5}{12} = \frac{5}{32}$
	d. $\frac{1}{7} \times \frac{7}{15} = \frac{1}{15}$	e. $\frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$	f. $\frac{3}{8} \times \frac{4}{15} = \frac{1}{10}$
3.	a. $\frac{3}{5} \times \frac{5}{7} = \frac{3}{7}$	b. $\frac{5}{6} \times \frac{12}{25} = \frac{2}{5}$	c. $\frac{4}{5} \times \frac{13}{16} = \frac{13}{20}$
	d. $\frac{7}{8} \times \frac{6}{7} = \frac{3}{4}$	e. $\frac{5}{6} \times \frac{4}{15} = \frac{2}{9}$	f. $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

Set 69. Cancellation in multiplying fractions

Cancel if you can before you multiply.

1.	a. $9 \times \frac{2}{3} = 6$	b. $3 \times 4\frac{1}{2} = 13\frac{1}{2}$	c. $\frac{3}{4} \times 15 = 11\frac{1}{4}$
	d. $\frac{5}{8} \times \frac{3}{10} = \frac{3}{16}$	e. $\frac{3}{7} \times 17\frac{1}{2} = 7\frac{1}{2}$	
2.	a. $4\frac{1}{3} \times 19 = 82\frac{1}{3}$	b. $2\frac{3}{8} \times \frac{2}{5} = \frac{19}{20}$	c. $1\frac{4}{5} \times 2\frac{2}{9} = 4$
	d. $8 \times \frac{3}{4} = 6$	e. $4 \times 3\frac{5}{16} = 13\frac{1}{4}$	
3.	a. $\frac{5}{6} \times 14 = 11\frac{2}{3}$	b. $\frac{6}{7} \times \frac{14}{15} = \frac{4}{5}$	c. $\frac{7}{8} \times 18\frac{2}{3} = 16\frac{1}{3}$
	d. $5\frac{4}{9} \times 12 = 65\frac{1}{3}$	e. $4\frac{4}{5} \times 4\frac{1}{6} = 20$	
4.	a. $1\frac{4}{7} \times \frac{7}{11} = 1$	b. $3\frac{3}{8} \times 1\frac{7}{9} = 6$	c. $6 \times \frac{5}{12} = 2\frac{1}{2}$
	d. $7 \times 1\frac{5}{14} = 9\frac{1}{2}$	e. $8\frac{1}{3} \times \frac{9}{10} = 7\frac{1}{2}$	

Set 70. Cancellation in multiplying fractions

Cancel if you can before you multiply.

1.	a. $7 \times \frac{5}{14} = 2\frac{1}{2}$	b. $4 \times 12\frac{1}{2} = 50$	c. $\frac{5}{7} \times 14 = 10$
	d. $\frac{3}{8} \times \frac{7}{12} = \frac{7}{32}$	e. $\frac{4}{7} \times 16\frac{1}{3} = 9\frac{1}{3}$	
2.	a. $3\frac{1}{8} \times 16 = 50$	b. $5\frac{3}{5} \times \frac{5}{7} = 4$	c. $1\frac{3}{4} \times 3\frac{1}{7} = 5\frac{1}{2}$
	d. $9 \times \frac{8}{15} = 4\frac{4}{5}$	e. $6 \times 9\frac{5}{8} = 57\frac{3}{4}$	
3.	a. $\frac{3}{4} \times 18 = 13\frac{1}{2}$	b. $\frac{7}{8} \times \frac{4}{15} = \frac{7}{30}$	c. $\frac{5}{9} \times 7\frac{1}{5} = 4$
	d. $4\frac{2}{3} \times 15 = 70$	e. $2\frac{5}{8} \times 5\frac{1}{3} = 14$	
4.	a. $5\frac{3}{7} \times \frac{14}{17} = 4\frac{8}{17}$	b. $4\frac{3}{8} \times 3\frac{1}{5} = 14$	c. $8 \times \frac{7}{12} = 4\frac{2}{3}$
	d. $5 \times 7\frac{9}{10} = 39\frac{1}{2}$	e. $3\frac{3}{4} \times \frac{8}{9} = 3\frac{1}{3}$	

Teaching Pages 152 and 153

Pupil's Objective. To use cancellation in multiplying fractions when there are more than two factors.

Background. Page 152 introduces multiplication involving fractions when there are three or more factors. The new thing here is that any number, even a whole number, may be written as a fraction. Then, using cancellation when possible, the product may be obtained by multiplying denominators to find the fractional unit for the product and multiplying numerators to find the total number of equal parts in the product. If pupils attempt one or two examples of this kind without cancelling, then with cancelling, they will quickly discover how much this procedure reduces the work in multiplying fractions.

Now that most of the understandings, skills, and abilities involving multiplying fractions have been introduced, a page of testing and practice is desirable as a "breather." The short-answer test at the top of page 153 is of the multiple-choice type in which one of the answers following each example is the correct one and the pupil is to use his judgment in deciding which.

The written practice at the foot of the page contains work involving both whole numbers and fractions. It is, therefore, an important part of the maintenance program.

Teacher's Preparation. Obtain a flannel board and accompanying fractional pieces to be used for the pre-book lesson.

Multiplying with More than Two Factors

[O]

Now that you know how to find the product of two fractions and how to use cancellation, you can easily find the product of three or more fractions.

A

$\frac{5}{7} \times \frac{1}{2} \times \frac{3}{4} = \frac{15}{56}$

B

$\frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{3}^{3}}{\cancel{16}_{4}} \times \frac{\cancel{5}^{1}}{\cancel{12}_{4}} = \frac{3}{16}$

1. Study the work in box A. Can you cancel? No Explain how to find the product.

2. Explain all the work in box B. To check, work the example on the board without using cancellation.

(✓=cancellation possible) [W]

Copy and find **n** in rows 3 and 4. Cancel when you can.

	a	b	c
3.	✓ $\frac{1}{5} \times \frac{1}{4} \times \frac{2}{3} = n$ 1/30	$\frac{1}{2} \times \frac{3}{8} \times \frac{4}{5} = n$ 3/20	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = n$ 1/27
4.	✓ $\frac{8}{9} \times \frac{3}{4} \times \frac{2}{5} = n$ 4/15	$\frac{7}{8} \times \frac{4}{5} \times \frac{10}{21} = n$ 1/3	✓ $\frac{3}{5} \times \frac{2}{9} \times \frac{15}{16} \times \frac{4}{7} = n$ 1/14

5. For the example $4 \times \frac{9}{16} \times 2\frac{1}{3}$, you can multiply just as in boxes A and B. When one of the factors in an example of this kind is a fraction, it is better to write all the factors in fraction form before you multiply. Remember that any whole number can be written as a fraction with the denominator 1, and that any mixed number can be written as an improper fraction. Then this example becomes $\frac{4}{1} \times \frac{9}{16} \times \frac{7}{3}$. Work the example on your paper. (Don't forget to cancel!) 5 1/4

Change whole and mixed numbers to fractions and find **n**.

6.	$\frac{3}{4} \times \frac{5}{8} \times 2\frac{1}{6} = n$ 1 11/64	$\frac{3}{10} \times \frac{5}{9} \times 10 = n$ 1 1/3	$\frac{1}{2} \times \frac{2}{9} \times 7 \times 6 = n$ 4 2/3
7.	$2\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6} = n$ 1 1/4	$\frac{5}{6} \times 1\frac{4}{5} \times \frac{2}{3} = n$ 1	$\frac{5}{8} \times 3\frac{1}{5} \times 4 \times \frac{3}{4} = n$ 6
8.	$\frac{3}{4} \times \frac{4}{7} \times 3\frac{1}{2} = n$ 1/2	$3\frac{1}{7} \times \frac{3}{4} \times 21 = n$ 49 1/2	$\frac{7}{12} \times 5 \times \frac{6}{7} \times 2\frac{4}{5} = n$ 7

Extra Practice. For more practice, work Set 71.

Pre-book Lesson (page 152)

1. To help pupils make the discovery that a whole number equals a fraction whose numerator is itself and whose denominator is 1, use a flannel board and proceed somewhat as follows:

a. Place $\frac{1}{4}$ on the flannel board and ask what fraction is thus shown.

b. Place 2 more fourths on the flannel board and ask what fraction is shown.

c. Place 4 fourths on the flannel board and ask what fraction is shown.

d. Place 1 whole circle on the flannel board and ask what is represented (one 1, or $\frac{1}{1}$).

e. Place 3 whole circles on the flannel board and ask how many ones are shown (three ones, or $\frac{3}{1}$).

2. Have pupils write 5, 6, and 9, as fractions. After these experiences, lead them to generalize that a whole number equals a fraction whose numerator is itself and whose denominator is 1. Write this on the board.

3. Now have pupils look at the example $\frac{5}{6} \times 6$. Ask in what other way the whole number might be written (as the fraction $\frac{6}{1}$).

4. Send pupils to the board to see if they can multiply the following examples by writing the whole numbers as fractions with denominator 1:

$\frac{5}{6} \times 60 \quad \frac{2}{9} \times 3 \quad \frac{7}{12} \times 4 \quad 25 \times \frac{4}{5} \quad 72 \times \frac{7}{8}$

Book Lesson (page 152). *Ex. 1 and 2:* Oral work. *Rows 3–8:* Written work. For the oral work, ask questions such as the following:

a. In example 1, can any numbers be cancelled?

Short Answer Test

[W]

One of the answers in parentheses after each example is correct. For each example, write the answer you think is correct.

1. $\frac{1}{2}+\frac{3}{4}$ $(\frac{4}{6})$ $(1\frac{1}{4})$ $(\frac{3}{8})$
2. $\frac{7}{8}+\frac{2}{8}$ $(\frac{9}{11})$ $(1\frac{1}{8})$ $(1\frac{3}{24})$
3. $4-1\frac{3}{4}$ $(3\frac{3}{4})$ $(2\frac{1}{4})$ $(3\frac{1}{4})$
4. $6\frac{5}{8}-2\frac{1}{2}$ $(4\frac{1}{48})$ $(4\frac{1}{2})$ $(4\frac{1}{4})$
5. $3\times\frac{7}{8}$ $(2\frac{5}{8})$ $(\frac{7}{24})$ $(3\frac{7}{8})$
6. $5\times\frac{3}{4}$ $(5\frac{3}{4})$ $(3\frac{3}{4})$ $(1\frac{1}{4})$
7. $4\times3\frac{1}{2}$ (14) $(12\frac{1}{2})$ $(7\frac{1}{2})$
8. $\frac{4}{5}\times2\frac{3}{4}$ $(1\frac{8}{5})$ $(8\frac{1}{2})$ $(2\frac{1}{5})$

9. How many partial products are there when we multiply 25×426? (1) (2) (3) [Product = 10,650]

10. What is the first partial dividend in the division example $43\overline{)36,195}$? (36) (361) (3619) [Quotient = 841, R32]

11. What is the smallest common denominator for $\frac{2}{3}+\frac{4}{5}+\frac{1}{2}$? (10) (15) (30) [Sum = $1\frac{29}{30}$]

12. Which of the fractions in Ex. 11 is smallest? $(\frac{1}{2})$ $(\frac{4}{5})$ $(\frac{2}{3})$

Written Practice

	a	b	c	d	e	f
1.	$\frac{3}{4}$	$4\frac{5}{8}$	$3\frac{7}{8}$	$6\frac{1}{2}$	7	$4\frac{1}{6}$
	$+\frac{1}{5}$ $\frac{19}{20}$	$+6\frac{3}{16}$ $10\frac{13}{16}$	$+5\frac{2}{11}$ $9\frac{24}{44}$	$-2\frac{1}{4}$ $4\frac{1}{4}$	$-3\frac{4}{5}$ $3\frac{1}{5}$	$-3\frac{1}{2}$ $\frac{3}{3}$
2.	56	498	293	301	652	600
	+97 153	+307 805	+146 439	−92 209	−148 504	−518 82
3.	72	65	78	305	38	401
	×30 2,160	×83 5,395	×96 7,488	×806 245,830	×27 1,026	×260 104,260

Write any remainder with R.

4. $26\overline{)832}$ 32
5. $19\overline{)6,035}$ 317, R12
6. $82\overline{)32,895}$ 401, R13

7. Change to improper fractions: $6\frac{1}{2}$; $4\frac{2}{3}$; $7\frac{3}{4}$; $5\frac{3}{8}$. $\frac{13}{2}$ $\frac{14}{3}$ $\frac{31}{4}$ $\frac{43}{8}$

8. Change to best form: $\frac{17}{5}$; $\frac{14}{16}$; $\frac{9}{12}$; $\frac{23}{7}$; $\frac{16}{24}$; $\frac{14}{6}$; $\frac{31}{8}$; $\frac{5}{10}$. $3\frac{2}{5}$ $\frac{7}{8}$ $3\frac{2}{7}$ $\frac{2}{3}$ $2\frac{1}{3}$ $3\frac{7}{8}$ $\frac{1}{2}$

b. How do you find the product in box A?

c. Can you explain the cancelling in box B?

d. What is the first thing to do in each example in rows 3 and 4?

e. In example 5, what must be done before you can cancel? (Change 4 and $2\frac{1}{3}$ to improper fractions.)

f. In some of the examples in rows 6, 7, and 8, what must be done before you can cancel?

Book Lesson (top of page 153). *Ex. 1–12:* Written work. Be sure pupils realize that they are not to guess, but are to estimate or work the examples in order to select the correct answer.

Book Lesson (bottom of page 153). *Ex. 1–8:* Written work. If necessary, review the meaning of improper fractions and the meaning of changing improper fractions to best form before pupils attempt the written work in Ex. 7 and 8.

Differentiations and Extensions

1. For *slower learners*, you may want to limit the assignment on page 152 by asking them to do rows 3, 6, and 7 only.

2. Assign **Extra Practice Set 71** as needed.

Set 71. Several factors in multiplication of fractions

Cancel if you can before multiplying.

1. a. $\frac{1}{2}\times\frac{2}{3}\times2\frac{1}{4}$ $\frac{3}{4}$ **b.** $3\frac{1}{7}\times\frac{7}{8}\times\frac{3}{4}$ $2\frac{1}{16}$

c. $18\times\frac{2}{3}\times\frac{7}{8}$ $10\frac{1}{2}$

2. a. $\frac{1}{4}\times2\times3\frac{1}{7}\times\frac{7}{8}$ $1\frac{3}{8}$ **b.** $\frac{5}{6}\times7\frac{1}{2}\times\frac{4}{5}$ 5

c. $\frac{3}{8}\times\frac{2}{9}\times3\frac{3}{5}$ $\frac{3}{10}$

3. a. $16\times\frac{5}{8}\times\frac{5}{6}$ $8\frac{1}{3}$ **b.** $\frac{2}{5}\times\frac{3}{8}\times2\frac{7}{9}\times3\frac{3}{5}$ $1\frac{1}{2}$

c. $\frac{1}{2}\times2\frac{4}{7}\times\frac{3}{4}\times1\frac{5}{9}$ $1\frac{1}{2}$

4. a. $8\frac{1}{3}\times3\frac{1}{5}\times\frac{1}{2}\times\frac{3}{5}$ 8 **b.** $9\times\frac{2}{3}\times12\times\frac{7}{8}$ 63

c. $10\times4\frac{4}{9}\times\frac{1}{6}\times\frac{3}{5}$ $4\frac{4}{9}$

Teaching Pages 154 and 155

Pupil's Objectives. (*a*) To discover how the size of a product is related to the size of its multiplier and multiplicand; (*b*) to learn how to round numbers by developing an ability to tell quickly whether a fraction is less than, equal to, or more than $\frac{1}{2}$.

Background. In multiplying whole numbers, products are always equal to or larger than multiplicands. On page 154 in the text, pupils will see summarized the eight different kinds of multiplication involving fractions and learn that when the multiplier is less than 1, the product is less than the multiplicand. This understanding (along with the idea that a whole-number or mixed-number multiplier gives a product larger than the multiplicand) is extremely important in practical situations, since it provides a basis for estimating products in working with fractions. In turn, as pupils attempt to refine estimates, they often discover ways of arriv-

A Discovery about Products

Size of product related to size of multiplier [O]

1. Check each product in the box below. You will need to work Ex. b, c, g, and h on the board because some steps are not shown, but you can check the other products "in your head."

2. In Ex. a, the multiplier is less than 1. The product is _?_ (the same as, more than, less than) the multiplicand.

3. In Ex. d, the multiplier is _1_. The product is _?_ (the same as, more than, less than) the multiplicand.

4. In Ex. f, the multiplier is more than 1. The product is _?_ (the same as, more than, less than) the multiplicand.

5. Read each of Ex. a–h and tell why the product is more than, less than, or the same as the multiplicand. *(See dark print below table.)*

Multiplier × Multiplicand = Product	Size of Multiplier	Size of Product
a. $\frac{4}{5} \times \frac{5}{8} = \frac{20}{40}$, or $\frac{1}{2}$	Less than 1	Less than $\frac{5}{8}$
b. $\frac{1}{4} \times 6 = \frac{3}{2}$, or $1\frac{1}{2}$	Less than 1	Less than 6
c. $\frac{3}{4} \times 2\frac{1}{2} = 1\frac{5}{8}$, or $1\frac{7}{8}$	Less than 1	Less than $2\frac{1}{2}$
d. $1 \times \frac{3}{4} = \frac{3}{4}$	1	Equal to $\frac{3}{4}$
e. $1 \times 4\frac{1}{2} = 4\frac{1}{2}$	1	Equal to $4\frac{1}{2}$
f. $3 \times \frac{1}{2} = \frac{3}{2}$, or $1\frac{1}{2}$	More than 1	More than $\frac{1}{2}$
g. $1\frac{1}{4} \times 4 = 5$	More than 1	More than 4
h. $2\frac{1}{3} \times 1\frac{1}{2} = \frac{7}{2}$, or $3\frac{1}{2}$	More than 1	More than $1\frac{1}{2}$

If the multiplier is less than 1, the product will be less than the multiplicand.

If the multiplier is 1, the product will be equal to the multiplicand.

If the multiplier is more than 1, the product will be more than the multiplicand.

For each example in rows 6–8, tell whether the product, **n**, will be less than, equal to, or more than the multiplicand.

a	b	c	d	e
6. $\frac{1}{4} \times 8 = n$	$\frac{2}{3} \times \frac{7}{8} = n$	$4 \times \frac{1}{8} = n$	$1 \times 4\frac{5}{6} = n$	$1\frac{1}{2} \times \frac{1}{3} = n$
7. $5 \times \frac{2}{3} = n$	$2\frac{1}{2} \times 4 = n$	$\frac{3}{4} \times 1\frac{7}{8} = n$	$\frac{4}{5} \times 7 = n$	$3\frac{3}{4} \times \frac{1}{3} = n$
8. $4\frac{1}{2} \times \frac{3}{4} = n$	$1 \times \frac{7}{8} = n$	$1\frac{2}{3} \times 6 = n$	$2\frac{1}{2} \times \frac{3}{4} = n$	$2 \times 1\frac{3}{4} = n$

[W]

Now go back and find **n** in each example.

Rounding Mixed Numbers

[O]

1. Each of the fractions $\frac{4}{8}$, $\frac{5}{10}$, and $\frac{3}{6}$ equals $\frac{1}{2}$. In each fraction, does the numerator equal $\frac{1}{2}$ of the denominator, more than $\frac{1}{2}$ of the denominator, or less than $\frac{1}{2}$ of the denominator?

2. Each of the fractions $\frac{9}{16}$, $\frac{7}{12}$, and $\frac{5}{8}$ equals more than $\frac{1}{2}$. Does each numerator equal $\frac{1}{2}$ of the denominator, more than $\frac{1}{2}$ of the denominator, or less than $\frac{1}{2}$ of the denominator?

3. Each of the fractions $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{4}{10}$ equals less than $\frac{1}{2}$. Does each numerator equal $\frac{1}{2}$ of the denominator, more than $\frac{1}{2}$ of the denominator, or less than $\frac{1}{2}$ of the denominator?

To tell whether a fraction is less than or equal to or more than $\frac{1}{2}$, compare its numerator with $\frac{1}{2}$ of its denominator.

4. Tell whether each of these fractions is less than, equal to, or greater than $\frac{1}{2}$: $\frac{3}{8}$; $\frac{7}{14}$; $\frac{5}{7}$; $\frac{7}{16}$; $\frac{3}{5}$; $\frac{8}{16}$; $\frac{5}{12}$; $\frac{6}{11}$; $\frac{4}{9}$.

5. To round $7\frac{5}{8}$ to the nearest whole number, we *think*, "$\frac{5}{8}$ is more than $\frac{1}{2}$, so the whole number nearest to $7\frac{5}{8}$ is 8."

6. Tell what to think in rounding $8\frac{2}{5}$ to the nearest whole number; (8) in rounding $5\frac{6}{7}$; (6) in rounding $3\frac{7}{10}$; (4) in rounding $9\frac{1}{3}$. (9)

7. When the fraction in a mixed number is $\frac{1}{2}$, is there any "nearest whole number"? Would you round $8\frac{1}{2}$ to 8 or to 9?

ing at exact products without using pencil and paper.

Pupils have already learned the principle of rounding mixed numbers and, by referring to a chart, used it in estimating sums and remainders (page 122 in the text). On page 155 pupils learn how to tell whether a fraction is less than, more than, or equal to $\frac{1}{2}$ without using a chart. This they do by comparing the numerator of a fraction with $\frac{1}{2}$ of its denominator.

Book Lesson (page 154 and top of page 155). *Ex. 1–8:* Oral work. *Bottom of lesson:* Written work.

1. Have each example in the box read and the process explained, making sure pupils note how the size of the multiplier affects the size of the product.

2. Put examples **b**, **c**, **g**, and **h** from the chart on the board so that the intervening steps in solving can be shown.

3. In their own words, let pupils summarize the relationship between the size of the multiplier and that of the product.

4. For rows 6–8, pupils should decide whether their written products are reasonable by comparing them with the size of the multiplicands.

Teacher's Preparation (bottom of page 155). Obtain or construct a flannel board and several circles about 8″ in diameter, some of which have been cut into $\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, etc. You may use thin, colored poster paper for the circles if you wish, since this will adhere to a flannel board, especially if the board is tipped slightly.

Pre-book Lesson (bottom of page 155)

1. Arrange on the flannel board a whole circle, a half circle, 3 sixths of a circle, 4 eighths of a circle, 5 tenths of a circle, and 2 thirds of a circle. Point to the 3 sixths and ask if this equals $\frac{1}{2}$. Let pupils show that it does by holding $\frac{1}{2}$ of a circle over the 3 sixths. Repeat the same procedure for $\frac{4}{8}$, $\frac{5}{10}$, and $\frac{2}{3}$ of a circle, making sure for the last case that pupils realize that $\frac{2}{3}$ is larger than $\frac{1}{2}$.

2. Write the following on the board and use the questions below in discussing them: $\frac{3}{6} = \frac{1}{2}$; $\frac{4}{8} = \frac{1}{2}$; $\frac{5}{10} = \frac{1}{2}$; $\frac{2}{3}$ is larger than $\frac{1}{2}$

a. In the fraction $\frac{3}{6}$, is the numerator 3 equal to one half of the denominator 6?

b. In the fraction $\frac{4}{8}$, is the numerator 4 equal to one half of the denominator 8?

c. In the fraction $\frac{5}{10}$, is the numerator 5 equal to one half of the denominator 10?

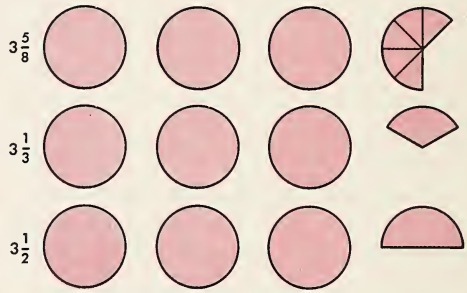
d. In the fraction $\frac{2}{3}$, is the numerator 2 equal to one half of the denominator 3?

e. How do you think we tell when a fraction is equal to $\frac{1}{2}$? (The numerator is $\frac{1}{2}$ of the denominator.)

3. Write the fractions in the next column on the board and ask which are more and which less than $\frac{1}{2}$. As pupils give reasons in each case, see if you can lead them to state in their own words the generalization given in dark print on page 155 in the text.

$\frac{6}{10}$ $\frac{3}{8}$ $\frac{8}{12}$ $\frac{2}{4}$ $\frac{5}{8}$ $\frac{3}{6}$ $\frac{2}{3}$ $\frac{1}{3}$

4. Show the three mixed numbers below on a flannel board and ask pupils how they would round each one to the nearest whole number. For $3\frac{5}{8}$, they should see that $\frac{5}{8}$ is more than $\frac{1}{2}$, so $3\frac{5}{8}$ rounds to 4. For $3\frac{1}{3}$, they should see that $\frac{1}{3}$ is less than $\frac{1}{2}$, so $3\frac{1}{3}$ rounds to 3. If pupils hesitate in rounding $3\frac{1}{2}$ to 4, tell them that they should round a mixed number to the next higher whole number when the numerator is *equal to* or *more than* $\frac{1}{2}$ of the denominator.



Book Lesson (bottom of page 155). *Ex. 1–7:* Oral work.

NOTES

So You Won't Forget!

[O]

1. Which of these drawings shows the mixed number $2\frac{1}{2}$? a.

a. b. c.

2. In multiplying 731 and 43 which number would you use for the multiplier?

3. Is $\frac{2}{3}$ of 12 the same as $\frac{2}{3} \times 12$? Yes

4. Will the product of $24 \times 2\frac{7}{8}$ be more than or less than the product of 24×3?

5. Will the product of $\frac{7}{8} \times 1\frac{5}{8}$ be more than or less than $1\frac{5}{8}$?

6. What is a common denominator for $\frac{5}{8}$ and $\frac{2}{3}$? 24

7. In multiplying $2\frac{3}{4}$ by $1\frac{1}{2}$, what changes would you make before multiplying? $\frac{11}{4} \times \frac{3}{2}$

8. Which fractions are greater than $\frac{1}{2}$? $\frac{3}{8}$ $\frac{7}{9}$ $\frac{5}{8}$ $\frac{3}{8}$

9. Estimate the product of $6\frac{1}{2} \times 10\frac{7}{8}$. 77

Practice in Multiplying with Fractions

[W]

	a	b	c	d
.	$\frac{1}{2} \times 16$ 8	$\frac{2}{3} \times 8$ $5\frac{1}{3}$	$8 \times \frac{3}{4}$ 6	$9 \times \frac{5}{6}$ $7\frac{1}{2}$
.	$4\frac{1}{2} \times 8$ 36	$3 \times 16\frac{2}{3}$ 50	$\frac{1}{2} \times \frac{3}{4}$ $\frac{3}{8}$	$\frac{7}{8} \times \frac{2}{3}$ $\frac{7}{12}$
.	$1\frac{3}{4} \times \frac{1}{6}$ $\frac{7}{24}$	$6\frac{1}{2} \times \frac{2}{3}$ $4\frac{1}{3}$	$\frac{7}{8} \times 4\frac{1}{2}$ $3\frac{15}{16}$	$1\frac{1}{5} \times 4$ $4\frac{4}{5}$
.	$2\frac{1}{2} \times 2\frac{1}{2}$ $6\frac{1}{4}$	$3\frac{1}{4} \times 1\frac{1}{2}$ $4\frac{7}{8}$	$6\frac{1}{4} \times 1\frac{2}{3}$ $10\frac{5}{12}$	$7\frac{1}{8} \times 2\frac{1}{3}$ $16\frac{5}{8}$
.	$6\frac{1}{4} \times 2\frac{1}{3}$ $14\frac{7}{12}$	$1\frac{5}{6} \times 2\frac{4}{5}$ $5\frac{2}{15}$	$4\frac{1}{3} \times 1\frac{2}{3}$ $7\frac{2}{9}$	$\frac{3}{4} \times 2\frac{1}{5}$ $\frac{13}{20}$

a	b	c	d	e	f
$80\frac{1}{2}$	$96\frac{3}{4}$	$70\frac{1}{2}$	$50\frac{1}{2}$	$23\frac{2}{3}$	$14\frac{1}{2}$
$\times 6$	$\times 8$	$\times 3$	$\times 12$	$\times 6$	$\times 8$
483	774	$211\frac{1}{2}$	606	142	116
16	42	54	21	36	14
$\times 4\frac{1}{2}$	$\times 8\frac{3}{4}$	$\times 4\frac{2}{3}$	$\times 5\frac{1}{8}$	$\times 6\frac{4}{5}$	$\times 7\frac{3}{4}$
72	$367\frac{1}{2}$	252	$107\frac{5}{8}$	$244\frac{4}{5}$	$108\frac{1}{2}$

Does the Answer Make Sense?

Testing the answer [O]

Often you can use the things you know about estimating and about the size of products to help you decide whether or not your answer to a multiplication example makes sense.

1. For $\frac{6}{7} \times 4\frac{2}{3}$, will the product be more than or less than $4\frac{2}{3}$? How can you tell? Do you think 4 is a reasonable answer? (4) Yes multiplier is less than 1

2. For $3\frac{1}{3} \times \frac{3}{4}$, John got the answer $4\frac{1}{4}$. Just by looking at the two factors that are multiplied, you can tell that this is not a reasonable answer. Explain. (2½) Call $\frac{3}{4}$ the multiplier; then since the multiplier is less than 1, the product will be less than $3\frac{1}{3}$.

3. A large package of dry cereal weighs $\frac{7}{8}$ lb. Nancy said, "Ten of these packages will weigh about 12 pounds." How can you tell right away that her answer was too large? (8⅜) $\frac{7}{8}$ is less than 1, so the product must be less than 10.

4. If either of two factors is less than 1, why is their product less than the other factor? The factor less than 1 may be thought of as the multiplier; then a fractional part of a number must be less than the number.

5. Claire earned $9. She had $\frac{1}{5}$ of it left after buying Christmas presents. Her brother Joe said, "You must have about 2 ($1\frac{4}{5}$) left." Does Joe's estimate make sense? Explain.

HELPER. *Think*, "$\frac{1}{5}$ of $10 = $2; $9 is less than $10, so $\frac{1}{5}$ of $9 must be less than $2."

6. Tony's answer for the example $5\frac{3}{4} \times 6$ was $30\frac{1}{2}$. ($34\frac{1}{2}$) To test this answer, think whether the product should be nearer to 6×6, or 36, or to 5×6, or 30. How can you tell? $\frac{3}{4}$ more than 5, so round $5\frac{3}{4}$ to 6. Is Tony's answer nearer to 36 or to 30? Is it a sensible answer? How do you know that the product should not be *more* than 36? $5\frac{3}{4}$ less than 6, so $5\frac{3}{4} \times 6$ is less than 6×6, or 36.

7. Why is 75 not a reasonable answer for $9 \times 7\frac{1}{3}$? (66) $7\frac{1}{3}$ is less than 8 and $9 \times 8 = 72$, so $7\frac{1}{3} \times 9$ must be less than 72.
HELPER. How can you use the M. fact $9 \times 8 = 72$?

8. For $5\frac{2}{3} \times 3\frac{1}{4}$, Don got the answer $10\frac{1}{12}$. (18⅝) Is this a reasonable answer? How would it have helped Don if he had rounded each factor to the nearest whole number and found an estimated product before he worked the example? $6 \times 3 = 18$ which is much larger than 10½.

Teaching Pages 156 and 157

Pupil's Objectives. (*a*) To maintain, through practice, various understandings and skills that have been developed in working with fractions; (*b*) to increase facility in multiplying fractions; (*c*) to use skills in rounding and in estimating the size of products to tell whether answers to multiplication problems are correct.

Book Lesson (page 156). *Ex. 1–9:* Oral work. *Rows 1–7:* Written work.

1. Be sure pupils give reasons for their answers for the oral work at the top of the page in the text.

2. Now that *all pupils* know how to relate the size of the product to the size of the multiplicand and how to round mixed numbers, let them estimate answers orally for rows 1–7, then write the work.

Book Lesson (page 157)

1. In Ex. 5–7 pupils are helped to refine their estimates. For example, in Ex. 6 they are led to see how an estimate may be more accurately determined by bracketing (in this case, the range of the bracket extends from the upper limit, $6 \times 6 = 36$, to the lower limit $5 \times 6 = 30$). This is an important ability in everyday affairs, so be sure to give *all pupils* an opportunity to discover these useful estimating techniques.

2. In case of doubt, pupils should work the examples for proof that their estimates are reliable.

At Zeno's

Estimating answers [O]

Tell which estimate after each problem seems best, and why.

1. At the Zeno Fruit Stand, the average weight of a seedless grapefruit is $1\frac{1}{4}$ lb. What would you expect 16 of these grapefruit to weigh? $16\frac{1}{4}$ lb. 160 lb. [20 lb] *20 lb.*

2. On the counter Mrs. Parks displayed $\frac{1}{2}$ of a grapefruit to show how good the fruit was. She gave Jim $\frac{1}{2}$ of the half grapefruit to taste. What part of a whole grapefruit did Jim get? $\frac{1}{2}$ [$\frac{1}{4}$] $\frac{2}{2}$, or 1 *$\frac{1}{4}$*

3. If tangerines weigh $\frac{1}{4}$ lb. each, on the average, about how much will a dozen of them weigh? 48 lb. [3 lb] 16 lb. *3 lb.*

4. At the Zeno stand, Temple oranges are $3 a bushel. How much will Jim pay for 3 pecks ($\frac{3}{4}$ bu.)? $9 $12 [$2] *$2.25*

5. Mrs. Parks sells pecans at $\frac{1}{2}$ a pound. Alice bought $1\frac{1}{2}$ lb. How much should she pay? [$\frac{3}{4}$] $2 1\frac{1}{4}$ *$\frac{3}{4}$*

6. Mr. James bought $3\frac{1}{2}$ lb. of orange-blossom honey. Now he has $\frac{1}{4}$ of it left. How many pounds are left? $2\frac{1}{2}$ 3 [$1\frac{7}{8}$ lb.] *$1\frac{7}{8}$ lb.*

[W]

Now go back and write your work for each problem.

Teaching Page 158

Pupil's Objective. To choose an appropriate estimate for problems involving fractions.

Background. Notice that pupils are asked to tell why the estimate they selected is the best of the three following each problem. Oral discussion of this kind allows children to help one another while skills are being sharpened.

Teacher's Preparation. If possible, obtain a set of household scales on which pupils can weigh grapefruit, tangerines, apples, and the like.

Pre-book Lesson

1. Have pupils weigh several grapefruit and write the weights on the board. From these weights they may estimate an average and write that on the board. Then pupils may estimate the weight of a half dozen, a dozen, or a dozen and a half grapefruit. (Be sure that all pupils realize that these may be sold individually or by the pound.)

2. Using the above estimates, make up problems for oral discussion and board solution, such as:

a. If grapefruit cost $7\frac{1}{2}$¢ per pound, how much will 5 grapefruit cost?

b. If oranges cost $9\frac{1}{2}$¢ per pound, how much will $3\frac{1}{2}$ pounds cost?

Book Lesson. *Ex. 1–6:* Oral work. *Bottom of the lesson:* Written work.

Differentiations and Extensions. *More capable children* may find the cost of a fruit salad which might be served to a group of 30, using current market prices and estimating the amount of fruit needed for 30 servings. They may also compare the cost of using fresh oranges for juice with the cost of using condensed frozen juice.

Teaching Page 159

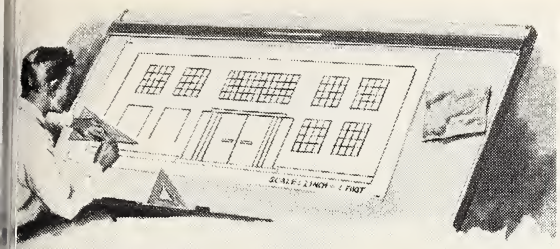
Pupil's Objectives. (*a*) To learn how to read and interpret a scale drawing; (*b*) to discover the value and efficiency of scale drawings.

Background. This lesson is the first of a group of lessons dealing with scale drawings and graphs. A firm understanding of the idea that a short line may stand for a long distance is necessary for the work on graphs which comes next in the text.

Note that in changing actual measurements to scale measurements, division is necessary, whereas changing from measurements read on a plan to actual measurements necessitates multiplication.

Teacher's Preparation. Be sure that you are able to provide yardsticks and rulers as needed.

Pre-book Lesson. Pupils may measure the length and width of the classroom. Discussion should follow as to how the measurement of the room can be expressed on paper. Out of this will come an understanding of the need for using a



Representing Real Things

Meaning of scale [O]

1. The drawing above shows the front of a building 38 ft. ng and 18 ft. high. The draftsman wanted his drawing to be .e same shape as the front of the real building, with every part :curately placed, so he used a **scale**—that is, he let 1 inch in s drawing stand for 1 foot in the real building. Then his :awing is 38 in. long and 18 in. high. We call this a **scale rawing** to the scale *1 in. = 1 ft.*

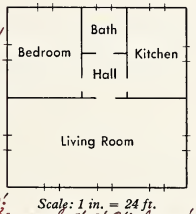
2. Suppose the draftsman had used the scale *1 in. = 6 ft.* hen, since every inch in the drawing would stand for 6 ft. in .e building, the drawing would be as many inches high as there :e 6's in 18, or 3 in.

3. How long would the drawing be if the draftsman used the :ale 1 in. = 6 ft.? (38 ÷ 6 =) 6⅓ in.

4. The floor plan at the right is also scale drawing. What is the scale? 1" = 24'

5. Find the dimensions of the living oom. HELPER. On the plan, it is in. wide and 1½ in. long. ¾ × 24 ft. = 18 ft., and 1½ × 24 ft. = 36 ft.

6. Tell how to find the real dimen- ions of the other rooms. (Bedroom, 15'x18'; Bath and Hall, each 9'x9'; Kitchen 12'x18')



Scale Drawing in Science

Using a scale [W]

Miss Pike's class looked in their encyclopedia for some scale drawings of well-known animals. They found this drawing of an elk to the scale $\frac{1}{60}$; that is, *1 in. = 60 in.*

1. In the drawing, the elk's shoulder height is 1 in. What is the true shoulder height? 60" How many feet? 5'

2. The class found other scale drawings of the deer family to the scale $\frac{1}{72}$. Explain this scale. 1 in. = 72 in.

3. If the scale drawing ($\frac{1}{72}$) of the white-tailed deer had a shoulder height of $\frac{1}{2}$ inch, what is the actual shoulder height? 36"

Find actual heights and lengths if scale drawings ($\frac{1}{72}$) show these measurements (Give your answers in feet and inches.)

	Height	Length
4. Black-tailed deer	½ in. 3'	1 in. 6'
5. Moose	1 in. 6'	1 7/16 in. 8' 7½"
6. Woodland caribou	¾ in. 4'6"	1 in. 6'

7. If the scale for the drawing of a whale is $\frac{1}{64}$ (that is, *1 in. = 64 in.*), how long will the actual measurement be for
   a. a right whale, shown with length 7½ in.? 40'
   b. a sperm whale, shown with length 10½ in.? 56'

8. Draw a line to stand for 60 ft. Scale: $\frac{1}{8}$ *in. = 10 ft.* (¾" line)

9. Draw a line to stand for 850 mi. Scale: $\frac{1}{4}$ *in. = 50 mi.* (3¼" line)

10. Make a scale drawing of a square which is 10 ft. on a side. Use the scale $\frac{1}{4}$ *in. = 1 ft.* Use the corners of a piece of paper to help you make square corners. (2½" on a side)

scale. See if the class can decide on a scale that might be used for this purpose.

Book Lesson. *Ex. 1–6:* Oral work. It may be helpful to reproduce the scale drawing in the book on the chalkboard by magnifying it many times (e.g., allowing 1 ft. to equal 24 ft.). Follow the questions in Ex. 1–4 in discussing the drawing.

Differentiations and Extensions. *More capable children* may draw a floor plan of their own homes, while *slower learners* draw floor plans of their own rooms.

Teaching Page 160

Pupil's Objective. To find the actual size of various animals by using scales.

Background. Now that pupils have learned the meaning of a scale, they use it in deciding the actual sizes of various objects drawn to scale. This is the kind of thing that pupils will often have to do in science and in working with encyclopedias or dictionaries. Several different scales are used, and for the first time pupils see a scale written as a fraction (e.g., 1 in. = 60 in. appears as $\frac{1}{60}$).

In Ex. 8–10 pupils make a few simple drawings to scale. In determining lengths for their drawings, they will have to divide first and then multiply a fraction.

Teacher's Preparation. If possible, borrow several stuffed animals or birds to provide motivation for the lesson. Pictures of the statues of the dinosaurs in Rapid City, South Dakota, will be very useful if obtainable. Also, provide a very simple toy, such as a sailboat, or a stuffed animal.

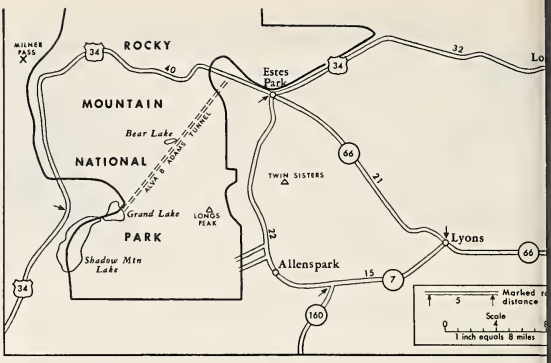
Pre-book Lesson

1. Show the class pictures of the dinosaurs at Rapid City, South Dakota, and ask how the sculptors would know how to make them the correct size. Guide the discussion so that the pupils will suggest that the sculptors can study actual skeletons in museums and then make scale drawings or scale models from which to work.

2. Using half-inch graph paper, have pupils make a half-size drawing of a very simple toy or stuffed animal. Measurements should be made first and listed on the board in inches. Lead pupils to see that by using one block on their graph paper for each inch of actual measurement they will produce a drawing to the scale $\frac{1}{2}$ in. = 1 in. Be sure pupils show the scale somewhere on their drawing. Ask them why this is necessary.

Book Lesson. *Ex. 1–10:* Written work. If necessary for *slower learners*, provide a thorough discussion of examples 1–3, making sure that they understand the meaning of the scales $\frac{1}{60}$ and $\frac{1}{72}$. Then assign Ex. 4–10 to these pupils for written work, giving individual help as needed.

Differentiations and Extensions. *More capable pupils* may make drawings to scale using models of stuffed birds, of a toy animal, or of a pet dog or cat. If desired, a papier-mâché model of the same article may be made in an art period. The same scale should be used as for the drawing.



On a Road Map

Using a scale

For Ex. 1–6, use road distances when given and measure other distances, using the scale (*1 in. = 8 mi.*).

1. In going from Loveland to Estes Park by route 34 and back over routes 66 and 87, about how far would you travel?

2. To Joe Lane, who lives 2 mi. west of Grand Lake, Long's Peak seems only 3 or 4 mi. away. How far is it?

3. Alva Adams tunnel, the world's longest irrigation tunnel, carries water from Grand Lake to a point near Estes Park. Measure to find about how many miles long the tunnel is.

4. The distance from Milner Pass to Long's Peak is about $2\frac{3}{8}$ in. on the map. About how many miles is that?

5. Which is the shorter road from Allenspark to Loveland, the road by way of Estes Park or the road by way of Lyons?

6. Many people start to climb Long's Peak from the nearest point on route 7. It is estimated that they walk 3 times as far as the straight-line distance. About how far do they walk?

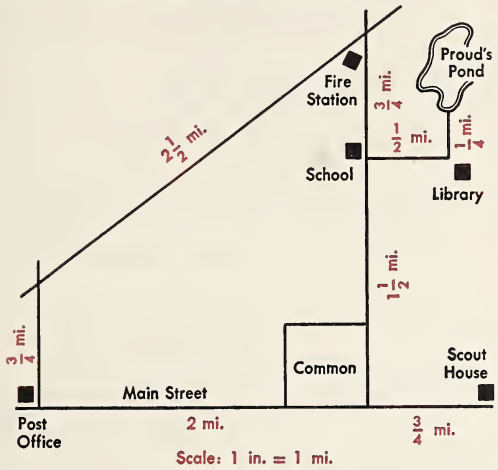
Teaching Page 161

Pupil's Objective. To learn how to find distance on a road map by using a scale.

Background. Another important use of scale is in reading maps. Although most pupils have probably had some familiarity with commercial maps, do not assume that they can use a scale properly in reading distances on a map. It probably will be best to start with the simple map given in the text and then progress to the more complicated commercial maps which the children might bring in for display. This lesson will also give you an opportunity to correlate arithmetic with social studies.

Teacher's Preparation. Have drawn on the board a map showing the school and local environs with symbols to locate important community buildings. This ought not to be any more complicated than the map shown on the next page. Pupils may name roads, buildings, and other landmarks.

The map may be drawn to scale, but since this is an introduction it may help if you have actual distances written on the map. (A town map is useful for obtaining accurate distances.)



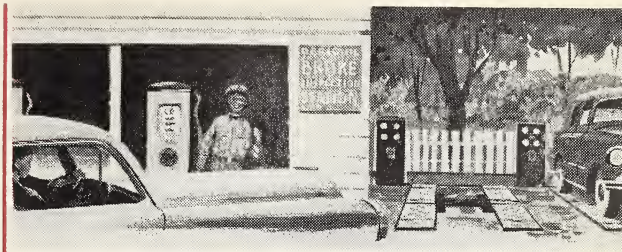
Pre-book Lesson. Have pupils label the roads and buildings on the map described above. The distances between places may be measured on the map and then the actual distances can be figured. (The scale used for such a map must depend upon the size of the area that is shown. One foot to one mile is good for a local neighborhood.)

Book Lesson. *Ex. 1–6:* Written work. Make sure that *all pupils* understand that the scale for the map is 1 in. to 8 mi. and that this means that the number of inches measured on the map must be multiplied by 8 to find the number of miles that are represented. Each child should realize that the small arrows designate the places between which the actual mileage is given on the map. Allow some pupils to read the distances between certain towns. Then have the same distances measured and the result multiplied by 8 to show that the map was really made to scale.

Differentiations and Extensions

1. *More capable children* may correlate this lesson with their social studies. A few pupils may use road maps in determining actual distances between cities. Others may use a flat wall map to determine air miles between various points.

2. *Slower learners* may draw to scale a map of a play area, such as a baseball diamond, or the like.



What Is a Table?

Making a table from tallies [O]

Jack likes to make records of things he does and sees. One day when he was working at the brake inspection station, he kept a record by tallies (marks) of the cars which came in, according to the year in which each was made.

1946	/
1947	
1948	/
1949	
1950	//
1951	////////
1952	//////
1953	////////
1954	//////////
1955	///////////////

Jack's tallies looked like those made at the left. Jack made one mark for each car opposite the year in which the car was made.

1. Opposite which year in the list did Jack make the most marks? 1955 How many did he make? 15

2. Opposite which of the years was there just one mark? 1946, 1948, 1947 no mark? 1949

1946	1
1947	0
1948	1
1949	0
1950	2
1951	8
1952	6
1953	8
1954	12
1955	15

In order to make the record easier to use, Jack made the **table** shown at the right above.

3. In the table, explain the numbers beside 1946 to 1950 which stand for the tallies in the tally table.

4. What numbers should Jack write for 1951 to 1955? see table.

162

Teaching Page 162

Pupil's Objective. To discover how to use tallies in making a table.

Background. In order to read graphs with any degree of understanding, pupils should know how they are constructed.

Usually the first step in making a graph is the recording of information. One method for doing this is shown on page 162 in the text. The next step is usually the conversion of the record, in this case the marks or tallies, into table form. In

the text both the talley sheet and the table are shown side by side so that pupils can note their differences and the advantages of each. Pupils should realize that sometimes the presentation of information in table form is sufficient and no further steps need be taken in refining the data.

Pre-book Lesson. Discuss efficient ways of keeping track of objects, scores, and the like. If the class does not offer the information that tally marks are useful for this purpose, ask if they have seen such marks used.

Book Lesson. *Ex. 1–4:* Oral work. Through oral discussion lead *all pupils* to understand that it is usually more accurate to make a mark for each item than it is to count and then record a total.

Differentiations and Extensions

1. Assign *slower learners* to tally scores in races or games that are played at recess or during physical education periods.

2. *More capable pupils* may create an original tally project. For example, if a boy has a poultry project at home, he can tally and record the number of eggs gathered daily.

Teaching Page 163

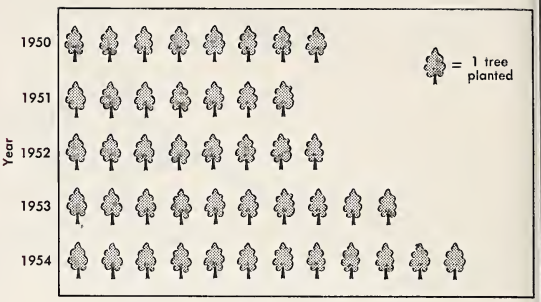
Pupil's Objective. To learn how to read and interpret a picture graph.

Background. The method of presenting information is usually determined by the use to which the information is to be put. If it has been decided that a talley sheet or a table does not display the facts clearly and concisely, a picture graph might be considered.

You will notice that a picture graph is very much like a talley sheet except that pictures instead of slash marks are used for each item. The picture graph, as at the top of page 163, also shows a scale or key which indicates the number of objects represented by one symbol. In the graph in the text, one picture of an object stands for one real object. This is the simplest kind of key and probably the best to use in introducing picture graphs. However, keys which show that one picture of an object stands for more than one real object are probably far more common. Such a key will be introduced and used by your pupils later (page 166 in the text).

Reading a Picture Graph

[O]



New Trees Planted by the Junior Conservation Club

Sometimes the information that we have recorded in a table can be shown by a picture, called a **picture graph.**

1. What is the name, or title, of this picture graph?

2. For what years does this graph give us some information? 1950-195

In order to read a graph, we must look for the **key** which explains how to read it.

3. From the key in the upper right corner of the graph, what do you find that the picture of 1 tree means? 1 tree planted

4. For the year 1950 the top line in the graph shows 8 trees. Then the number of new trees planted during 1950 by the Junior Conservation Club was 8.

5. Explain about the number of new trees planted by the Conservation Club during each of the years 1951 to 1954. (See graph.)

[W]

6. Write down two things you could show by picture graphs. Make up the key to be used for each graph.

163

Book Lesson. *Ex. 1–5:* Oral work. *Ex. 6:* Written work. Be sure that *all pupils* understand that the key and the title of a picture graph really explain what the graph is about. After discussing exercises 1–5, it might be well to have the pupils make a table showing the same information.

Differentiations and Extensions

1. *All pupils* may make a picture graph of the population of several streets in their neighborhood.

2. *More capable pupils* may make a picture graph as part of a social-studies activity. For example, these pupils might make a picture graph showing the population of their town, allowing one man to stand for 100, 1,000, or 10,000 people.

Making Picture Graphs

[W]

) show what is asked for in each exercise below, make a ›er table like the one on page 162. Then from the facts in table, make a picture graph to go with the table.

. The number of cents you have spent for candy each day g 1 week. Use a circle ◯ to stand for 1 cent.

. The number of glasses of milk you drank each day during /eek. Use a glass 🥛 to stand for 1 glass of milk.

. The number of children absent from your room each day week. Let a stick man 🧍 stand for 1 child.

. The number of runs scored in each World Series game ›ear by the team that won the Series. (You can get this nation from the *World Almanac*.) Use a baseball ⚾ to for 1 run.

Can You Do These?

[W]

/rite your work for these. Give your answers in best form.

$\frac{3}{4} + 5\frac{2}{8}$ 10 5/12 2. $1\frac{7}{8} + \frac{1}{6}$ 2 1/24 3. $2\frac{1}{2} - 1\frac{2}{8}$ 5/8 4. $5 - 1\frac{7}{8}$ 3 5/8

$\times 8$ 6 6. $1\frac{1}{2} \times \frac{4}{5}$ 1 1/5 7. $2\frac{3}{4} \times 1\frac{1}{4}$ 3 7/16 8. $6\frac{1}{2} \times 24$ 156

6)1,438 55,R8, or 55 1/3 10. 97)3,814 39,R31 11. 160×207 33,120

2. Write in words the number 17,204,583,906.

3. Write the figures which are in these places in the number x. 12: **a.** million's; 4 **b.** ten's; 0 **c.** thousand's; 3 **d.** billion's; 7 ındred million's. 2

4. Write a number that shows 203 thousands. 203,000; 6,203,081 etc.

5. What number means 26 hundreds and 5 ones? 2,605

6. In the number 576,103,825, 825 is in one's period.

So You Won't Forget!

[W]

1. Round to the nearest whole number: $7\frac{3}{8}$; 7 $2\frac{5}{9}$; 3 $3\frac{1}{4}$; 3 $3\frac{7}{10}$. (9 7/12)

2. To check the sum of $2\frac{3}{4}$, $5\frac{1}{2}$, and $1\frac{1}{3}$, what round numbers will you use? 3+6+1=10

3. On a city map, a park is $\frac{5}{8}$ in. long. The scale of the map is *1 in. = 1,000 ft.* How many feet long is the park? 625 ft.

4. If $\frac{1}{4}$ *in. = 1 ft.*, how long a line represents 23 ft.? 5 3/4"

5. Find the sum of 2 mi., $1\frac{7}{8}$ mi., and $2\frac{1}{2}$ mi. 6 3/8 miles

6. Show by a number line that $\frac{3}{4} \times 12 = 9$.

7. One day Mr. Lane's time on three jobs was: $2\frac{1}{3}$ hr., $3\frac{3}{4}$ hr., $2\frac{1}{6}$ hr. At $2 an hour, how much did he earn? $16.50

8. Mrs. Fine says her roast should be in the oven for $3\frac{1}{4}$ hr. She put it in at 10:50 A.M. It is now 12:05 P.M. How much longer should the roast remain in the oven? 2 hr.

9. Cheese weighs $\frac{1}{8}$ as much as the milk used to make it. 1,296 lb. of milk will make how many pounds of cheese? 162 lb.

10. In a store window, a sign said: "Potatoes, 6¢ a pound." What was the price of $12\frac{1}{2}$ lb. of potatoes? 75¢

11. What is the price of $5\frac{3}{4}$ lb. of beef if 1 lb. costs 90¢? $5.18

12. In $312 \div 18$, what is the final remainder? 6

13. Round to the nearest thousand:

a. 1,429,824 1,430,000 **b.** 854,175 854,000 **c.** 4,059 4,000 **d.** 2,952 3,000

14. Change: **a.** 14 ft. to inches; **b.** 75 ft. to yards; **c.** 15 qt. to gallons; **d.** 25 lb. to ounces. 168" 25 yd. 3 3/4 gal. 400 oz.

15. Change: **a.** $6\frac{1}{2}$ lb. to ounces; **b.** $5\frac{1}{3}$ yd. to feet; **c.** $9\frac{1}{2}$ bu. to pecks; **d.** $17\frac{3}{4}$ gal. to quarts. 104 oz. 16' 38 pk. 71 qt.

16. Write the number that has 406 in million's period, 137 in one's period, and 801 in thousand's period. 406,801,137

Teaching Pages 164 and 165

Pupil's Objectives. (*a*) To become proficient in making tables and picture graphs; (*b*) to review processes and techniques learned to date.

Background. Now that pupils have learned about tables and picture graphs, they are given practice in constructing both. In each case a symbol is suggested that can be used for the picture in the picture graph.

The work at the bottom of page 164 and all of page 165 is provided so that you can be sure that meanings, skills, and abilities are kept alive. From these pages you should be able to tell how well your pupils are retaining previous learnings.

Pre-book Lesson (page 164, top). Have pupils make a number table on the board to illustrate the number of children in each row or group in the classroom. From the number table a picture graph may be made using a stick figure to represent each child.

Book Lesson (page 164, top). *Ex. 1–4:* Written work. Make sure that each child realizes that he should make a number table like the one on the board before attempting to make a picture graph. It might be well to have each problem read aloud and to discuss the way in which each graph should be labeled. Give individual help to those pupils who have any difficulty.

Book Lesson (page 164, bottom). *Ex. 1–16:* Written work.

Book Lesson (page 165). *Ex. 1–16:* Written work. This is a written test to determine which processes or techniques will require reteaching. Therefore, each child must work independently except in the case of slow readers who should be given necessary help with the actual reading of the problems.

Teaching Pages 166, 167, and 168

Pupil's Objectives. (*a*) To learn how to read horizontal and vertical bar graphs; (*b*) to construct bar graphs for given situations; (*c*) to discover that bar graphs save time in reading and digesting statistical data.

Background. Another way of refining information for display is to make a bar graph. A horizontal bar graph is shown on page 166 and a vertical bar graph on page 167. It can be seen from the bar graph on page 166 that there is very little difference between it and a picture graph. One important difference is that a key is not given for a bar graph; it must be interpreted from the scale shown along the bottom of the graph. An important point to make in this connection is that the key for the graph on page 166 is such that one division stands for two tickets, whereas the key for the graph on page 167 is such that one division stands for ten inches. While discussing this consideration, you might be able to lead pupils to see that in the latter case there is more need for estimating in reading bars that fall within division spaces.

On page 168 in the text is a series of graphs which can be used in providing practice in reading both picture and bar graphs. This work can be supplemented also by using graphs that pupils have brought from home. Be selective, however, for some graphs are much too complicated for children in Grade 6 to attempt to read.

Teacher's Preparation. Collect picture and bar graphs from magazines, newspapers, and other sources. Also have half-inch graph paper on hand.

Pre-book Lesson (pages 166 and 167). Display picture graphs and bar graphs. Have the class tell what symbol is used in each picture graph. Discuss differences in appearances and names for picture and bar graphs.

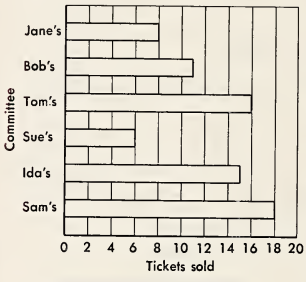
Book Lesson (pages 166 and 167). *Ex. 1–4:* Oral work. *Ex. 5 and 6.* Written work.

1. Have pupils read the table on the chalkboard before attempting to read the graph. Be sure that each pupil understands what each small space on the bar graph stands for. Have the spaces on each bar counted and compared with the number of tickets listed in the table.

Showing Records with Bars

Horizontal-bar graph [O]

The table on the board gives the number of tickets for the play sold by committees during one week. We could show this information by a picture graph, but there is an easier way.



Ticket Sales for One Week

At the left, the numbers in the table are shown by number lines, or bars. This picture is called a **bar graph.** The key for reading this graph is that *the distance shown by each of the numbered divisions stands for 2 tickets.*

1. Compare the lengths of the bars with the numbers in the table. (*Lengths represent numbers in table.*)

2. During another week, tickets sold by other committees were: Mary's 10, Joe's 14, Cora's 13, Helen's 17, Jack's 14. On the board, make a graph for these numbers like the one above, letting *each 4-inch space equal 2 tickets.* ★

These bars are horizontal, that is, they go across from left to right. This kind of graph is called a **horizontal-bar graph.**

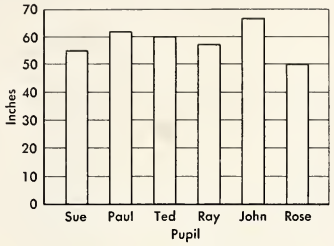
166

(★*See graph on page 183*)

Vertical-bar graph

The pupils in Sue's row made a number table and a graph of their heights in inches. They made their bars vertical (straight up and down). This kind of graph is called a **vertical-bar graph.** They used this key: *The distance shown by each small space equals 10 in.*

List of Heights in Sue's Row	
Sue	55 in.
Paul	62 in.
Ted	60 in.
Ray	57 in.
John	66 in.
Rose	50 in.



Heights of Pupils in Sue's Row

3. Are the bars the right heights? Yes Check from the table.

4. In a vertical-bar graph, which is the important thing—the width of a bar or the height? In Ex. 3, how did you check the heights of bars that did not end exactly on the scale lines? by estimating We often have to estimate in using graphs.

[W]

5. Make a vertical-bar graph for these heights in inches: Ted 62 in., Tom 68 in., Betty 58 in., Dot 60 in. Use the scale: *1 vertical space = 10 in.* Make the bars equal distances apart, so your graph will look neat. ★

6. Make a horizontal-bar graph to show these ages: Sam 11 yr., Jack 12 yr., Peter 10 yr., Ed 13 yr., Ray 14 yr. Use the scale: *1 horizontal space = 2 yr.* ★

See if you can find in a newspaper or a magazine either a horizontal-bar or a vertical-bar graph which you can explain. Bring it to class.

(★*See graphs at right*)

2. Discuss the meaning of the terms *horizontal* and *vertical* so that all pupils will realize the difference between a vertical- and a horizontal-bar graph.

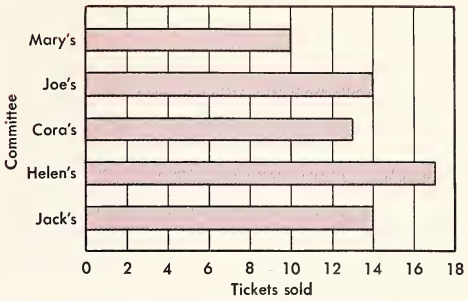
3. Give individual help to pupils who have difficulty in constructing the graphs for Ex. 5 and 6.

Book Lesson (page 168). *Ex. 1–4:* Oral work. Various pupils should identify each type of graph and then make complete explanations of the data shown.

(*Continued on page 184*)

★Answers Not on Reproduced Page 166

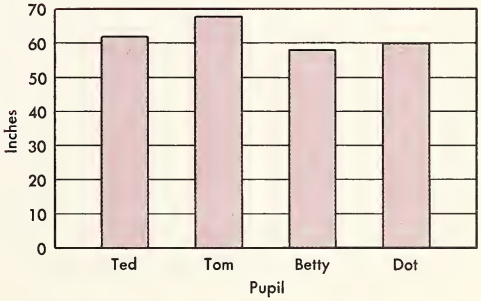
2.



Ticket Sales for One Week

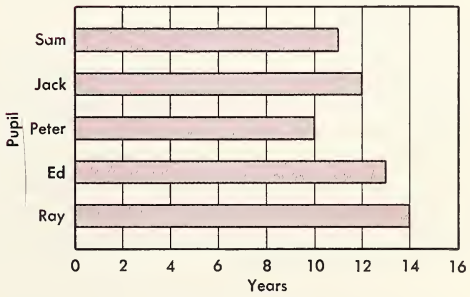
★Answers Not on Reproduced Page 167

5.



Heights of Pupils

6.



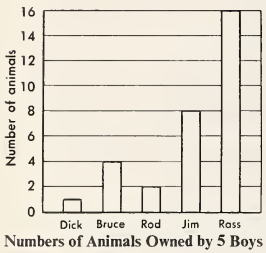
Ages of Pupils

Reading Picture and Bar Graphs

[O]

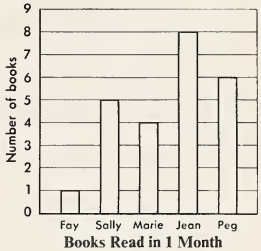
Tell the story shown by each of these graphs:

1.



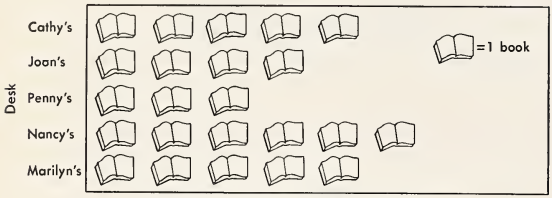
Numbers of Animals Owned by 5 Boys

2.



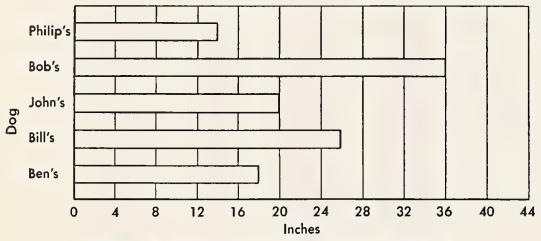
Books Read in 1 Month

3.



Books in the Desks of 5 Girls at School

4.



Lengths of Five Boys' Dogs from Nose to Tip of Tail

Differentiations and Extensions

1. *All pupils:*

a. May look through magazines and newspapers for examples of bar graphs. A bulletin board may be arranged from their findings or the graphs may be put into a scrapbook.

b. May make a graph showing the heights or weights of members of the class. The heights or weights may be put on the board. The girls may make a graph showing their heights or weights and the boys may do likewise.

2. *Slower pupils* may construct simple bar graphs to show such things as the number of races won by each team during the physical education period.

3. *More capable children* may make a graph showing the areas of the seven continents using the data below. They should first round the numbers to the nearest million. Then, allowing 1 space for each 1,000,000 square miles, they may construct a horizontal bar graph.

	Square Miles	Square Miles Rounded to Millions
Africa	11,500,000	12,000,000
Antarctica	6,000,000	6,000,000
Asia	17,000,000	17,000,000
Oceania	4,000,000	4,000,000
Europe	3,750,000	4,000,000
North America	8,300,000	8,000,000
South America	6,800,000	7,000,000

After they have completed the graph, pupils will profit from discussing its value. They may also make a graph showing the population of the seven continents. They may obtain these figures for themselves. This group will probably find it interesting to draw comparisons between the population and the area of each continent.

Teaching Page 169

Pupil's Objective. To experiment with different ways of multiplying fractions.

Background. The work on page 169, and other similar pages directed to "Young Experimenters," contains enrichment material. It may be that you will want to assign this work to *more capable children* only, or perhaps not assign it at all. Whatever the case, you know the needs of your pupils better than anyone else and so you are the one who should make the decision.

If you have been following the suggestions for previous lessons by proposing that your pupils attempt to find ways of working without paper and pencil, you will find that some of the ways of working shown in the boxes on page 169 are already known. If this is the case, you may want to write Jack's, Linda's, and Sam's examples on the board. Pupils may then tell, in their own words, different ways for working these examples.

Book Lesson. *Ex. 1–5:* Oral work. *Ex. 6–9:* Written work.

Differentiations and Extensions.

1. Some of your *more capable pupils* may want to apply the techniques of multiplying with fractions shown on this page to a variety of other examples.

Work for Young Experimenters

Enrichment material [O]

Have you ever been a Young Experimenter? Young Experimenters find different ways of working with numbers.

See if you can understand how these Experimenters multiplied with fractions by using ways they thought of themselves:

Jack $4 \times 5\frac{3}{4} = 23$

$4 \times 6 = 24$; $4 \times \frac{1}{4} = 1$; $24 - 1 = 23$. So $4 \times 5\frac{3}{4} = 23$.

1. When Jack multiplied 4×6, where did he get the multiplier, 6? *We rounded $5\frac{3}{4}$ to 6.*

2. Why did Jack then need to multiply the fraction $\frac{1}{4}$ by 4 and subtract that product from 24? *$5\frac{3}{4}$ is $\frac{1}{4}$ less than 6, so product will be $(4 \times \frac{1}{4})$ less than 24.*

Linda $6 \times 3\frac{1}{8} = 18\frac{3}{4}$

$6 \times 3 = 18$; $6 \times \frac{1}{8} = \frac{6}{8}$, or $\frac{3}{4}$; $18 + \frac{3}{4} = 18\frac{3}{4}$. So $6 \times 3\frac{1}{8} = 18\frac{3}{4}$.

3. Was Linda multiplying parts of the multiplicand? *yes* Explain how she worked. *She multiplied whole number first $(6 \times 3 = 18)$; then she multiplied fraction $(6 \times \frac{1}{8} = \frac{6}{8})$ and added the products.*

Sam $1\frac{7}{16} \times 14 = 20\frac{1}{8}$

$1\frac{1}{2} \times 14 = 1 \times 14$, or 14, plus $\frac{1}{2} \times 14$, or 7, $= 21$

$\frac{1}{\cancel{16}_{8}} \times \cancel{14}^{7} = \frac{7}{8}$; $21 - \frac{7}{8} = 20\frac{1}{8}$. So $1\frac{7}{16} \times 14 = 20\frac{1}{8}$.

4. What did Sam do to get his first multiplier, $1\frac{1}{2}$? *Rounded $\frac{7}{16}$ to nearest $\frac{1}{2}$, that is $1\frac{8}{16}$, or $1\frac{1}{2}$.*

5. Why did he have to multiply 14 by $\frac{1}{16}$ and subtract that product from the product of $1\frac{1}{2} \times 14$? *$\frac{7}{16}$ is $\frac{1}{16}$ less than $\frac{1}{2}$, so product will be $(\frac{1}{16} \times 14)$ less than 21.* [W]

Try to find products for these in different ways:

5 × $8\frac{2}{3}$ *$43\frac{1}{3}$* 7. $\frac{7}{8} \times 92$ *$80\frac{1}{2}$* 8. 65 × $3\frac{4}{5}$ *247* 9. 192 × $2\frac{15}{16}$ *564*

Do You Understand?

Test of Information and Meaning 4

1. In multiplying 4 by $\frac{1}{3}$ (that is, in $\frac{1}{3} \times 4$), can you get the answer by finding $\frac{1}{3}$ of 4? *Yes*

2. Copy $\frac{1}{2} \times 6 = 3$ and label the product, *3* the multiplier, *$\frac{1}{2}$* and the multiplicand. *6* Also label the factors of the product. *$\frac{1}{2}$ and 6*

3. When the numerator of a fraction is larger than the denominator, what kind of fraction is it? *Improper*

4. Find the answer for $4 \times \frac{7}{8}$ without multiplying. *$3\frac{1}{2}$* *(Either add $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$, or use a number line.)*

5. What are two ways of multiplying a mixed number by a whole number? Use $24 \times 3\frac{1}{8}$ to show the two ways. *Multiply parts, or change mixed number to improper fraction; 84*

6. In which of these three multiplication examples can n be found by adding? *C*

(a) $\frac{1}{3} \times \frac{1}{4} = n$ (b) $\frac{1}{3} \times 5\frac{1}{2} = n$ (c) $5 \times \frac{1}{3} = n$

7. How would you read $\frac{3}{4} \times 8$? *three fourths of eight*

8. What part of 1 yd. is $\frac{1}{3}$ of 2 yd.? *$\frac{2}{3}$*

9. Is $\frac{3}{5}$ equal to, greater than, or less than $\frac{1}{2}$?

10. Which of these are greater than 1: $\frac{3}{4}$? $\frac{15}{13}$? $\frac{7}{8}$? $\frac{9}{8}$?

11. $\frac{5}{8}$ is how many $\frac{1}{8}$'s? *5* Draw a picture to show your answer.

12. How many $\frac{1}{5}$'s are there in 1? *5* in $2\frac{2}{5}$? *12* in $\frac{4}{5}$? *4*

13. Round to the nearest whole number:

$1\frac{1}{2}$ *2* $7\frac{1}{5}$ *7* $4\frac{3}{4}$ *5* $17\frac{2}{3}$ *18* $25\frac{4}{5}$ *26*

14. Will the product be more or less than the multiplicand?

a. $1\frac{3}{4} \times 2$ *more* b. $\frac{7}{8} \times 46$ *less* c. $\frac{2}{3} \times \frac{7}{12}$ *less*

15. When the scale is $\frac{1}{4}$ *in.* = *6 ft.*, how long a line will be needed to represent 24 ft.? *1"*

16. In which direction do horizontal bars go? *across* vertical bars? *up and down*

17. What does the key on a graph do? *explains how to read the graph*

Throughout their work in arithmetic, they should be encouraged to devise methods of solution which appeal to them. Often they can work out ingenious short-cuts which should be regarded favorably as long as they understand the meanings involved.

2. Extra Practice Sets 56–70 are a series of sets containing multiplication examples involving fractions. You may want to let pupils leaf through these sets looking for examples which they think they can work either orally or by a technique different from that used for the standard algorism. See if you can stimulate pupils to think of different ways of working.

Teaching Pages 170, 171, 172, and 173

Pupil's Objectives. (*a*) To take the four regular end-of-chapter tests; (*b*) to obtain mixed practice with whole numbers and fractions; (*c*) to review the terms *multiplier* and *multiplicand* when used to refer to examples containing fractions; (*d*) to find *n* in a variety of addition, subtraction, and multiplication examples, some of which contain fractions and mixed numbers.

Background. You have given the end-of-the-chapter tests three times now. Refer to the teach-

Practice Exercises

[W]

For Ex. 1 to 5, do what the signs tell you to do.

	a	b	c	d	e
1.	560	385	738	3,005	5,800
	×97	×240	×895	×408	×4,500
	54,320	92,400	660,510	1,226,040	26,100,000
2.	876	609	400	846	6,200
	+958	+392	−93	−547	−5,818
	1,834	1,001	307	299	382
3.	82)416	29)206	16)286	37)924	48)1,208
	5, R6	7, R3	17, R14	24, R36	25, R8

4. $5\frac{1}{2} + 2\frac{3}{4}$ 8 1/4 $\quad 3\frac{1}{8} - 1\frac{1}{2}$ 5/8 $\quad 6 - 2\frac{7}{10}$ 3 3/10 $\quad 3\frac{4}{5} + 2\frac{7}{10}$ 6 1/2 $\quad 3\frac{1}{8} - 2\frac{5}{8}$ 1/2

5. $\frac{3}{4} \times \frac{8}{2}$ 1/2 $\quad \frac{5}{2} \times 6$ 15 $\quad 4 \times \frac{3}{8}$ 1/2 $\quad 4\frac{5}{8} \times \frac{2}{5}$ 17/20 $\quad 3\frac{1}{8} \times 5\frac{8}{3}$ 17/24

6. Write with words the numbers 16,042,830 and 76,095.

7. Write in figures: one hundred seven thousand seven; 107,007
four billion twenty-two million eight hundred five thousand. 4,022,805,000

Do You Make Mistakes?

Diagnostic Test 4

	Use cancellation when you can.	Study Pages	Practice Sets
1.	a $7 \times 2\frac{8}{8}$ 16 5/8 $\quad$ b $12 \times \frac{5}{8}$ 7 1/2 $\quad$ c $6 \times \frac{3}{16}$ 1/8 $\quad$ d $4 \times 2\frac{8}{3}$ 10 2/3	131, 132 134, 135	54, 55
2.	$1\frac{1}{4}$ ×5 6 2/4 $\quad$ $3\frac{2}{5}$ ×4 13 3/5 $\quad$ $6\frac{1}{4}$ ×8 50 $\quad$ $4\frac{2}{3}$ ×7 32 2/3	136	56
3.	$\frac{2}{5} \times 6$ 4 $\quad \frac{3}{4} \times 9$ 6 3/4 $\quad \frac{5}{8} \times 6$ 3 3/4 $\quad \frac{1}{5} \times 8$ 1/5	138, 139	57, 58
4.	$4\frac{1}{5} \times 3$ 12/25 $\quad 2\frac{3}{4} \times 5$ 13 3/4 $\quad 1\frac{1}{2} \times 3\frac{5}{6}$ 5 5/6 $\quad 2\frac{1}{2} \times \frac{4}{5}$ 2	147, 148	63–67
5.	$\frac{3}{4} \times \frac{3}{4}$ 9/16 $\quad \frac{7}{8} \times \frac{2}{5}$ 7/20 $\quad \frac{3}{8} \times 4\frac{1}{2}$ 11/16 $\quad \frac{7}{8} \times \frac{2}{3}$ 7/12	140–142 150	59–62 68–71
6.	Use drawings to solve Ex. a, b, and c. 1/2 a. $\frac{2}{3} \times \frac{1}{2}$ 1/3 $\quad$ b. $\frac{3}{4}$ of 2 1/2 $\quad$ c. $6 \times \frac{1}{4}$		

How Well Can You Figure?

Computation Test 4

	a	b	c	d	e
1.	400	853	3,052	197	308
	−192	−147	−2,164	203	476
	208	706	888	856	349
2.	186	973	4,239	980	436
	×24	×300	×1,080	+76	+270
	4,464	291,900	4,578,120	2,312	1,839
3.	64)232	26)221	48)2,432	36)2,936	18)7,344
	3, R40	8, R13	50, R32	81, R20	408

10. $2\frac{1}{4}$ +6 1/2 8 3/4 $\quad 4\frac{7}{8}$ +1 1/2 5 8/8 $\quad 5\frac{2}{3}$ +4 1/4 9 11/12 $\quad 7\frac{3}{8}$ −2 1/3 5 3/24 $\quad 6\frac{1}{5}$ −2 4/5 3 5/5

11. $4\frac{5}{8}$ +6 3/8 11 $\quad \frac{4}{5}$ +7 3/10 8/10 $\quad 3$ −1 2/3 1 3/3 $\quad 5\frac{1}{4}$ −2 2/3 2 7/12 $\quad 8\frac{1}{2}$ −7/10 7 7/5

12. $6\frac{3}{4}$ ×5 33 3/4 $\quad 9\frac{3}{8}$ ×6 56 4/8 $\quad 16\frac{5}{6}$ ×2 33 3/3 $\quad 5\frac{2}{3}$ ×7 39 2/3 $\quad 2\frac{1}{2}$ ×4 10

13. 25 ×3 2/5 85 $\quad$ 24 ×2 1/3 56 $\quad$ 28 ×5 3/4 161 $\quad$ 24 ×2 1/2 60 $\quad$ 400 ×1 3/4 700

14. $5 \times 2\frac{5}{8}$ 13 1/8 $\quad 4\frac{1}{2} \times 2\frac{3}{3}$ $\quad 6\frac{1}{4} \times 4$ 25 $\quad 7 \times 2\frac{2}{3}$ 18 2/3 $\quad 3 \times 2\frac{1}{4}$ 6

15. $\frac{3}{8} \times 1\frac{1}{2}$ 9/16 $\quad \frac{5}{6} \times \frac{3}{4}$ 5/8 $\quad 4\frac{2}{3} \times 1\frac{3}{5}$ 7 7/15 $\quad \frac{1}{8} \times \frac{2}{3}$ 1/12 $\quad \frac{2}{5} \times \frac{3}{4}$ 3/10

Written Practice

1. In $4 \times 1\frac{1}{2}$, which number tells the size of the equal groups? 1/2 Do we call it "multiplier" or "multiplicand"?

2. In $3\frac{1}{2} \times 8$, which number counts the equal groups? 3 1/2 Do we call it "multiplier" or "multiplicand"?

3. Find n in each of the following:

a. $24 \times n = 37$ 1 7/24 $\quad$ b. $43 \times n = 1{,}118$ 26 $\quad$ c. $n - 75 = 129$ 204

d. $4\frac{1}{2} \times 16 = n$ 72 $\quad$ e. $3\frac{1}{2} + n = 7\frac{3}{8}$ 3 7/8 $\quad$ f. $\frac{5}{16} \times \frac{7}{10} = n$ 35/32

ing suggestions given for the Chapter 1 tests on pages 61–63 in this Manual. You may find that some of these suggestions will supplement your own testing methods effectively.

Pre-Book Lesson. Remind pupils once again that the purpose of any test is to help them evaluate their achievement, to pin-point areas of difficulty, and to correlate the important facts, skills, and ideas contained in a specific unit of work.

Book Lesson (page 170). *Ex. 1–17:* Written work. Test of Information and Meaning 4.

Book Lesson (top of page 171). *Ex. 1–7:* Written work. Notice that in addition to the practice in computation, children receive practice in writing numbers in words and figures.

Book Lesson (bottom of page 171). *Rows 1–6:* Written work. Diagnostic Test 4. Practice sets with answers are reproduced on previous pages in the manual. For their location, refer to the heading **Practice** in the index for this Manual.

Book Lesson (top of page 172). *Rows 1–15:* Written work. Computation Test 4. If necessary, this test may be used for practice, since there is an alternate computation test on page 302 of this Manual.

Book Lesson (bottom of page 172). *Ex. 1–3:* Written work.

Can You Solve Problems?

Problem Test 4

1. For re-silvering small mirrors, a shop charges 1 cent per
are inch. Find the cost for a mirror $6\frac{1}{2}$ in. × $4\frac{1}{2}$ in. *30¢*

2. On every paper sold, James makes $3\frac{1}{4}$¢. How much does
earn by selling 16 papers? *52¢*

3. At an average speed of 50 miles an hour, how far will a
travel in $\frac{3}{4}$ hr.? *37½ mi.*

4. How much will 2 yd. of drapery material cost if the price
$3.75 per yard? *$7.50*

5. At $8\frac{1}{2}$¢ a pound what will be the cost of 4 lb. of dog food? *34¢*

6. At 40¢ a dozen what will $2\frac{1}{2}$ doz. oranges cost? *$1.00*

7. John made a door from 4 boards. Each board was $6\frac{5}{8}$ in.
le. If the boards were used vertically and touching each other,
w wide was the door? *26½ in., or 2 ft. 2½ in.*

8. How many square miles in a school district which is
$\frac{1}{2}$ mi. long and $8\frac{1}{2}$ mi. wide? *123½ sq. mi.*

9. In a month Nancy's little sister grew $\frac{3}{16}$ in. At that rate
w much would she grow in a year? *2¼ in.*

10. A store buys $\frac{1}{2}$ page of advertising space in a magazine
d allows each of its four departments $\frac{1}{4}$ of this space. What
rt of a page does each department get? *⅛*

11. Edna knits on her sweater $\frac{3}{4}$ hr. a day. How many hours
ll she knit in 30 days? *22½ hr.*

12. A certain kind of steel tubing weighs $\frac{3}{8}$ lb. to the foot.
ow much will $7\frac{1}{2}$ ft. weigh? *2 13/16 lb., or 2 lb. 13 oz.*

13. Three fourths of the 8,640 lb. of coal delivered to a home
s hauled in the morning. This was how many pounds? *6,480 lb.*

14. In a shipment of 72 cups, $\frac{1}{12}$ were broken. How many
ps were broken? *6*

Book Lesson (page 173). *Ex. 1–14:* Written work. Problem Test 4. You may want to group the slow readers and check each problem as they work it.

Differentiations and Extensions. Use the table below as an aid in entering the scores on the test record cards.

TABLE OF PER CENTS FOR CHAPTER 4 SCORES

Score	Per Cents for Problem Test 4
1	7
2	14
3	21
4	29
5	36
6	43
7	50
8	57
9	64
10	71
11	79
12	86
13	93
14	100

Score	Per Cents for Computation Test 4	Score	Per Cents for Computation Test 4
1	2	23	53
2	5	24	56
3	7	25	58
4	9	26	60
5	12	27	63
6	14	28	65
7	16	29	67
8	19	30	70
9	21	31	72
10	23	32	74
11	26	33	77
12	28	34	79
13	30	35	81
14	33	36	84
15	35	37	86
16	37	38	88
17	40	39	91
18	42	40	93
19	44	41	95
20	47	42	98
21	49	43	100
22	51		

Term Test

An optional term test for the first portion of Grade 6 is provided on Manual pages 304–306. It consists of three parts to test separately learning outcomes of different kinds: information and meanings taught during the first portion of the year; problem-solving ability; and computational skills. The test should probably be mimeographed so that each child may have his own set of questions. As they are set up, the three parts of this term test are intended to be given at three sittings.

Madison Public Schools

Making and using graphs offer unlimited possibilities for integrating arithmetic with other subject-matter areas.

Teaching Chapter 5 of Grade 6

Introduction

Just as Chapter 4 was devoted to introducing multiplication involving fractions, so, for the most part, Chapter 5 is devoted to introducing division of fractions. Except for a few pages in work with graphs, Chapter 5 is mostly new work. You will find, therefore, that it is interspersed with many pages designed to maintain previously-learned skills and abilities.

I. Learning Outcomes in Chapter 5

1. Understanding and ability in division involving fractions
2. Ability to estimate quotients in dividing fractions
3. Increased understanding and ability in work with tables and graphs
4. Ability to solve problems
5. Understanding of various specific concepts, generalizations, relationships, and skills
6. Desirable emotionalized responses (attitudes, appreciations, values)

Understanding and Ability in Division Involving Fractions (pages 175–179, 186–189, 193–197, 199)

Pupils are now ready for division involving fractions. Their work with whole numbers and fractions provides excellent preparation for this new work. As in multiplying fractions, there are several different kinds of examples in dividing fractions. These are summarized on page 199 in the text. The sequence for the presentation of these various types of examples is given in the outline below.

a. Whole number divided by a fraction
b. Whole number divided by a mixed number
c. Fraction divided by a whole number
d. Mixed number divided by a whole number
e. Fraction divided by a fraction, first with like-fractions; then with unlike-fractions
f. Fraction divided by a mixed number
g. Mixed number divided by a mixed number
h. Mixed number divided by a fraction

Pupils start off by dividing a whole number by a fraction or a mixed number using the common-denominator method and always obtaining a quotient less than the dividend. The transition from dividing whole numbers to dividing fractions is made quite easily in this way, since the common-denominator method of dividing emphasizes meaning and because pupils are accustomed to obtaining quotients smaller than dividends.

After this introduction to dividing fractions, pupils examine the idea of ratio. The interjection of this idea is exceedingly important at this time, since in the division of fractions, unlike the usual case in the division of whole numbers, a quotient may be either smaller or larger than the dividend.

Ratios often occur in comparison division in the following guise:

> Fran had $2 and Frank had $4. Use a fraction to compare the amount Fran had with the amount Frank had. Also compare the amount Frank had with the amount Fran had.

Using a situation such as this and the ratio idea just introduced, pupils receive much practice in telling whether a quotient is a how-many-times number or a what-part-of number. They are then ready to learn how to divide a fraction by a whole number.

Examples that require dividing a fraction by a whole number occur frequently in situations such as the following:

> A half-pound of candy is to be divided into 4 equal parts. How many pounds will there be in each equal part?

Since the divisor is larger than the dividend, it is obvious that the quotient must be a what-part-of number. Pupils are used to thinking of dividing a number into 4 equal parts by taking $\frac{1}{4}$ of the number (fractional-part division). This is a good place to introduce inversion, for pupils will be able to see quite easily how $\frac{1}{2} \div 4$ can be thought of as $\frac{1}{4}$ of $\frac{1}{2}$, or, reversing the factors, $\frac{1}{2} \times \frac{1}{4}$. This latter form is what would be obtained if the divisor had been inverted and the division sign changed to multiplication. (For another explanation of how inversion may be made meaningful, turn to pages 202–204 in this Manual and see the Background section for Teaching Pages 186 and 187.)

Once pupils have learned to divide a fraction by a whole number, they can divide a mixed number by a whole number simply by changing the mixed

number to an improper fraction. Of course, in this type of example the quotient may be either a what-part-of number or a how-many-times number, and pupils are shown why this is so and how the two kinds of examples occur in problem situations.

The next step in the development takes up dividing a fraction by a fraction. When the fractions are alike, the common-denominator method of dividing is probably the easier, since it is only necessary to divide the numerators and place this quotient over the denominator common to both the dividend and the divisor. Either method may be used when the fractions are not alike, but the inversion method is probably the quicker. Of course, the quotient may be either more or less than 1, and cancellation may frequently be involved when the inversion method is used.

Learning how to divide the remaining three types of examples (*f*, *g*, and *h* at the beginning of this section) should cause pupils no difficulty, since nothing new occurs. Mixed numbers should be changed to improper fractions when necessary, and the division done by inversion. Whenever cancellation is possible, it should be used.

Ability to Estimate Quotients in Dividing Fractions (pages 182–185, 191)

This topic has been given a special heading, since it is an important ability in practical affairs and in learning how to divide fractions. Actually, the basic skill — telling whether a quotient is a how-many-times or what-part-of number—has been discussed under the section above, for it is very much a part of ability to divide fractions.

There are, however, other techniques for estimating quotients in dividing fractions, such as telling the range within which a quotient must fall. Many of these techniques, some of which lead pupils to discover ways of working without pencil and paper, are discussed in relation to appropriate lessons in the teaching section of this chapter of the Manual. For example, see pages 196, 197, 206, and 215.

Increased Understanding and Ability in Working with Tables and Graphs (pages 208–211)

In this chapter pupils extend whatever ability they developed in working with tables and graphs in the previous chapter. For example, they learn how to make a line graph from a bar graph and are led to realize that the important function of a line graph is to show data that is subject to change. They also work with larger numbers in tables and in bar and line graphs. In the latter cases they learn how the rounding of larger numbers will increase the usefulness and readability of graphs.

Ability to Solve Problems (pages 180, 189, 190, 198, 200, 201, 204–206)

As new understandings, abilities, and skills are added, they are applied in problem-solving situations. At the same time, other aids and helpful activities are presented. For example, pupils use a number line as an aid in solving problems involving division of fractions. They also write original problems, and they differentiate multiplication and division problems involving fractions.

A very important summary of the three kinds of problems that can exist in multiplication and division also occurs in this chapter. The three types of problems are first labeled *Type I*, *Type II*, and *Type III* in work with whole numbers, and then in work with fractions. In a Type-I problem we find a *product* by multiplying; in a Type-II problem we find a *ratio factor* by dividing; and in a Type-III problem we find the *other factor* by dividing. Whenever a problem involves multiplication and division, one of these three types may be identified.

In the chart which summarizes the three types of problems for fractions on page 205 in the text, the letter *n* is used to represent a missing number. In this way emphasis may be placed on whether it is a product, a ratio factor, or the other factor that is to be found. Immediately after this summary, pupils are asked to tell whether problems are Type I, Type II, or Type III.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

how-many-times number
invert, inversion
line graph
ratio
three kinds of problems (Type I, Type II, Type III)
what-part-of number

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 5 will be found on the following pages in the text, and on the corresponding pages in this Manual: pages 178, 179, 183, 184, 185, 187, 199, 203, 205.

3. *Skills beyond those already listed:* Ability to find n whenever it represents a missing factor (pages 202, 203).

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

You are now quite familiar with this outcome, but are you doing all you can about it? Remember that without this outcome there is little point to the others. Refer to the suggestions given under this heading for Chapter 4 to help you decide what must be done in the future in this area.

5

Dividing a Whole Number by a Fraction

Meaning [O]

1. When the clerk opened one of the 4 boxes of straws, Jane asked, "How long will those 4 boxes of straws last?"

"We use about $\frac{2}{3}$ of a box a day," said the clerk. "You figure it out."

2. Jack said, "The first day you use $\frac{2}{3}$ of a box, the second day $\frac{2}{3}$ of a box, and so on. The straws will last as many days as the number of times you can take $\frac{2}{3}$ out of 4."

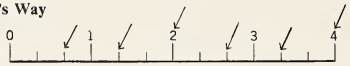
On a scrap of paper, Jack subtracted as at the right. How many times did he take $\frac{2}{3}$ from 4? 6 How many $\frac{2}{3}$'s are there in 4? 6

Jack's Way

4
$-\frac{2}{3}$ (1)
$3\frac{1}{3}$
$-\frac{2}{3}$ (2)
$2\frac{2}{3}$
$-\frac{2}{3}$ (3)
2
$-\frac{2}{3}$ (4)
$1\frac{1}{3}$
$-\frac{2}{3}$ (5)
$\frac{2}{3}$
$-\frac{2}{3}$ (6)
0

3. On the back of Jack's paper, Jane made a number line, as shown below. What size are the parts shown on this line? $\frac{1}{3}$ When Jane counted the $\frac{2}{3}$'s, how many did she find in 4? 6

Jane's Way



4. Did Jack and Jane get the same answers? Yes

5. How long would 8 boxes last? 12 days On the board, find the answer in two ways—by subtracting and by making a number line. [W]

6. By subtracting, find how many

a. $\frac{1}{2}$'s in 4; 8 b. $\frac{3}{5}$'s in 9; 15 c. $\frac{3}{4}$'s in 6; 8 d. $\frac{4}{5}$'s in 8.

7. By using number lines, find how many

a. $\frac{1}{5}$'s in 3; 15 b. $\frac{5}{8}$'s in 5; 8 c. $\frac{2}{5}$'s in 8; 20 d. $\frac{3}{8}$'s in 6.

8. For one of the special chocolate marshmallow sundaes, the clerk uses about $\frac{1}{6}$ quart of ice cream. How many of these sundaes can he make from a gallon of ice cream? 24 (Find the answer by subtracting.)

Teaching Pages 174 and 175

Pupil's Objective. To learn how to divide a whole number by a fraction by subtracting and by using a number line.

Background. A large proportion of Chapter 5 is devoted to the division of fractions. In the initial stages of this work, pupils are led to discover that dividing fractions has the same basic meaning as dividing whole numbers. In the box on page 175, they see how a fraction can be repeatedly subtracted in finding a quotient. They also see, using the number line given in the text, how it is possible to count the groups which stand

for the parts of a whole in order to derive a quotient. Repeated practice with a number line will clarify this concept.

Teacher's Preparation. Obtain 2 apples and 3 slices of bread, or other similar items that can be divided as suggested in the pre-book lesson.

Pre-book Lesson

1. Divide 2 apples in half so that pupils will see that there are four halves in two. Then divide 3 slices of bread into fourths and write on the board the notations shown below.

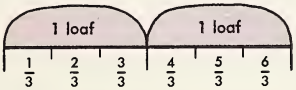
There are 4 halves in 2 — $2 \div \frac{1}{2} = 4$

There are 12 fourths in 3 — $3 \div \frac{1}{4} = 12$

2. Present a problem such as the following and ask pupils to find the answer by subtracting in a manner similar to that shown at the left of the problem:

$$\begin{array}{r} 2 \\ -\ \frac{1}{3}\ (1) \\ \hline 1\frac{2}{3} \\ -\ \frac{1}{3}\ (2) \\ \hline 1\frac{1}{3} \\ -\ \frac{1}{3}\ (3) \\ \hline 1 \\ -\ \frac{1}{3}\ (4) \\ \hline \frac{2}{3} \\ -\ \frac{1}{3}\ (5) \\ \hline \frac{1}{3} \\ -\ \frac{1}{3}\ (6) \\ \hline 0 \end{array}$$

Mrs. Smith buys 2 long loaves of bread for "submarine" sandwiches. Each sandwich takes $\frac{1}{3}$ of a loaf of bread. How many sandwiches can be made from 2 loaves?



3. Show the same solution on a number line, as above, and discuss the two ways of finding the answer. Pupils may discuss the relative merits of these methods.

Book Lesson. *Ex. 1–5:* Oral work. *Ex. 6–8:* Written work.

Differentiations and Extensions. Have pupils continue with examples of the kind given in Ex. 6 and 7 in the text. Quotients may be found by subtracting or by using a number line, but the written work should be recorded on the board. Ask pupils to examine the examples with quotients to see if they can make any discoveries concerning the size of the quotient in relation to the divisor and dividend. Some pupils may see immediately that in dividing a whole number by a proper fraction, the quotient is always a whole number larger than the dividend.

Dividing a Whole Number by a Fraction

Meaning; common-denominator method [O]

1. When Jack and Jane were finding the number of $\frac{2}{3}$ boxes in 4 boxes of straws, a quicker way to work would have been to divide by the fraction. We write the division example this way: "$4 \div \frac{2}{3} = ?$" This means "How many $\frac{2}{3}$'s are there in 4?"

In finding the number of $\frac{2}{3}$'s in 4, the work is easy if we change 4 to $\frac{1}{3}$'s. Why does 4 equal $\frac{12}{3}$? $4 = 4 \times 1 = 4 \times \frac{3}{3} = \frac{4 \times 3}{3} = \frac{12}{3}$

If $\frac{12}{3}$ is written "12 thirds" and $\frac{2}{3}$ is written "2 thirds," we can write, "$4 \div \frac{2}{3} = 12$ thirds $\div$ 2 thirds." Why do we know that 12 thirds $\div$ 2 thirds $= 6$? In 12 thirds there are 6 groups of 2 thirds each ($12 \div 2 = 6$).

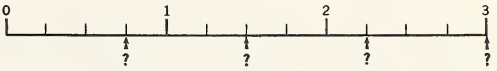
Now explain why $\frac{12}{3} \div \frac{2}{3} = 6$.

Changing both numbers before we divide so that they will have a common fractional unit, is called dividing by using a **common denominator.**

2. On the board find how many times the fraction $\frac{3}{4}$ is contained in the whole number 3

by subtracting $\frac{3}{4}$ from 3 as many times as you can; then (4 times)

by using a number line like the one below; and then (4 groups of $\frac{3}{4}$'s)



by dividing with the use of a common denominator.

$3 = \frac{12}{4}$. $\frac{12}{4} \div \frac{3}{4} = 12 \div 3$, or 4

3. Find on the board how many times the fraction $\frac{2}{3}$ is contained in the whole number 6 (9)

by subtracting $\frac{2}{3}$ from 6 as many times as you can; then

by using a number line; and then

by using a common denominator in a division example.

Teaching Pages 176 and 177

Pupil's Objective. To learn how to divide a whole number by a fraction by using the common-denominator method (changing the whole number to a fraction which has the same fractional unit as the divisor).

Background. Now that pupils have divided a whole number by a fraction by using a number line or by repeated subtraction, they are ready to learn how to use the common-denominator method in working with figures.

Essentially the common-denominator method is a way of changing both the dividend and divisor to

like-numbers (i.e., fractions having the same fractional unit, and, hence, the same denominator). This should not be a difficult concept, since pupils are used to finding common denominators in adding fractions. Once both numbers in a division example are like-numbers, the division may be done just as with whole numbers (i.e., 12 thirds ÷ 2 thirds = 6, just as 12 birds ÷ 2 birds = 6).

After pupils learn the common-denominator method of dividing, they will know three ways to find quotients. They may subtract repeatedly, use a number line, or use the common-denominator method. All three of these ways of working are presented in the text and used as a check on one another.

Teacher's Preparation. Provide four boxes containing a dozen pencils each.

Pre-book Lesson. Present a problem such as the following:

To replace worn-out pencils, Miss Brown gave members of her class $\frac{2}{3}$ of a box of new ones in one week. If her class needed $\frac{2}{3}$ of a box each week, how long would 4 boxes of pencils last?

Let pupils find the answer to this problem by counting out $\frac{2}{3}$ of the pencils in each box and then combining the remaining thirds to find out how many two-thirds there are in all in 4 boxes. Have other pupils solve the same problem on a number line. After both ways of solving have been clearly illustrated, tell the pupils that they are about to learn a much shorter way of doing the same work.

Book Lesson. *Ex. 1–6:* Oral work.

1. Be sure all pupils realize that the problem solved in the pre-book lesson is similar to the problem concerning the straws. Follow each step as outlined in Ex. 1. Make sure that everyone understands that $4 = 1\frac{2}{3}$. If some pupils do not seem to understand, take 4 circles that have been cut into thirds and put them together on a flannel board to show the 12 thirds that make 4 wholes.

2. Have several pupils explain why it is necessary to have common denominators in order to divide numerators. After Ex. 6, let pupils tell whether using repeated subtraction, a number line, or the common-denominator method for dividing fractions is the easiest.

Differentiations and Extensions. *All pupils* may attempt to solve the problems at the right. The first set is for the girls and the last set for the boys. The problems may be solved by subtraction, by division, or by using a number line.

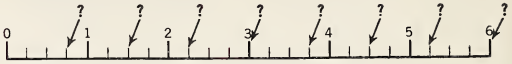
The Girl Scouts were having a sale to raise money for camp. Solve these problems about their sale:

a. The Girl Scouts made 9 lb. of fudge. They put $\frac{3}{4}$ of a pound in each box. How many boxes did they fill? (12)

b. The girls dressed dolls. Each dress took $\frac{3}{8}$ yd. of material. They bought 9 yd. How many dresses could they make? (24)

4. Peg wanted to make ribbon streamers $\frac{3}{4}$ yd. long for her bicycle handlebars. She had 6 yd. of ribbon. How many streamers $\frac{3}{4}$ yd. long could Peg get from 6 yd.?

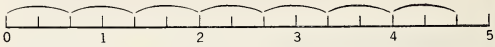
To find out, Peg made a number line like the one below. The line shows 6 yards, each divided into 4. Explain. Count to find how many streamers Peg could get. 8



Using the common-denominator method, divide 6 by $\frac{3}{4}$ on the board. Which do you think is easier, dividing this way or drawing a number line and counting $\frac{3}{4}$'s?

5. When we divide a whole number by a fraction, we do not always get a whole number for an answer. $5 \div \frac{2}{3} = 7\frac{1}{2}$

Look at the number line below. It shows 15 thirds in all. The 15 thirds are divided into groups of 2. There are 7 whole groups and 1 extra third. Since each group has 2 thirds, the 1 extra third is half of another group. Then the quotient for $5 \div \frac{2}{3}$ is $7\frac{1}{2}$.



To find the quotient by the common-denominator method, we work this way: $5 \div \frac{2}{3} = \frac{15}{3} \div \frac{2}{3}$, or $15 \div 2$, or $7\frac{1}{2}$

6. Tell in your own words how to divide a whole number by a fraction when you use a common denominator.

c. They had 3 yd. of ribbon to use in making bows for the dolls. Each bow took $\frac{1}{6}$ yd. How many bows could they make? (18)

d. The girls made bean bags. They used $\frac{1}{4}$ lb. of beans for each bag. How many bean bags could they make from 5 lb. of beans? (20)

The Boy Scouts were working for their bookbinding badge. Solve these problems about their project:

a. They had 5 yd. of stiff cotton to put under the binding. Each piece had to be $\frac{1}{4}$ yd. long. How many pieces could they get out of this strip? (20)

b. Each cover required $\frac{3}{8}$ yd. of book cloth. They had a strip of green book cloth which was 12 yd. long. How many covers could they bind from that piece? (32)

c. To sew the back of each book, $\frac{2}{3}$ yd. of thread was needed. Out of 18 yd. of thread, how many books could be sewed? (27)

d. They had 21 yd. of book cloth for the binding. Each binding would take $\frac{7}{8}$ yd. How many binding strips could they get out of 21 yd.? (24)

Teaching Page 178

Pupil's Objective. To obtain practice in dividing a whole number by a fraction, using the common-denominator method.

Background. In actually using the common-denominator method to divide a whole number by a fraction, the most important ability is the changing of the whole number to an improper fraction with the same denominator as the fraction that is the divisor. This should not be difficult for your pupils, since it is very much like changing a mixed number to an improper fraction and pupils have already been called upon many times to make this kind of change in multiplying fractions. Moreover, changing a whole number to an improper fraction has already been presented (see page 134 in the text) and so the work in Ex. 1 and 2 on page 178 represents review or reteaching.

Pre-book Lesson. If this page is not used on the same day as pages 176 and 177, one or two exercises from those pages should be reviewed on the board before the new work is attempted.

Book Lesson. *Ex. 1–4:* Oral work. *Rows 5–8:* Written work. If some pupils have difficulty in changing the whole numbers to improper fractions in rows 3 and 4, have a few examples done out on the board to show each step.

Using a Common Denominator in Dividing

Whole number ÷ fraction [O]

1. In order to use the common-denominator method in finding the quotient for $8 \div \frac{1}{2}$, the whole-number dividend 8 must be changed to a fraction with the same fractional unit as the fraction divisor has.

To change 8 to halves, *think*, "In 1 whole there are 2 halves, or $\frac{2}{2}$, so in 8 wholes there are $8 \times \frac{2}{2}$, or $\frac{16}{2}$."

2. Explain what was done in Ex. a–c to get the fraction equivalents. For Ex. d–o, give the missing numerators.

a.	$6 = \frac{18}{3}$	d.	$12 = \frac{48}{4}$	g.	$9 = \frac{54}{6}$	j.	$14 = \frac{56}{4}$	m.	$21 = \frac{168}{8}$
b.	$5 = \frac{50}{10}$	e.	$8 = \frac{32}{4}$	h.	$12 = \frac{36}{3}$	k.	$16 = \frac{80}{5}$	n.	$18 = \frac{54}{3}$
c.	$4 = \frac{32}{8}$	f.	$5 = \frac{40}{8}$	i.	$10 = \frac{50}{5}$	l.	$15 = \frac{45}{3}$	o.	$24 = \frac{120}{5}$

Explain how to change the whole number in each example so that you could divide by the common-denominator method.

	a	b	c	d	e
3.	$7 \div \frac{2}{3}$	$8 \div \frac{3}{4}$	$5 \div \frac{7}{8}$	$9 \div \frac{5}{6}$	$14 \div \frac{3}{4}$
4.	$4 \div \frac{3}{8}$	$24 \div \frac{3}{5}$	$18 \div \frac{2}{3}$	$21 \div \frac{5}{8}$	$16 \div \frac{4}{5}$

[W]

Using common denominators, divide in rows 5–8.

5.	$16 \div \frac{4}{5}$ 20	$18 \div \frac{5}{9}$ 25 $\frac{1}{5}$	$21 \div \frac{3}{4}$ 28	$45 \div \frac{5}{8}$ 72	$72 \div \frac{8}{9}$ 81
6.	$12 \div \frac{3}{8}$ 32	$24 \div \frac{8}{9}$ 27	$25 \div \frac{7}{8}$ 28 $\frac{4}{7}$	$54 \div \frac{9}{11}$ 66	$64 \div \frac{3}{8}$ 170 $\frac{2}{3}$
7.	$15 \div \frac{2}{9}$ 67 $\frac{1}{2}$	$28 \div \frac{4}{5}$ 35	$36 \div \frac{9}{10}$ 40	$28 \div \frac{7}{8}$ 32	$63 \div \frac{9}{10}$ 70
8.	$14 \div \frac{2}{5}$ 35	$32 \div \frac{4}{7}$ 56	$42 \div \frac{6}{7}$ 49	$12 \div \frac{5}{6}$ 14 $\frac{2}{5}$	$49 \div \frac{7}{12}$ 84

Extra Practice. For more practice work Sets 72 and 73.

To divide a whole number by a fraction, you can first change the whole number so that the dividend and the divisor will have a common denominator. Then divide the numerator of the dividend by the numerator of the divisor.

178

Differentiations and Extensions

1. For an example like $9 \div \frac{3}{4}$, use the following questions and activities in working with *slower learners*:

a. How many whole circles do we have? (9)

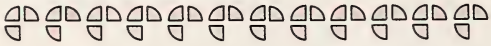
Ask a pupil to select 9 circles from a pile and place them on a flannel board.

b. How will we go about finding the number of $\frac{3}{4}$'s in 9? (Change each of the 9 circles into fourths)

c. Ask 9 pupils each to fold and cut a circle into quarters and replace the pieces on the flannel board so that the result is as follows:

d. How many groups of $\frac{3}{4}$ have we? (12)

Pupils may arrange the pieces on the flannel board in units of $\frac{3}{4}$ as shown below.



e. Do we have more groups than we started with? Why? (We made smaller groups so we have more of them.)

If all seem to understand the meaning of the above example, pupils may show on manila paper diagrams and the written work for the following examples:

$6 \div \frac{1}{3}$ $= \frac{18}{3} \div \frac{1}{3} = 18$

$2 \div \frac{1}{8}$ $= \frac{16}{8} \div \frac{1}{8} = 16$

$6 \div \frac{1}{4}$ $= \frac{24}{4} \div \frac{1}{4} = 24$

$6 \div \frac{2}{3}$ $= \frac{18}{3} \div \frac{2}{3} = 9$

2. *More capable pupils* may construct number lines and show the written work for the following examples:

$5 \div \frac{1}{4}$ $6 \div \frac{3}{8}$ $14 \div \frac{7}{10}$ $12 \div \frac{2}{3}$ $15 \div \frac{5}{6}$

3. It was suggested in the work involving multiplication of fractions that pupils be given an opportunity to estimate answers. Continue the same kind of program for division of fractions. For rows 3–8 in the text, pupils ought to be able to see that dividing a whole number by a number less than one will result in a quotient larger than the dividend. This understanding gives pupils a better feeling for the meaning of quotients and gives them a rough check of the accuracy of their computation. By this time, your pupils should be accustomed to using an estimate as a check on the actual computation.

4. Continue also the program designed to help children discover ways of working without pencil and paper in dividing fractions. This is usually quite difficult in dividing whole numbers by common fractions except in very special cases such as in dividing 9 by $\frac{1}{2}$, where it is only necessary to double the whole number since each whole in 9 contains two halves.

Sometimes exceptional children develop keen insight into useful relationships. For Ex. 4**c** in the text, one or two pupils might see that three groups of $\frac{2}{3}$ each make 2 wholes and that there are 9 groups of 2's in 18. Therefore, 9×3, or 27, gives the exact answer.

5. Assign **Extra Practice Sets 72 and 73** as needed.

Set 72. Dividing a whole number by a proper fraction; common-denominator method

Divide, using common denominators.

	a	b	c	d	e	f
1.	$12 \div \frac{1}{2}$ **24**	$8 \div \frac{3}{8}$ **$21\frac{1}{3}$**	$10 \div \frac{2}{5}$ **25**	$6 \div \frac{4}{7}$ **$10\frac{1}{2}$**	$20 \div \frac{2}{3}$ **30**	$19 \div \frac{1}{4}$ **76**
2.	$4 \div \frac{5}{8}$ **$6\frac{2}{5}$**	$14 \div \frac{5}{6}$ **$16\frac{4}{5}$**	$30 \div \frac{5}{7}$ **42**	$9 \div \frac{3}{5}$ **15**	$2 \div \frac{1}{6}$ **12**	$12 \div \frac{3}{4}$ **16**
3.	$16 \div \frac{4}{5}$ **20**	$25 \div \frac{7}{8}$ **$28\frac{4}{7}$**	$15 \div \frac{5}{9}$ **27**	$11 \div \frac{3}{7}$ **$25\frac{2}{3}$**	$18 \div \frac{2}{9}$ **81**	$13 \div \frac{2}{3}$ **$19\frac{1}{2}$**

Set 73. Dividing a whole number by a proper fraction; common-denominator method

Divide, using common denominators.

1.	$8 \div \frac{4}{5}$ **10**	$10 \div \frac{2}{3}$ **15**	$9 \div \frac{3}{8}$ **24**	$30 \div \frac{8}{9}$ **$33\frac{3}{4}$**	$12 \div \frac{1}{6}$ **72**	$6 \div \frac{2}{7}$ **21**
2.	$7 \div \frac{5}{9}$ **$12\frac{3}{5}$**	$6 \div \frac{7}{8}$ **$6\frac{6}{7}$**	$20 \div \frac{5}{7}$ **28**	$8 \div \frac{5}{8}$ **$12\frac{4}{5}$**	$2 \div \frac{7}{9}$ **$2\frac{4}{7}$**	$4 \div \frac{5}{6}$ **$4\frac{4}{5}$**

Dividing by a Mixed Number

Dividend a whole number [O]

1. If $1\frac{1}{2}$ oranges make a small glass of orange juice, how many glasses of juice will you get from 6 oranges?

What is the fractional unit for the oranges in the picture? $\frac{1}{2}$

Explain how to find the quotient from the picture. Count groups of $1\frac{1}{2}$ (4).

We can do the changing with figures to make the dividend and the divisor have the same fractional unit.

$6 = \frac{12}{2}$ $\quad 1\frac{1}{2} = \frac{3}{2}$ $\quad \frac{12}{2} \div \frac{3}{2} = 12 \div 3$, or 4.

Why is it easy to divide a whole number by a mixed number after you have learned to divide a whole number by a fraction? Same thing after mixed number is changed to improper fraction.

To divide a whole number by a mixed number, you can change the whole number and the mixed number to fractions with a common denominator. Then divide, using only the numerators.

[W]

Work the examples below by using common denominators.

a	b	c	d	e
$5 \div 2\frac{1}{2}$ 2	$9 \div 1\frac{1}{3}$ 6 $\frac{3}{4}$	$3 \div 1\frac{1}{2}$ 2	$8 \div 4\frac{1}{2}$ / $\frac{7}{9}$	$4 \div 1\frac{1}{3}$ 3
$10 \div 1\frac{1}{4}$ 8	$15 \div 7\frac{1}{2}$ 2	$5 \div 3\frac{1}{8}$ / $\frac{3}{5}$	$6 \div 1\frac{1}{3}$ 4 $\frac{1}{2}$	$8 \div 1\frac{1}{3}$ 6
$10 \div 8\frac{1}{3}$ / $\frac{1}{5}$	$12 \div 5\frac{1}{2}$ 2 $\frac{2}{11}$	$4 \div 2\frac{1}{3}$ / $\frac{5}{7}$	$3 \div 2\frac{1}{2}$ / $\frac{1}{5}$	$2 \div 1\frac{2}{3}$ / $\frac{1}{5}$
$8 \div 6\frac{2}{3}$ / $\frac{1}{5}$	$8 \div 5\frac{1}{7}$ / $\frac{5}{9}$	$7 \div 2\frac{2}{5}$ 2 $\frac{11}{12}$	$2 \div 1\frac{1}{3}$ / $\frac{1}{2}$	$12 \div 2\frac{2}{3}$ 4 $\frac{1}{2}$
$15 \div 4\frac{1}{2}$ 3 $\frac{1}{3}$	$9 \div 1\frac{3}{5}$ 5 $\frac{5}{8}$	$12 \div 1\frac{4}{5}$ 6 $\frac{2}{3}$	$8 \div 3\frac{1}{3}$ 2 $\frac{2}{5}$	$12 \div 1\frac{2}{3}$ 7 $\frac{1}{5}$

Check your quotients for row 2 by multiplying quotient and divisor.

Extra Practice. For more practice, work Set 74.

Teaching Page 179

Pupil's Objectives. (*a*) To learn how to divide a whole number by a fraction using the common-denominator method; (*b*) to check work in dividing fractions by multiplying quotient and divisor.

Background. Dividing a whole number by a mixed number is exactly the same as dividing a whole number by a fraction once the mixed number has been changed to an improper fraction. Pupils are led to make this discovery and the point is then made in a generalization.

Teacher's Preparation. Obtain a flannel board and prepare 6 orange circles for each pupil.

Book Lesson. *Ex. 1:* Oral work. *Rows 2–6:* Written work.

1. After reading the first problem, ask pupils to let their orange circles represent the six oranges. Then ask what they can do with their circles to show how many glasses of juice the six oranges would make using $1\frac{1}{2}$ oranges to each glass. They should see that some of the circles would have to be cut in two so that they could be arranged in groups of $1\frac{1}{2}$.

2. Have pupils study the picture at the top of the page in the text and compare it with the work they have done with the circles.

3. Draw from the class observations concerning the method for dividing a whole number by a mixed number. After the class has discovered the method, let them read together the rule which follows Ex. 1.

4. Have some examples from row 2 done on the board. After the computation, ask a pupil to label the dividend, divisor, and quotient. Then ask how such a division example could be checked.

Differentiations and Extensions

1. *More capable pupils* may solve the following problems:

a. A bus driver takes $2\frac{1}{4}$ hours to make a round trip from the city to the fair grounds. How many round trips can he make in 9 hours? (4)

Circle each subtrahend as you check your answer by subtraction.

b. Mr. Smith paid \$5 for $2\frac{1}{2}$ dozen roses. How much did each dozen cost? (\$2)

2. Let *all pupils* attempt to estimate quotients for the examples in rows 2–6 in the text.

For Ex. 2**a**, pupils ought to use a bracketing technique in estimating, thinking "$5 \div 2 = 2\frac{1}{2}$. $5 \div 3 = 1\frac{2}{3}$. The exact answer is somewhere between $1\frac{2}{3}$ and $2\frac{1}{2}$." For Ex. 2**b**, they should see that the answer will be *less than* $9 \div 1$.

3. Assign **Extra Practice Set 74** as needed.

Set 74. Dividing a whole number by a mixed number; common-denominator method

Divide, using common denominators.

1. a. $6 \div 2\frac{1}{2}$ $2\frac{2}{5}$ $\quad$ **b.** $2 \div 2\frac{2}{3}$ $\frac{3}{4}$ $\quad$ **c.** $3 \div 1\frac{1}{3}$ $2\frac{1}{4}$

d. $10 \div 2\frac{5}{8}$ $3\frac{17}{21}$ $\quad$ **e.** $4 \div 3\frac{3}{5}$ $1\frac{1}{9}$

2. a. $16 \div 9\frac{1}{3}$ $1\frac{5}{7}$ $\quad$ **b.** $6 \div 6\frac{6}{7}$ $\frac{7}{8}$ $\quad$ **c.** $18 \div 1\frac{2}{3}$ $10\frac{4}{5}$

d. $9 \div 6\frac{3}{4}$ $1\frac{1}{3}$ $\quad$ **e.** $7 \div 9\frac{1}{3}$ $\frac{3}{4}$

Teaching Pages 180 and 181

Pupil's Objectives. (*a*) To use the common-denominator method in solving problems involving a whole number and a mixed number or a fraction; (*b*) to review processes already learned.

Teacher's Preparation. Have on hand a piece of yarn 15 feet long, thirty-four circles to represent pennies, an aquarium or other container that holds just 9 quarts, and a $1\frac{1}{2}$ quart pail.

Pre-book Lesson (page 180)

1. Present problems such as the following:

a. We have a piece of red yarn 15 feet long which we will use in binding our notebooks. Each notebook will take a $1\frac{1}{4}$ foot piece. How many notebooks can we bind from this 15 foot piece? (12)

b. Mary paid 34¢ for some oranges. They were marked $8\frac{1}{2}$¢ per pound. How many pounds did she buy? (4)

c. How many pails of water each containing $1\frac{1}{2}$ quarts of water will be needed to fill a 9-quart aquarium? (6)

For problem *a*, pupils may cut $1\frac{1}{4}$-foot lengths of yarn from a piece 15 feet long.

For problem *b*, children may lay out 34 circles to represent pennies, divide them into groups of $8\frac{1}{2}$ (cutting circles in half when necessary); and then count groups as a way of finding the number of pounds bought.

For problem *c*, pupils should find the answer by using the $1\frac{1}{2}$ qt. pail as many times as necessary to fill the aquarium.

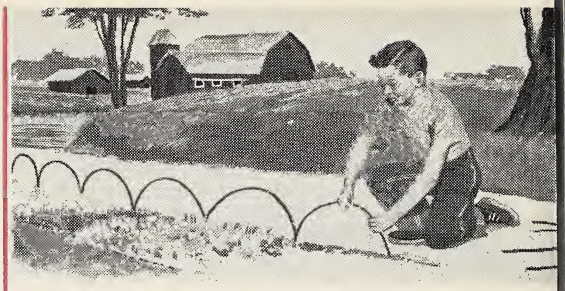
2. After working each of the problems above concretely, pupils should divide using figures. The written work may be placed on the board and compared with the concrete work.

Book Lesson (page 180). *Ex. 1–7:* Written work. If the pre-book lesson is worked out with objects and by division, your pupils should be ready to do this page independently. Remind them to draw diagrams whenever they need help.

Book Lesson (page 181). *Ex. 1–18:* Oral work. *Bottom of the page:* Written work.

1. As pupils give orally their estimated answers for Ex. 1–7, let other pupils tell why the estimates are or are not sensible.

2. *Slower learners* might be allowed to omit the last two examples in rows 1, 2, and 3.



Alan and Betsy on Grandfather's Farm

[W]

Use the common-denominator method in working these problems. Write all your work. If you need help with any problem, draw a diagram.

1. Grandfather asked Alan to make a fence along the front walk by setting up wire wickets. The walk was just 21 ft. long, and each of the wickets filled a space of $1\frac{1}{2}$ ft. How many wickets did Alan use in all? 14

2. Grandfather said that his power lawn mower would operate $2\frac{1}{2}$ hours on a gallon of gasoline. How many gallons of gasoline would it use in 24 hours of operation? $9\frac{3}{5}$

3. Alan found a piece of wire 9 feet long. How many $1\frac{1}{2}$-foot pieces can he cut from it? 6

4. Betsy picked 21 quarts of cherries in $3\frac{3}{4}$ hours. That was an average of how many quarts per hour? $5\frac{3}{5}$

5. How many roofing boards, just the right length and $\frac{1}{2}$ foot wide, should Grandfather order for a roof on a small lean-to shed which is 16 ft. wide? 32

6. Alan carried water in a pail holding $2\frac{1}{2}$ gallons. To fill a 40-gallon tub, how many pailfuls did he carry? 16

7. At the store Betsy paid 5¢ for a candy bar that weighed $1\frac{1}{3}$ ounces. How much did the candy cost per ounce? $3\frac{3}{4}$¢

NOTES

Just to Remind You!

[O]

Estimate answers for Ex. 1–7. ★

a	b	c	d	e	f
1,460	3,608	8,916	1,036	7,412	4,061
+1,812	+7,395	−703	−359	−3,385	−2,384
3,272	11,003	8,213	677	4,027	1,677
306	476	196	429	876	600
×18	×38	×205	×300	×893	×210
5,508	18,088	40,180	128,700	782,268	126,000
$2\frac{1}{4}$	$6\frac{1}{8}$	$7\frac{1}{2}$	$5\frac{3}{4}$	8	$4\frac{1}{2}$
$+3\frac{1}{2}$	$+1\frac{2}{3}$	$+\frac{3}{4}$	$-\frac{1}{2}$	$-1\frac{2}{5}$	$-2\frac{2}{3}$
$5\frac{3}{4}$	$7\frac{7}{24}$	$8\frac{1}{4}$	$5\frac{1}{2}$	$6\frac{3}{5}$	$1\frac{5}{6}$

7) 1,703 (243, R2) 5. 8) 6,000 (750) 6. 41) 1,264 (30, R34) 7. 72) 5,814 (80, R54)

8. How many square miles are there in a ranch that is 8 miles ng and 4 miles wide? 32 sq. mi.

9. In 12,638,507, what figure is in ten-million's place? 1

10. Which is larger, $\frac{5}{10}$ or $\frac{5}{11}$? $\frac{4}{5}$ or $\frac{3}{8} \times 2$?

11. What is a common denominator for thirds and fifths? 75

12. Which product will be larger?

) 3×567 or 12×567 (b) $\frac{1}{4} \times 768$ or $\frac{1}{8} \times 768$

13. If $6 \times 17 = 102$, how can you find what 12×17 is ithout multiplying by 12? Multiply 102 by 2. Divide 1008 by 2

14. From $14 \times 72 = 1{,}008$, tell how to find quickly 7×72.

15. In 73)4,073 what is the first partial dividend? 407 (tens)

16. If $\frac{3}{4} \times 3\frac{1}{3} = 2\frac{1}{2}$, what does **n** equal in $3\frac{1}{3} \times \frac{3}{4} = n$? $2\frac{1}{2}$

17. Will the product be more or less than the multiplicand?

$\frac{1}{7} \times 4{,}284$ less b. $5\frac{1}{2} \times 492$ more c. $\frac{7}{3} \times 84$ more

18. Which fractions can be reduced to better form?

$\frac{3}{6}$ $\frac{1}{2}$ $\frac{12}{14}$ $\frac{6}{7}$ $\frac{8}{12}$ $\frac{2}{3}$ $\frac{4}{16}$ $\frac{1}{4}$ $\frac{5}{9}$ $\frac{5}{25}$ $\frac{1}{5}$ $\frac{10}{15}$ $\frac{2}{3}$ $\frac{6}{9}$ $\frac{2}{3}$

[W]

Copy the examples in Ex. 1–7 and find the exact answers. (See above.)

Bob and Jim Go Fishing

Comparing by using a fraction; ratio [O]

1. Jim has a string of 7 fish. Bob's string has only 2 fish. How do you compare Jim's fish with Bob's fish?

You can subtract. Then you can say that Jim caught 5 more fish than Bob; or that Bob caught 5 fewer fish than Jim.

But you can compare also by using an improper fraction and a proper fraction.

As an improper fraction	As a proper fraction
$\frac{\text{Jim's } 7}{\text{Bob's } 2} = \frac{7}{2}$	$\frac{\text{Bob's } 2}{\text{Jim's } 7} = \frac{2}{7}$

When we use a fraction to compare two groups or numbers, we show relationship (ratio). For the fish, we say:

The ratio of Jim's 7 fish to Bob's 2 fish is $\frac{7}{2}$ (seven to two).
The ratio of Bob's 2 fish to Jim's 7 fish is $\frac{2}{7}$ (two to seven).

Tell the two ratios you would use in comparing the two groups in each of the following:

2. Bob's pole is 8 ft. long. Jim's is 12 ft. long. $\frac{8}{12}$; $\frac{12}{8}$

3. Jim lives 2 mi. from the lake. Bob lives 1 mi. from it. $\frac{2}{1}$; $\frac{1}{2}$

4. Bob brought 4 sandwiches. Jim brought 3 sandwiches. $\frac{4}{3}$; $\frac{3}{4}$

5. Bob had 23 worms to use for bait. Jim had 17 worms. $\frac{23}{17}$; $\frac{17}{23}$

★Answers Not on Reproduced Page 181

1. **a.** 1,000 + 2,000 = 3,000 **d.** 1,000 − 400 = 600
   b. 4,000 + 7,000 = 11,000 **e.** 7,400 − 3,400 = 4,000
   c. 8,900 − 700 = 8,200 **f.** 4,000 − 2,000 = 2,000
2. **a.** 20 × 300 = 6,000 **d.** 300 × 400 = 120,000
   b. 40 × 500 = 20,000 **e.** 900 × 900 = 810,000
   c. 200 × 200 = 40,000 **f.** 200 × 600 = 120,000
3. **a.** 2 + 4 = 6 **c.** 8 + 1 = 9 **e.** 8 − 1 = 7
   b. 6 + 2 = 8 **d.** 6 − 0 = 6 **f.** 5 − 3 = 2
4. Between 200 and 300
5. Between 700 and 800
6. About 30 (1,200 ÷ 40 = 30)
7. About 80 (5,600 ÷ 70 = 80)

Teaching Pages 182 and 183

Pupil's Objectives. (*a*) To learn that groups or numbers may be compared either by subtraction or by division; (*b*) to discover that a fraction may be used to express a comparison or a ratio (i.e., the relation between two groups or numbers).

Background. In Grade 5 pupils were introduced to the idea that a fraction may mean division; that comparisons made by division are best expressed as fractions; and that these fractions indicate relationships and are called ratios. In

Grade 6 the meaning of a fraction as division has already been reviewed and comparison by division will now be reviewed as a preface to introducing (pages 184 and 185) the idea that a quotient may be a how-many-times or a what-part-of number.

The ratio idea is a very important but often neglected idea. In working with scale drawings on pages 160 and 161 in the text, pupils encountered ratios, although they did not call them by this name. Since comparisons always involve two separate groups, the use of a fraction in this case is quite different from the meaning a fraction has when it refers to part of a whole or a group.

Teacher's Preparation. Obtain 15 or 20 marbles for use in the pre-book lesson.

Pre-book Lesson

1. Give one pupil 8 marbles and another pupil 3 marbles. Ask the class how the marbles held by the two pupils could be compared. Lead them to suggest that one pupil has 5 more than the other or that the second pupil has 5 less than the first. Continue to give other pairs of pupils different numbers of marbles, using the combinations 9 and 5, 7 and 4, and 5 and 2. Have other pupils compare the number of marbles held by each pair of pupils. Then give pupils marbles in these combinations: 6 and 3; 8 and 4; 9 and 3. Lead the class to express these comparisons as twice as many, half as many, three times as many, and one-third as many.

2. With a ruler draw a 4 ft. line on the board and directly beneath it draw a 2 ft. line. This will help pupils to see the relation of 2 ft. to 4 ft. and 4 ft. to 2 ft. Let pupils explain the comparisons.

Book Lesson. *Ex. 1–5:* Oral work. *Ex. 6–11:* Written work.

1. Let pupils study the picture; then have the first problem read aloud. Follow each step on page 182, writing on the board the ratios for Ex. 2–5.

2. Have the generalization on page 183 read aloud. Then write the word *ratio* on the board for the class to pronounce as a group.

3. Continue step by step on page 183, making sure that the pupils get the idea of comparing sizes of objects. If there is difficulty, have pupils draw diagrams or number lines to help them visualize the various relationships.

(*Continued on page 201*)

We often use a fraction to show the relation between two groups or numbers. This relation is called a ratio.

Many times the fraction which expresses a ratio is not in best form and should be reduced.

Look back at Ex. 2 on page 182. Tell again the two ratios you would use. To state the ratios in best form, we should reduce both fractions as in these boxes:

As a proper fraction:	As an improper fraction:
$\frac{\text{Bob's, 8 ft.}}{\text{Jim's, 12 ft.}} = \frac{8}{12}$, or $\frac{2}{3}$	$\frac{\text{Jim's, 12 ft.}}{\text{Bob's, 8 ft.}} = \frac{12}{8}$, or $\frac{3}{2}$, or $1\frac{1}{2}$

With the fractions reduced, it is easier to understand what the ratios mean when we compare this way:

Bob's pole is $\frac{2}{3}$ as long as Jim's pole.
Jim's pole is $1\frac{1}{2}$ times as long as Bob's pole.

[W]

Write in best form the two ratios by which you can compare the numbers in each of the following exercises. Then, as is done for the ratios above, write out the comparisons in sentence form.

6. A lake is 8 mi. wide and 10 mi. long. $\frac{4}{5}; 1\frac{1}{4}$
7. Jack walked 5 mi. Sam walked 10 mi. $\frac{1}{2}; 2$
8. Claire's story was 5 pages long. Hope's was 2 pages. $2\frac{1}{2}; \frac{2}{5}$
9. Line *a* is 2 in. long. Line *b* is 6 in. long. $\frac{1}{3}; 3$
10. Plain ice cream cost 15¢. A sundae cost 25¢. $\frac{3}{5}; 1\frac{2}{3}$
11. A factory makes 3 trucks for every 7 cars. $\frac{3}{7}; 2\frac{1}{3}$

HELPER. This is one way of saying that the ratio of the number of trucks made by this factory to the number of cars is $\frac{3}{7}$. Then the ratio of the number of cars to the number of trucks is $\frac{7}{3}$. The factory makes $\frac{3}{7}$ as many trucks as cars, and $2\frac{1}{3}$ times as many cars as trucks.

★Answers Not on Reproduced Page 183

6. The lake is $\frac{4}{5}$ as wide as it is long.
   The length of the lake is $1\frac{1}{4}$ times its width.
7. Jack walked $\frac{1}{2}$ as far as Sam walked.
   Sam walked 2 times as far as Jack walked.
8. Claire's story had $2\frac{1}{2}$ times as many pages as Hope's.
   Hope's story was $\frac{2}{5}$ as long as Claire's.
9. Line *a* is $\frac{1}{3}$ as long as line *b*.
   Line *b* is 3 times as long as line *a*.
10. Plain ice cream cost $\frac{3}{5}$ as much as a sundae.
    A sundae cost $1\frac{2}{3}$ times as much as plain ice cream.
11. The factory makes $\frac{3}{7}$ as many trucks as cars.
    The factory makes $2\frac{1}{3}$ times as many cars as trucks.

4. Assign exercises 6–11 for written work. Give individual help to *slower learners*, working the more difficult exercises on the board if they continue to have trouble.

Differentiations and Extensions

1. *All pupils* might help to measure the length and width of the classroom. They should round the lengths to the nearest foot, draw comparison lines to scale, and write the appropriate ratios in best form.

2. *More capable pupils* may find the area of the window space in the classroom and the area of the wall space. Then they may compare these areas by writing the correct ratios in best form.

Teaching Pages 184 and 185

Pupil's Objectives. (*a*) To learn how to compare whole numbers and fractions by division; (*b*) to learn how to tell whether a quotient will be a how-many-times number (a number larger than 1) or a what-part-of number (a number smaller than 1).

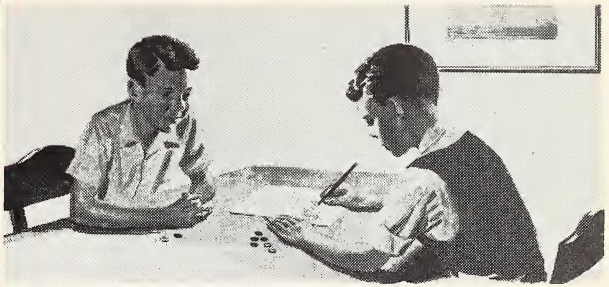
Background. Here pupils are reintroduced to the idea that a quotient may be a how-many-times number or a what-part-of number and that in both cases this may be indicated by a fraction called a ratio. This idea first appears in conjunction with whole numbers. Pupils learn that the improper fraction which shows a how-many-times number and the common fraction which shows a what-part-of number both indicate the special relationship which exists between the dividend and the divisor.

When the concept is extended to include division of fractions, *all pupils* will have a way of deciding whether quotients will be larger or smaller than 1. This will be a useful method of checking the reasonableness of their answers.

Book Lesson. *Ex. 1–12:* Oral work.

1. After discussing Ex. 1 according to the steps outlined in the text, have several pupils explain what is meant by a *how-many-times* number. In the same manner have a *what-part-of* number explained after discussing Ex. 2.

2. For the examples in rows 3–7, have pupils tell *why* the quotient will be a how-many-times number or a what-part-of number.



Comparison by Division

With whole numbers [O]

1. When Bill compared his 24¢ with Joe's 6¢, he wrote "$\frac{24}{6} = 4$." The quotient means that 24 is 4 times as large as 6. So when we divide to compare a larger number with a smaller ($24 \div 6$), the quotient is a **how-many-times number,** that is, a number larger than 1. $\frac{24}{6}$ means $24 \div 6$, or 4.

2. Joe compared his money with Bill's by writing "$\frac{6}{24} = \frac{1}{4}$." The $\frac{1}{4}$ means that 6 is $\frac{1}{4}$ of 24. When we divide to compare a smaller number with a larger number ($6 \div 24$), the quotient is a **what-part-of number,** that is, a number smaller than 1.

3. For $8 \div 8 = 1$, why is the 1 a how-many-times number? *It means that 8 is 1 times 8.*

Without dividing, tell for each example whether the quotient will be a how-many-times number or a what-part-of number.

	a	b	c	d	e
4.	$36 \div 4$	$5 \div 25$	$35 \div 7$	$8 \div 24$	$6 \div 18$
5.	$4 \div 16$	$42 \div 6$	$9 \div 36$	$7 \div 21$	$15 \div 5$
6.	$8 \div 32$	$40 \div 5$	$6 \div 24$	$9 \div 45$	$3 \div 27$
7.	$25 \div 4$	$31 \div 7$	$4 \div 12$	$29 \div 7$	$44 \div 6$

When you compare two numbers by division, the quotient answers one of two questions:
(1) The dividend is how many times the divisor?
(2) The dividend is what part of the divisor?

3. Have pupils explain the generalizations at the bottom of page 185. To prove that they understand, they should give examples to illustrate each generalization.

Differentiations and Extensions

1. *Slower learners* may continue telling whether quotients will be how-many-times numbers or what-part-of numbers. The following exercises may be used:

$8 \div \frac{1}{2} =$ $\quad \frac{1}{3} \div \frac{1}{15} =$ $\quad 4 \div 2\frac{1}{2} =$ $\quad 6\frac{2}{3} \div 8 =$
$\frac{5}{8} \div 4 =$ $\quad \frac{5}{6} \div \frac{1}{3} =$ $\quad 8\frac{1}{2} \div 4 =$ $\quad \frac{3}{8} \div 3\frac{1}{8} =$

The statement at the bottom of page 184 is always true, no matter whether you are comparing whole numbers, fractions, or mixed numbers. Although you do not yet know how to do all kinds of division with fractions, you can tell by looking at any example whether the quotient will be a how-many-times number or a what-part-of number. All you have to do is to look at the dividend and the divisor and decide which is larger.

8. Dick walks $\frac{1}{3}$ mi. to school, and Ray walks $\frac{4}{5}$ mi. To compare Dick's distance with Ray's, we can write the ratio $\frac{\frac{1}{3}}{\frac{4}{5}}$ or $\frac{1}{3} \div \frac{4}{5}$. Which is larger, $\frac{1}{3}$ or $\frac{4}{5}$? Will the quotient be a how-many-times number or a what-part-of number?

When we compare Ray's distance with Dick's, we can write $\frac{4}{5} \div \frac{1}{3}$. Then which kind of number will the quotient be? how-many-times

9. In the following examples, tell which quotients are how-many-times numbers and which are what-part-of numbers:

a. $\frac{1}{8} \div \frac{3}{5} = \frac{5}{24}$; **b.** $\frac{3}{4} \div \frac{7}{9} = \frac{27}{28}$; **c.** $2 \div \frac{1}{2} = 4$; **d.** $\frac{5}{8} \div \frac{2}{7} = 2\frac{3}{16}$.

For each example below, tell which is larger, the dividend or the divisor. Then, without dividing, tell whether the quotient will be a what-part-of number or a how-many-times number.

	a	b	c	d	e
10.	$6 \div \frac{3}{4}$	$\frac{1}{8} \div \frac{1}{8}$	$2\frac{1}{2} \div \frac{3}{4}$	$2 \div 5\frac{1}{2}$	$5\frac{5}{6} \div 4$
11.	$1\frac{1}{2} \div 4$	$\frac{7}{8} \div 5$	$\frac{2}{3} \div 1\frac{1}{2}$	$2\frac{1}{2} \div 8\frac{2}{3}$	$2 \div 1\frac{1}{2}$
12.	$2\frac{3}{4} \div 3\frac{1}{2}$	$6 \div 3\frac{1}{4}$	$7\frac{1}{2} \div 3$	$3\frac{3}{4} \div 5$	$\frac{2}{3} \div \frac{1}{4}$

When we compare two numbers by division:
If the dividend is larger than the divisor, the quotient is a how-many-times number (that is, a number larger than 1).
If the dividend is smaller than the divisor, the quotient is a what-part-of number (that is, a number smaller than 1).

2. *More capable pupils may:*

a. Solve problems such as the following:

Paul had a board 6 feet long. How many swing seats $1\frac{1}{2}$ feet long could he cut from it? (4)

There are about 150 calories in $2\frac{1}{2}$ apples. How many calories are there in one apple? (60)

b. Answer the following questions after solving the above problems:

In which example did you find the number of equal parts? (the first—measurement division)

In which problem did you find the size of a group? (the second—fractional-part division)

c. Write two original problems for each type of division (measurement and fractional-part division).

Teaching Pages 186 and 187

Pupil's Objective. To learn how to divide a fraction by a whole number by inverting the divisor and multiplying.

Background. Using the common-denominator method to divide a fraction by a whole number is the best way to show the meaning of division involving fractions. However, the best way to work with figures, since it is usually the quickest way, is the inversion method which pupils will be introduced to in this lesson.

A complete explanation of why the inversion method works might confuse pupils. Therefore, on page 186 in the text, division of a fraction by a whole number is related to fractional-part division. Pupils see in Ex. 1 that $\frac{3}{4} \div 6$ can be thought of as $\frac{1}{6}$ of $\frac{3}{4}$ (just as $8 \div 2$ may be thought of as $\frac{1}{2}$ of 8). Then, since factors may always be reversed in multiplication, pupils learn in Ex. 2 that $\frac{1}{6} \times \frac{3}{4}$ may be thought of as $\frac{3}{4} \times \frac{1}{6}$. Pupils are now at the point they would be if they used inversion in dividing $\frac{3}{4}$ by 6. They are then led to see that this inversion method would be a good short way of working.

Another way of explaining why inversion may be used in dividing fractions is as follows:

The example $\frac{1}{3} \div \frac{2}{5}$ can be written as a complex fraction in the form shown in box A below. This expression could be simplified by multiplying both the numerator and the denominator by some number that will make the denominator equal to 1. Multiplying both terms by $\frac{5}{2}$ will accomplish this purpose, as shown in box B.

A	B
$\frac{\frac{1}{3}}{\frac{2}{5}}$	$\frac{\frac{5}{2} \times \frac{1}{3}}{\frac{5}{2} \times \frac{2}{5}} = \frac{\frac{5}{2} \times \frac{1}{3}}{1} = \frac{5}{2} \times \frac{1}{3}$

If the factors of the expression obtained in box B are reversed, $\frac{1}{3} \times \frac{5}{2}$ results. Of course, this latter form is the one that would be obtained if the divisor had been inverted and the division sign changed to multiplication.

Pre-book Lesson. Use the following problem to illustrate the division of a fraction by a whole number:

John had a large bar of chocolate. He had eaten part of it before his friends came to play. He wanted to cut the $\frac{3}{4}$ bar that was left into 6 equal pieces for his friends and himself. What part of the bar of chocolate would each boy have?

A Busy Office

Fraction ÷ whole number; inversion [O]

1. Carl, the office boy, used $\frac{3}{4}$ pint of ink in filling 6 inkwells of the same size. What part of a pint did each inkwell hold?

Must we divide $\frac{3}{4}$ pt. into 6 equal parts? yes To find the answer, we can *think*, "$\frac{1}{6}$ of $\frac{3}{4}$ = ?" Then we can work as in box A. Explain the work. Each inkwell held $\frac{1}{8}$ pt.

A

$$\frac{3}{4} \div 6 = \frac{1}{6} \text{ of } \frac{3}{4} = \frac{1}{6} \times \frac{3}{4} = \frac{3}{24}, \text{ or } \frac{1}{8}$$

2. Now look again at the third step in box A. Could we write $\frac{3}{4} \times \frac{1}{6}$ instead of $\frac{1}{6} \times \frac{3}{4}$? yes Explain. Then we could write $\frac{3}{4} \div 6 = \frac{3}{4} \div \frac{6}{1} = \frac{3}{4} \times \frac{1}{6}$. Factors may be reversed in multiplication.

When we make the change from $\frac{3}{4} \div \frac{6}{1}$ to $\frac{3}{4} \times \frac{1}{6}$, what happens to the divisor, $\frac{6}{1}$? It is turned upside down. When we change the divisor from $\frac{6}{1}$ to $\frac{1}{6}$, we say that we **invert** the divisor. Any fraction can be inverted.

The short way to find the quotient for $\frac{3}{4} \div 6$ is to write

$$\frac{3}{4} \div 6 = \frac{3}{4} \times \frac{1}{6} = \frac{3}{24}, \text{ or } \frac{1}{8}.$$

Could you use cancellation to make the work even shorter? yes Explain.

Tell what multiplication example you would write for each of Ex. 3–7 after inverting the divisor.

3. $\frac{2}{3} \div 8$ $\frac{2}{3} \times \frac{1}{8}$ 4. $\frac{7}{8} \div 3$ $\frac{7}{8} \times \frac{1}{3}$ 5. $\frac{4}{5} \div 8$ $\frac{4}{5} \times \frac{1}{8}$ 6. $\frac{5}{8} \div 10$ $\frac{5}{8} \times \frac{1}{10}$ 7. $\frac{3}{4} \div 6$ $\frac{3}{4} \times \frac{1}{6}$

The method of inverting the divisor and multiplying (the *inversion method*) is such a good short cut that we use it in all kinds of examples in division with fractions. Study Ex. 8 to see how we use it when we divide to compare two numbers.

8. Miss Read spends about 5 hours a day typing. After typing for $\frac{5}{6}$ hr., she has spent what part of her 5 hr.?

Will the quotient be a what-part-of number or a how-many-times number? Explain. We *think*, "$\frac{5}{6}$ hr. is what part of 5 hr., or $\frac{5}{6} \div 5$ = ?" You must find what part $\frac{5}{6}$ is of 5.

Explain the work in box B. $\frac{5}{6}$ hr. is what part of 5 hr.? $\frac{1}{6}$

How is the division checked in box B? Is this the same kind of check that we use for division with whole numbers? yes

B

$$\frac{5}{6} \div 5 = \frac{5}{6} \div \frac{5}{1} = \frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$$

Check

$$\frac{1}{6} \times 5 = \frac{5}{6}$$

Quotient · Divisor · Dividend

To divide a fraction by a whole number, you can invert the divisor and then multiply.

[W]

Work problems 9 and 10 as in box B above.

9. Miss Read left a margin of $\frac{7}{8}$ in. at the bottom of a 14-inch page. The margin was what part of the total length? $\frac{1}{16}$

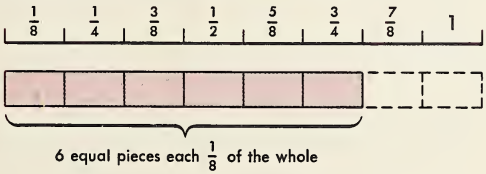
10. It is 2 mi. from Carl's house to the office. If he walks $\frac{4}{5}$ mi., what part of the 2 mi. has he walked? $\frac{2}{5}$

Divide by the inversion method. Check as in box B.

	a	b	c	d	e
11.	$\frac{9}{10} \div 3\frac{3}{10}$	$\frac{4}{7} \div 3\frac{4}{21}$	$\frac{3}{5} \div 4\frac{3}{20}$	$\frac{2}{5} \div 6\frac{1}{15}$	$\frac{2}{3} \div 2\frac{1}{3}$
12.	$\frac{2}{7} \div 4\frac{1}{14}$	$\frac{5}{6} \div 2\frac{5}{12}$	$\frac{3}{10} \div 6\frac{1}{20}$	$\frac{3}{4} \div 3\frac{1}{4}$	$\frac{1}{3} \div 2\frac{1}{6}$
13.	$\frac{1}{3} \div 4\frac{1}{12}$	$\frac{9}{16} \div 3\frac{3}{16}$	$\frac{7}{8} \div 5\frac{7}{40}$	$\frac{4}{5} \div 4\frac{1}{5}$	$\frac{5}{16} \div 3\frac{5}{48}$

Extra Practice. For more practice, work Sets 75–77.

Let pupils attempt to solve this problem by using a number line, as shown below.



Have the written form, $\frac{3}{4} \div 6 = \frac{1}{8}$; written on the board so that pupils can attempt to work the example using the common-denominator method. They may then turn to the text to discover how the inversion method would be used.

Book Lesson. *Ex. 1–8:* Oral work. *Ex. 9–13:* Written work.

1. Write Ex. 3–7 on the board with the multiplication examples that would be needed for solving. After the first paragraph on page 187 has been read aloud, write the term *inversion method* on the board. Have pupils explain what is meant by inversion and what is meant by the inversion method. Both explanations should be illustrated by an example already on the board.

2. After discussing the steps involved in Ex. 8, assign exercises 9–13 for written work. *Slower learners* may need help in setting up the correct multiplication examples.

Differentiations and Extensions

1. For rows 11 to 13, see if pupils first realize that all quotients will be what-part-of numbers smaller than the dividends. Then see if you can get some of your pupils to think in terms of fractional-part division for an example like 11**a**, for from the expression $\frac{1}{3}$ of $\frac{9}{10}$ they ought to be able to obtain the answer, $\frac{3}{10}$, as easily as they could derive the answer 3 tents for $\frac{1}{3}$ of 9 tents. In a similar manner, let pupils work out ways of finding quotients without pencil and paper for some of the other examples in rows 11 to 13.

2. *More capable pupils* may solve the following problems:

a. Mary spends 6 hours a week practicing on the violin. After she has practiced $\frac{2}{3}$ hr., she has spent what part of her 6 hours? ($\frac{1}{9}$)

b. Susan has a roll of ribbon which has $\frac{7}{8}$ yd. left on it. If she divides it into 7 pieces, what part of a yard will each piece be? ($\frac{1}{8}$)

c. Louise learned in cooking class that 16 tablespoons of butter weighed $\frac{1}{2}$ lb. What part of a pound will each tablespoon weigh? ($\frac{1}{32}$)

3. **Extra Practice Sets 75, 76, and 77** may be assigned as needed.

Set 75. Dividing a whole number by a mixed number; common-denominator method

Divide, using common denominators.

	a	b	c	d	e
1.	$8 \div 2\frac{2}{3}$ **3**	$10 \div 12\frac{1}{2}$ $\frac{4}{5}$	$9 \div 6\frac{3}{4}$ $1\frac{1}{3}$	$7 \div 18\frac{2}{3}$ $\frac{3}{8}$	$12 \div 5\frac{1}{4}$ $2\frac{2}{7}$
2.	$5 \div 10\frac{5}{8}$ $\frac{8}{17}$	$6 \div 13\frac{1}{3}$ $\frac{9}{20}$	$20 \div 5\frac{5}{7}$ $3\frac{1}{2}$	$4 \div 14\frac{2}{7}$ $\frac{7}{25}$	$16 \div 7\frac{1}{5}$ $2\frac{2}{9}$

Set 76. Dividing a fraction by a whole number; inversion method

Divide by inverting the divisor and multiplying.

	a	b	c	d	e	f
1.	$\frac{5}{6} \div 5$ $\frac{1}{6}$	$\frac{7}{8} \div 2$ $\frac{7}{16}$	$\frac{3}{4} \div 6$ $\frac{1}{8}$	$\frac{1}{8} \div 2$ $\frac{1}{16}$	$\frac{2}{3} \div 8$ $\frac{1}{12}$	$\frac{1}{2} \div 5$ $\frac{1}{10}$
2.	$\frac{3}{5} \div 9$ $\frac{1}{15}$	$\frac{4}{5} \div 3$ $\frac{4}{15}$	$\frac{5}{7} \div 4$ $\frac{5}{28}$	$\frac{2}{9} \div 8$ $\frac{1}{36}$	$\frac{5}{6} \div 9$ $\frac{5}{54}$	$\frac{3}{5} \div 7$ $\frac{3}{35}$
3.	$\frac{1}{3} \div 3$ $\frac{1}{9}$	$\frac{4}{9} \div 6$ $\frac{2}{27}$	$\frac{2}{5} \div 2$ $\frac{1}{5}$	$\frac{6}{7} \div 3$ $\frac{2}{7}$	$\frac{1}{5} \div 2$ $\frac{1}{10}$	$\frac{5}{9} \div 5$ $\frac{1}{9}$

Set 77. Dividing a fraction by a whole number; inversion method

Divide by inverting the divisor and multiplying.

1.	$\frac{7}{8} \div 7$ $\frac{1}{8}$	$\frac{3}{8} \div 9$ $\frac{1}{24}$	$\frac{3}{16} \div 6$ $\frac{1}{32}$	$\frac{5}{12} \div 2$ $\frac{5}{24}$	$\frac{1}{6} \div 3$ $\frac{1}{18}$	$\frac{7}{12} \div 2$ $\frac{7}{24}$
2.	$\frac{3}{10} \div 9$ $\frac{1}{30}$	$\frac{7}{9} \div 3$ $\frac{7}{27}$	$\frac{8}{9} \div 4$ $\frac{2}{9}$	$\frac{5}{8} \div 2$ $\frac{5}{16}$	$\frac{5}{16} \div 5$ $\frac{1}{16}$	$\frac{9}{10} \div 3$ $\frac{3}{10}$
3.	$\frac{2}{3} \div 4$ $\frac{1}{6}$	$\frac{3}{7} \div 6$ $\frac{1}{14}$	$\frac{7}{12} \div 3$ $\frac{7}{36}$	$\frac{1}{10} \div 3$ $\frac{1}{30}$	$\frac{8}{11} \div 4$ $\frac{2}{11}$	$\frac{1}{16} \div 2$ $\frac{1}{32}$

NOTES

Teaching Pages 188 and 189

Pupil's Objectives. (*a*) To learn how to divide a mixed number by a whole number; (*b*) to obtain further practice in identifying quotients according to whether they are how-many-times numbers or what-part-of numbers; (*c*) to solve problems in dividing a mixed number by a whole number.

Background. Pupils have already learned that a fraction can be used to show the relation between two groups or numbers and that this relation is called a ratio. The problem in Ex. 1 at the top of page 188 in the text involves ratios and offers an opportunity to introduce the dividing of a mixed number by a whole number in a practical situation that results in both a what-part-of and a how-many-times quotient.

As the explanation develops, pupils see in box A on page 188 how the two ratios may be written. They then tell why the quotient will be a how-many-times number for the first week and a what-part-of number for the second week. By this procedure they will be able to obtain a rough check of the correctness of the exact quotients.

In boxes B and C, pupils are led to discover that the mixed number must first be changed to an improper fraction. Then, calling on understandings developed in learning how to divide fractions, they explain how a mixed number may be divided by a whole number using the inversion method and cancellation. The boxes also show the written record for the work and the way of interpreting the how-many-times and what-part-of quotients for the problems that have been solved.

As the oral work continues at the top of page 189 in the text, pupils obtain further practice in identifying how-many-times and what-part-of quotients. Finally, for written work, they go back and work Ex. 2–13 using the inversion method first and the common-denominator method as a check of their work.

Teacher's Preparation. Have on hand a flannel board and three large circles to represent pies.

Pre-book Lesson. Have pupils use a flannel board and some large circles to solve the following problem:

Mother had $2\frac{1}{2}$ small pies in the house. If there were 10 people to be served, what part of a pie could Mother serve each one if she cuts pieces of equal size?

Dividing a Mixed Number

Mixed number ÷ whole number [O]

1. The first week of vacation, Sam worked in the garden $4\frac{1}{2}$ hr., and Ted worked 3 hr. The next week Sam worked $4\frac{1}{2}$ hr. again, but Ted worked 9 hr. Find the ratio of Sam's hours to Ted's for each week; that is, compare Sam's hours with Ted's.

Explain the way the two ratios are written in box A. Why will the first quotient be a how-many-times number, and the second a what-part-of number? $4\frac{1}{2}$ is more than 3. $4\frac{1}{2}$ is less than 9.

A

1st week: Sam / Ted $4\frac{1}{2}$ / 3 $= 4\frac{1}{2} \div 3 = 2\frac{1}{2}$, or $\frac{3}{2}$

2d week: Sam $4\frac{1}{2}$ / Ted 9 $= 4\frac{1}{2} \div 9 = \frac{1}{2}$

Before dividing in boxes B and C, what must you do to the mixed number $4\frac{1}{2}$? Change to an improper fraction. Then do you work just as you do when the dividend is a fraction? Yes Explain the work in the boxes.

B

1st week: How many times?

$4\frac{1}{2} \div 3 = ?$ $(4\frac{1}{2} = \frac{9}{2})$

$\frac{9}{2} \div 3 = \frac{\overset{3}{\cancel{9}}}{2} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{3}{2} = 1\frac{1}{2}$

Sam worked $1\frac{1}{2}$ times as long as Ted.

C

2d week: What part of?

$4\frac{1}{2} \div 9 = ?$ $(4\frac{1}{2} = \frac{9}{2})$

$\frac{9}{2} \div 9 = \frac{\overset{1}{\cancel{9}}}{2} \times \frac{1}{\underset{1}{\cancel{9}}} = \frac{1}{2}$

Sam worked $\frac{1}{2}$ as long as Ted.

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A pupil may cut one circle so that there will be $2\frac{1}{2}$ circles to place on the flannel board. Let the class decide, by inspection, just how the pies should be cut to make 10 equal pieces. After the pupils have recognized the fractional unit and cut the circles, write on the board $2\frac{1}{2} \div 10 = \frac{1}{4}$ pie. Then have them turn to the text to develop the steps involved in solving the example with figures.

Book Lesson (page 188 and top of page 189). *Ex. 1–13:* Oral work. *Bottom of the lesson:* Written work. After discussing each step involved in

Read Ex. 2–13 to show the meaning of each comparison. For Ex. 2, say, "$6\frac{1}{2}$ is how many times 2?" For Ex. 3, say "$1\frac{7}{8}$ is what part of 5?" (P)

2. $6\frac{1}{2} \div 2$ $3\frac{1}{4}$ 3. $1\frac{7}{8} \div 5$ $\frac{3}{8}$ 4. $3\frac{1}{4} \div 8$ $\frac{13}{32}$ 5. $2\frac{5}{8} \div 6$ $\frac{7}{16}$

6. $9\frac{1}{3} \div 4$ $2\frac{1}{3}$ 7. $5\frac{3}{4} \div 5$ $1\frac{3}{20}$ 8. $5\frac{4}{7} \div 13$ $\frac{3}{7}$ 9. $3\frac{1}{10} \div 10$ $\frac{31}{100}$

10. $2\frac{2}{5} \div 6$ $\frac{2}{5}$ 11. $4\frac{5}{16} \div 2$ $2\frac{5}{32}$ 12. $8\frac{1}{8} \div 15$ $\frac{13}{24}$ 13. $2\frac{3}{8} \div 5$ $\frac{6}{15}$ [W]

Now go back and work Ex. 2–13. Use the inversion method. To check your work in Ex. 2–5, divide again by the common-denominator method.

Extra Practice. For more practice, work Sets 78–80.

Problems in Dividing a Mixed Number

Divisor a whole number [W]

Write out your work for these problems:

1. Mary helped her mother label some glasses of jelly. Mary cut 5 labels of equal length from a strip of gummed paper $4\frac{3}{8}$ inches long. How long was each label? $\frac{7}{8}$"

2. Grandfather said that when he was a boy he used to buy licorice candy in strings $8\frac{1}{2}$ inches long and divide them with his brother. How long would half a string be? $4\frac{1}{4}$"

3. Mrs. Jeans wanted to serve 5 glasses of iced coffee. She had $1\frac{1}{4}$ cups of cream. If she divided the cream equally, how much cream did she put in each glass? $\frac{1}{4}$ c.

4. At a candy pull, Joan and Ruth pulled a piece of molasses taffy until it made a rope $8\frac{3}{4}$ inches long. They cut the rope into 10 equal pieces. How long was each piece? $\frac{7}{8}$"

5. In writing a report Miss Evans wants to make 5 columns of equal width in a space $6\frac{1}{2}$ inches wide. How wide can she make each column? $1\frac{3}{10}$"

solving Ex. 1, and after having pupils explain the work in boxes B and C, work out with figures the problem that was used in the pre-book lesson. Make sure that the pupils understand that the answer is a what-part-of number.

Book Lesson (bottom of page 189). *Ex. 1–5:* Written work. Before solving the problems individually, pupils might read each one to tell what the division example would be and what kind of number (how-many-times, or what-part-of) the answer would be. Individual attention should be given to any pupils who have difficulty in writing the examples or in solving. They may need a variety of experiences in this kind of work.

Differentiations and Extensions

1. *All pupils* may write a story that explains the answer for each of Ex. 2–5 at the top of page 189. For Ex. 2, the story should be: $6\frac{1}{2}$ is $3\frac{1}{4}$ times 2; therefore the quotient is a how-many-times number. For Ex. 3, the story should be: $3\frac{1}{2}$ is $\frac{7}{10}$ of 5; therefore the quotient is a what-part-of number.

2. As a continuation of the oral work, let pupils attempt to estimate quotients for Ex. 2 to 13 at the top of page 189 in the text. For Ex. 2 pupils should realize that they could round $6\frac{1}{2}$ to 7, obtaining an estimate of $3\frac{1}{2}$. Other pupils may see that they can consider this the upper limit of a range within which the quotient will fall, the lower limit being $6 \div 2$, or 3. For Ex. 3 they may estimate $\frac{2}{5}$ by rounding $1\frac{7}{8}$ to 2 and dividing by 5.

3. Let pupils continue exploring various ways of dividing fractions without using pencil and paper. For Ex. 2 at the top of page 189 some pupils will see right away that they can divide the whole number 6 by 2, obtaining a partial quotient of 3; and that they may then divide the $\frac{1}{2}$ by 2 obtaining a partial quotient of $\frac{1}{4}$. Combining the partial quotients will give the exact answer, $3\frac{1}{4}$. Some pupils may even be able to discover that for Ex. 13 the estimate $\frac{3}{5}$ would be too large by $\frac{1}{5}$ of $\frac{1}{3}$, or $\frac{1}{15}$. By subtracting $\frac{1}{15}$ from $\frac{3}{5}$, or $\frac{9}{15}$, pupils will obtain the exact answer, $\frac{8}{15}$.

4. *Slower learners* may solve these problems:

a. Tom walked $4\frac{1}{2}$ miles in 2 hours. What was his average speed per hour?

b. Mr. Smith used $5\frac{1}{4}$ gallons of oil every 3 days to heat his workshop. What was the average amount of oil he used per day?

c. A group of boys are going on a $4\frac{1}{2}$ mile hike. They plan to walk about 3 miles an hour. How long will it take them to complete the hike?

5. Assign **Extra Practice Sets 78, 79, and 80** as needed.

Set 78. Dividing a mixed number by a whole number; inversion method

Divide by inverting the divisor and multiplying.

1. a. $2\frac{1}{2} \div 2$ $1\frac{1}{4}$ **b.** $4\frac{3}{8} \div 10$ $\frac{7}{16}$ **c.** $5\frac{1}{3} \div 8$ $\frac{2}{3}$

d. $6\frac{2}{5} \div 16$ $\frac{2}{5}$ **e.** $9\frac{3}{5} \div 3$ $3\frac{1}{5}$

2. a. $1\frac{2}{3} \div 5$ $\frac{1}{3}$ **b.** $9\frac{1}{3} \div 4$ $2\frac{1}{3}$ **c.** $7\frac{1}{9} \div 16$ $\frac{4}{9}$

d. $8\frac{1}{4} \div 6$ $1\frac{3}{8}$ **e.** $4\frac{7}{8} \div 9$ $\frac{13}{24}$

3. a. $6\frac{1}{8} \div 7$ $\frac{7}{8}$ **b.** $7\frac{5}{7} \div 12$ $\frac{9}{14}$ **c.** $3\frac{3}{10} \div 9$ $\frac{11}{30}$

d. $2\frac{5}{8} \div 14$ $\frac{3}{16}$ **e.** $5\frac{5}{16} \div 2$ $2\frac{21}{32}$

Set 79. Dividing a mixed number by a whole number; inversion method

Divide by inverting the divisor and multiplying.

1. a. $8\frac{3}{4} \div 7$ $1\frac{1}{4}$ **b.** $15\frac{3}{8} \div 4$ $3\frac{27}{32}$ **c.** $6\frac{1}{2} \div 5$ $1\frac{3}{10}$

d. $9\frac{4}{5} \div 7$ $1\frac{2}{5}$ **e.** $14\frac{2}{5} \div 6$ $2\frac{2}{5}$

2. a. $16\frac{1}{4} \div 5$ $3\frac{1}{4}$ **b.** $7\frac{5}{9} \div 2$ $3\frac{7}{9}$ **c.** $10\frac{1}{8} \div 3$ $3\frac{3}{8}$

d. $12\frac{1}{2} \div 8$ $1\frac{9}{16}$ **e.** $6\frac{3}{7} \div 3$ $2\frac{1}{7}$

3. a. $7\frac{7}{8} \div 6$ $1\frac{5}{16}$ **b.** $5\frac{5}{6} \div 5$ $1\frac{1}{6}$ **c.** $8\frac{2}{3} \div 4$ $2\frac{1}{6}$

d. $3\frac{1}{7} \div 2$ $1\frac{4}{7}$ **e.** $4\frac{4}{9} \div 5$ $\frac{8}{9}$

Set 80. Dividing a mixed number by a whole number; inversion method

Divide by inverting the divisor and multiplying.

1. a. $3\frac{3}{4} \div 6$ $\frac{5}{8}$ **b.** $1\frac{5}{9} \div 8$ $\frac{7}{36}$ **c.** $4\frac{1}{2} \div 12$ $\frac{3}{8}$

d. $2\frac{2}{7} \div 4$ $\frac{4}{7}$ **e.** $5\frac{5}{8} \div 18$ $\frac{5}{16}$

2. a. $8\frac{1}{8} \div 15$ $\frac{13}{24}$ **b.** $3\frac{5}{6} \div 7$ $\frac{23}{42}$ **c.** $7\frac{7}{8} \div 9$ $\frac{7}{8}$

d. $6\frac{4}{5} \div 8$ $\frac{17}{20}$ **e.** $6\frac{2}{3} \div 10$ $\frac{2}{3}$

3. a. $4\frac{2}{5} \div 8$ $\frac{11}{20}$ **b.** $7\frac{1}{2} \div 5$ $1\frac{1}{2}$ **c.** $2\frac{2}{9} \div 4$ $\frac{5}{9}$

d. $1\frac{3}{7} \div 5$ $\frac{2}{7}$ **e.** $3\frac{1}{7} \div 6$ $\frac{11}{21}$

Problems in Division

[O]

Tell for each problem the division example to use.

1. What part of 5 qt. is $\frac{3}{4}$ of a quart? $\frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

2. At 90¢ a yard, what part of a yard of cloth can be bought for 60¢? $60 \div 90 = \frac{60}{90} = \frac{2}{3}$

3. It is 12 mi. to town and Mr. Hill has driven 8 mi. toward town. What part of the trip has he covered? $8 \div 12 = \frac{8}{12} = \frac{2}{3}$

4. What part of a pound (16 oz.) is 6 oz.? $6 \div 16 = \frac{6}{16} = \frac{3}{8}$

5. Mrs. Bundy has 10 in. more to knit on her afghan. If she knits $\frac{5}{8}$ in. an hour, how many hours will it take to finish it? $10 \div \frac{5}{8} = \frac{10}{1} \times \frac{8}{5} = \frac{80}{5} = 16$ hr.

6. Sam cut 15 pieces of equal length from 12 ft. of lumber. Find the length of each piece. $12 \div 15 = \frac{12}{15} = \frac{4}{5}$ of 9 $\frac{3}{5}$"

[W]

Now go back and solve problems 1–6.

Practice with Whole Numbers

[W]

Copy and work. Use R with any remainder in division.

1. 8,764	2. $869.61	3. 6,763	4. 4,352	5. $9.82
219	42.78	4,972	6,427	7.68
4,613	73.18	6,943	961	0.09
+723	+97.06	+307	+814	+0.56
14,319	$1082.63	18,985	12,554	$18.15
6. 817	7. 5284	8. 3,060	9. 6,059	10. $72.00
−469	×69	−891	×76	−48.75
348	364,596	2,169	460,484	$23.25
11. 716	12. 8,738	13. $5.49	14. 6,007	15. 2,786
×87	−4,965	×54	−428	×49
62,292	3,773	$296.46	5,579	136,514
16. 78)2,190 (28, R6)	17. 84)9,864 (117)	18. 69)5,082 (73, R45)	19. 25)$50.00 ($2.00)	
20. 64)5,136 (80, R16)	21. 56)2,252 (40, R12)	22. 46)3,496 (76)	23. 45)$31.50 ($0.70)	

Teaching Pages 190, 191, and 192

Pupil's Objectives. (*a*) To solve problems involving division of fractions; (*b*) to obtain practice in whole-number computation involving the four basic processes; (*c*) to maintain understandings and skills developed in working with fractions; (*d*) to discover a principle that can be used in multiplying a fraction by 1.

Book Lesson (top of page 190). *Ex. 1–6:* Oral work. *Bottom of lesson:* Written work. Have pupils tell what division example to use in each problem before attempting the written work. They might also tell whether they expect each answer to be a how-many-times or a what-part-of number.

Book Lesson (bottom of page 190). *Ex. 1–23:* Written work.

Book Lesson (top of page 191). *Ex. 1–13:* Written work. Before assigning the written work, it would be well to have pupils tell for rows 10–13 whether each quotient will be more or less than 1 and why.

Book Lesson (bottom of page 191). *Ex. 1 and 2:* Written work. After pupils have had an opportunity to study the work at the bottom of the page and to make a decision concerning an easy way to find the quotient when the dividend is 1 (when the dividend is 1, the quotient is the divisor inverted),

So You Won't Forget!

[W]

1. $3 \div \frac{3}{4} = 4$ Find the quotient by subtracting.

2. By using a number line divide 6 by $\frac{3}{4}$. 8

3. Divide $\frac{1}{4}$ by 3 by the inversion method. $\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

4. Divide 7 by $\frac{2}{3}$ by the common-denominator method. $7 \div \frac{2}{3} = \frac{21}{3} \div \frac{2}{3} = \frac{21}{2} = 10\frac{1}{2}$

5. Decide without dividing: Is the quotient more or less than 1 in

a. $5 \div \frac{5}{8}$? more b. $9 \div 10$? less c. $20 \div 18$? more d. $1 \div \frac{7}{8}$? more

6. Find what part a. 2 is of 6; $\frac{1}{3}$ b. $\frac{4}{5}$ is of 2. $\frac{2}{5}$

7. We check $8 \div 5 = 1\frac{3}{5}$ by writing "$1\frac{3}{5} \times 5 = 8$." Then for $9 \div 2 = 4\frac{1}{2}$, we write "$\frac{9}{2} \times 2 = 9$."

8. $12 \div \frac{3}{4} = 16$, so $16 \times \frac{3}{4} = 12$.

9. $\frac{1}{2} \div 6 = \frac{1}{12}$, so $\frac{1}{12} \times 6 = \frac{1}{2}$.

Work all the examples in rows 10–13 by inverting the divisor and then multiplying. Check to see if each quotient is reasonable by thinking, "Should the quotient be more than or less than 1?"

	a	b	c	d	e
10.	$5 \div 27\frac{5}{27}$	$\frac{5}{6} \div 9\frac{5}{54}$	$10 \div \frac{2}{9}45$	$1 \div \frac{2}{7}3\frac{1}{2}$	$7 \div 14\frac{1}{2}$
11.	$1 \div \frac{2}{3}1\frac{1}{2}$	$\frac{6}{7} \div 2\frac{3}{7}$	$5 \div \frac{2}{5}12\frac{1}{2}$	$130 \div 7\frac{4}{7}18$	$\frac{3}{5} \div 15\frac{1}{25}$
12.	$10 \div \frac{1}{6}60$	$1 \div 8\frac{1}{8}$	$2 \div \frac{1}{3}6$	$21 \div \frac{3}{4}28$	$18 \div \frac{2}{3}27$
13.	$\frac{4}{9} \div 2\frac{2}{9}$	$3 \div \frac{2}{3}4\frac{1}{2}$	$8 \div \frac{1}{8}64$	$\frac{7}{8} \div 7\frac{1}{8}$	$2 \div 10\frac{1}{5}$

When the Dividend Is 1

[W]

1. Study the exercises below. Then try to write an easy way for getting the quotient when the dividend is 1. Invert the divisor

a. $1 \div \frac{2}{3} = \frac{3}{2}$ b. $1 \div \frac{5}{4} = \frac{4}{5}$ c. $1 \div \frac{1}{5} = \frac{5}{1}$ d. $1 \div \frac{5}{1} = \frac{1}{5}$

2. Write just the quotients:

a. $1 \div \frac{4}{7} = \frac{7}{4}$, or $1\frac{3}{4}$ b. $1 \div 8 = \frac{1}{8}$ c. $1 \div \frac{3}{4} = \frac{4}{3}$, or $1\frac{1}{3}$

A Page of Practice

[O]

Give the sums without using pencil and paper.

	a	b	c	d	e	f
1.	22	54	26	45	63	38
	+7 = 29	+3 = 57	+4 = 30	+7 = 52	+9 = 72	+6 = 44
2.	19	47	28	36	17	98
	+8 = 27	+6 = 53	+5 = 33	+8 = 44	+7 = 24	+9 = 107

Copy and work. Use R with any remainder in division. [W]

3. 6,876 / 71,945 / 2,639 / 3,663 / 9,875 / +376 = 95,374

4. $76 \times 8{,}765 = 666{,}140$

5. $3{,}768 - 1{,}985 = 1{,}783$

6. $29{,}470 \div 76 = 387$, R 58

7. $5{,}200 - 647 = 4{,}553$

8. $5{,}207 - 4{,}958 = 249$

9. $486 \times 208 = 101{,}088$

10. $3{,}506 + 309 = 3{,}815$

11. $56\overline{)1{,}758} = 31$, R 22

12. $15\overline{)7{,}805} = 520$, R 5

13. $162\overline{)984} = 6$, R

	a	b	c	d	e
14.	$\frac{5}{6} + \frac{3}{8} = \frac{5}{24}$	$\frac{3}{10} + \frac{3}{4} = \frac{1}{20}\frac{1}{5}$	$-\frac{1}{8}\frac{1}{30}$	$\frac{7}{8} - \frac{5}{6}\frac{1}{24}$	$4\frac{3}{8} + \frac{2}{3}5\frac{1}{24}$
15.	$\frac{4}{9} + \frac{5}{8} = \frac{5}{18}$	$\frac{5}{8} - \frac{1}{12}\frac{13}{24}\frac{1}{4} + \frac{1}{6}\frac{5}{72}$		$\frac{7}{10} - \frac{4}{15}\frac{13}{30}$	$2 - \frac{7}{8} = \frac{1}{8}$
16.	$12\frac{7}{8} - 4\frac{3}{8}8\frac{1}{2}$	$5 - 1\frac{2}{3}3\frac{1}{3}$	$12\frac{1}{6} - 9\frac{2}{3}2\frac{1}{3}$	$8 - 4\frac{2}{3}3\frac{1}{3}$	$6\frac{7}{10} - \frac{9}{10}5$

17.	$9\frac{7}{8}$	$4\frac{2}{5}$	$8\frac{1}{6}$	$5\frac{9}{10}$	$2\frac{3}{8}$
	$8\frac{3}{10}$	$1\frac{1}{6}$	$6\frac{1}{9}$	$3\frac{3}{4}$	$5\frac{1}{2}$
	$+24\frac{5}{5} = 20\frac{32}{40}$	$+2\frac{9}{10} = 8\frac{7}{15}$	$+42\frac{3}{3} = 18\frac{17}{18}$	$+7\frac{2}{5} = 17\frac{2}{20}$	$+6\frac{3}{4} = 14\frac{3}{8}$
18.	$7 \times \frac{2}{3}4\frac{2}{3}$	$\frac{5}{6} \times 42\ 35$	$\frac{3}{8} \times \frac{2}{5}\frac{3}{20}$	$2\frac{2}{3} \times 1\frac{5}{8}4\frac{1}{3}$	$\frac{5}{6} \times 15\frac{3}{7}$
19.	$\frac{5}{6} \times 30\ 25$	$3\frac{2}{3} \times 15\ 55$	$3 \times \frac{1}{6}\frac{1}{2}$	$2 \times 3\frac{1}{3}6\frac{2}{3}$	$4\frac{1}{6} \times \frac{2}{5}1\frac{2}{3}$
20.	$6 \div \frac{3}{5}\ 10$	$27 \div 45\frac{3}{5}$	$1 \div \frac{5}{6}1\frac{1}{5}$	$\frac{8}{9} \div 4\frac{2}{9}$	$1\frac{4}{5} \div 7\frac{2}{15}$
21.	$4\frac{3}{8} \div 5\frac{7}{8}$	$10\frac{1}{2} \div 3\ 3\frac{1}{2}$	$\frac{7}{16} \div 2\frac{7}{32}$	$1\frac{7}{8} \div 5\frac{3}{8}$	$1\frac{2}{3} \div 10\frac{1}{6}$

22. $3\frac{1}{2}$ is what part of 14? $\frac{1}{4}$

23. 2 is how many times $\frac{3}{4}$? $2\frac{2}{3}$

24. $3\frac{1}{7}$ is what part of 11? $\frac{2}{7}$

25. 16 is how many times $2\frac{2}{3}$? 6

discuss the short cut so that you can be sure everyone understands it.

Book Lesson (page 192). *Rows 1 and 2:* Oral work. *Ex. 3–25:* Written work.

1. Assign examples 3–25 for written work. While the class is at work, call pupils to your desk one by one to do the oral work in rows 1 and 2. It would be helpful in planning the necessary reteaching to keep a check list of the kinds and number of errors made by individuals.

2. Since examples 3–25 provide for an evaluation of achievement, this page will provide a basis for diagnosing difficulties and for grouping pupils who are making similar types of errors.

Differentiations and Extensions

1. *All pupils* may make up other examples like those at the bottom of page 191. *Slower learners* might work at the board, while *more capable pupils* work on paper to solve these examples, first by inspection and then by the longer inversion process to prove that they are using the inspection method correctly.

2. *More capable pupils* may make up a set of problems involving many different kinds of measuring units, similar to those at the top of page 190 in the text. You may wish to save the best of these problems to use from time to time as review for the whole class in the use of denominate numbers.

3. *More capable pupils* may also fill in a cross-number puzzle similar to the one below or attempt to construct one like it.

	1. 3	2. 6	3. 5	
4. 1	6			5. 6
7		6. 1	4	4
2		6		0
7. 8		0		
	8. 4		9. 3	2

Across

1. 1 yr. = _?_ days
2. $\frac{1}{2}$ ft. = _?_ in.
3. $\frac{1}{12}$ min. = _?_ sec.
4. 1 lb . = _?_ oz.
5. 2 yd. = _?_ ft.
6. 1 sq. ft. = _?_ sq. in.
7. 1 pk. = _?_ qt.
8. 1 gal. = _?_ qt.
9. 1 bu. = _?_ qt.

Down

1. 1 yd. = _?_ in.
4. 1 cu. ft. = _?_ cu. in.
5. 1 sq. mi. = _?_ A.
6. 1 A. = _?_ sq. rd.

Teaching Pages 193, 194, and 195

Pupil's Objective. To learn how to divide a fraction by a fraction when the quotient is more than 1 and when it is less than 1.

Background. Pupils first encounter the necessity for dividing like-fractions in a problem situation (Ex. 1 at the top of page 193). They then use a ruler as a number line and count to find the number of eighths in 5 eighths. Next, they are led to discover that the quotient will be more than 1, since the dividend is larger than the divisor, and that it is easy to use the common-denominator method of dividing because $\frac{5}{8}$ and $\frac{1}{8}$ have the same fractional unit. If pupils do not see it themselves, it would probably be wise to point out that fractions with the same fractional unit are like-numbers which can be divided as easily as whole numbers which are alike (5 qt. ÷ 1 qt. = 5).

After pupils have explained the division for a problem that results in a quotient less than 1, they are asked to give orally quotients for several examples in which the fractional units and, therefore, the denominators, are alike. They will probably be pleased to see how easily this can be done using the common-denominator method of dividing.

Dividing a Fraction by a Fraction

Like-fractions; quotient more than 1 [O]

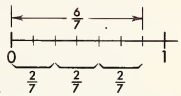
1. Joan's mother says that each of her candy dishes will hold about $\frac{1}{8}$ lb. How many dishes can Joan fill from $\frac{5}{8}$ lb. of mints? $\frac{5}{8} \div \frac{1}{8} = ?$ That is, how many $\frac{1}{8}$'s are there in $\frac{5}{8}$?

On your ruler find the $\frac{5}{8}$-inch mark. Count to find how many $\frac{1}{8}$'s there are in $\frac{5}{8}$. 5

In $\frac{5}{8} \div \frac{1}{8}$, is the dividend more or less than the divisor? Then will the quotient be more or less than 1?

Since $\frac{5}{8}$ and $\frac{1}{8}$ have the same fractional unit, why is it easy to use the common-denominator method? $\frac{5}{8} \div \frac{1}{8} = 5 \div 1 = 5$ only need to divide 5 by 1

2. $\frac{6}{7} \div \frac{2}{7} = 3$ In the diagram, count to find how many $\frac{2}{7}$'s there are in $\frac{6}{7}$. 3



Do $\frac{6}{7}$ and $\frac{2}{7}$ have the same fractional unit? Yes Does $\frac{6}{7} \div \frac{2}{7} = 6 \div 2$? Yes Why? both fractions have the same fractional unit

Then $\frac{6}{7} \div \frac{2}{7} = 3$

3. On the board, check your answers for Ex. 1 and 2 by using the inversion method.

Quotient less than 1

4. $\frac{3}{8}$ is what part of $\frac{5}{8}$? That is, $\frac{3}{8} \div \frac{5}{8} = \frac{3}{5}$

Will the quotient be more or less than 1? Why? Dividend is less than divisor.

$\frac{3}{8} \div \frac{5}{8} = 3 \div 5$, or $\frac{3}{5}$. Explain. Divide numerators when the fractional unit is the same.

5. Is the inversion method or the common-denominator method easier when the fractional unit is the same?

In rows 6 to 9, try to divide without pencil and paper.

	a	b	c	d	e
6.	$\frac{7}{8} \div \frac{1}{8}$ 7	$\frac{9}{10} \div \frac{1}{10}$ 9	$\frac{6}{7} \div \frac{1}{7}$ 6	$\frac{11}{4} \div \frac{1}{4}$ 11	$\frac{5}{3} \div \frac{1}{3}$ 5
7.	$\frac{8}{9} \div \frac{2}{9}$ 4	$\frac{9}{16} \div \frac{3}{16}$ 3	$\frac{1}{8} \div \frac{3}{8}$ $\frac{1}{3}$	$\frac{4}{9} \div \frac{8}{9}$ $\frac{1}{2}$	$\frac{2}{5} \div \frac{3}{5}$ $\frac{2}{3}$
8.	$\frac{2}{3} \div \frac{2}{3}$ 1	$\frac{4}{9} \div \frac{5}{9}$ $\frac{4}{5}$	$\frac{7}{12} \div \frac{5}{12}$ $\frac{2}{5}$	$\frac{1}{6} \div \frac{5}{6}$ $\frac{1}{5}$	$\frac{4}{5} \div \frac{4}{5}$ 1
9.	$\frac{1}{2} \div \frac{5}{25}$	$\frac{8}{3} \div \frac{2}{3}$ 4	$\frac{3}{20} \div \frac{9}{20}$ $\frac{1}{3}$	$\frac{25}{12} \div \frac{5}{12}$ 5	$\frac{2}{15} \div \frac{8}{15}$ $\frac{1}{4}$

On page 194 somewhat the same procedure is used in introducing the division of unlike-fractions when the quotient is more than 1. The need for this kind of division first appears in a problem situation, and pupils count on a number line to find the number of sixteenths in $\frac{3}{4}$. Then the written record for the inversion method is shown in the box, and pupils review the idea that the division can be checked by multiplying the divisor and the quotient. Finally, pupils are asked how they could use the common-denominator method. They should realize from the work in counting sixteenths on a number line that it is simply a matter of changing $\frac{3}{4}$ to sixteenths and dividing the numerators.

At the top of page 195, pupils use a diagram in discovering the meaning of dividing unlike-fractions when the quotient is a what-part-of number. The written work and the check is shown in the box on this page and pupils are asked to explain it fully.

Pre-book Lesson

1. As a prelude to the work on pages 193 and 194, introduce a problem similar to the one below but derived from a classroom activity.

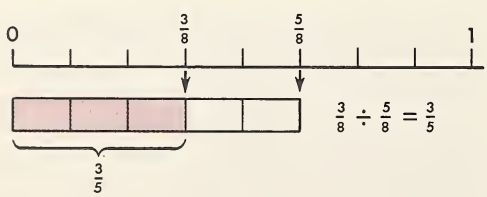
If lemonade is to be served in glasses which hold $\frac{1}{4}$ qt., how many glasses can be poured from $\frac{3}{4}$ qt. of lemonade? How many cups each holding $\frac{1}{8}$ qt. can be poured from $\frac{3}{4}$ qt. of lemonade?

Have pupils use real objects in demonstrating the solving of the problem. In this way, they will discover the meaning of dividing a fraction when the quotient is more than 1. As work with the objects proceeds, have the written work put on the board. Without further development, see if you can lead the children to tell how to use the common-denominator method for dividing a fraction by a fraction both when like- and unlike-fractions are involved.

2. Let *more capable children* turn to rows 6 and 7 at the bottom of page 193 and attempt to work the examples there without benefit of the development that precedes. Watch carefully for Ex. **7c-7e** to be sure these pupils can handle examples in which the quotient is less than 1.

Book Lesson (page 193). *Ex. 1–9:* Oral work.

1. Let pupils use a large cardboard ruler or the facsimile of one on the board for Ex. 1 and Ex. 4. The what-part-of situation in Ex. 4 may be shown as below. (For Ex. 2, use the diagram in the text.)

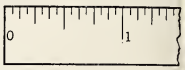


2. Before pupils attempt the examples in rows 6 to 9 on page 193, let them first tell whether the quotient for each example will be more than 1, or less than 1. As they do the examples, point out again that when the fractional unit is the same, the common-denominator method is the easier method for dividing.

Dividing a Fraction by a Fraction

Unlike-fractions; quotient more than 1 [O]

1. Estimating that a slice of bacon weighs 1 ounce ($\frac{1}{16}$ lb.), Bill tried to think out how many slices of bacon there would be to eat if he fried $\frac{3}{4}$ pound. Can you find out?



You will have to find how many $\frac{1}{16}$'s there are in $\frac{3}{4}$. On the section of a ruler shown here, find the $\frac{3}{4}$ mark and count the $\frac{1}{16}$'s in $\frac{3}{4}$. 12

The work with figures is done in the box. Explain the work.

$\frac{3}{4} \div \frac{1}{16} = \frac{3}{4} \times \frac{16}{1}$

$\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{1} = 12$

Check

$12 \times \frac{1}{16} = \frac{12}{16} = \frac{3}{4}$

You can check the division as usual by multiplying the divisor and the quotient, as in the box. Explain.

Could you use the common-denominator method here? 2/64 How? $\frac{3}{4} = \frac{12}{16}$; $\frac{12}{16} \div \frac{1}{16} = 12 \div 1 = 12$ [W]

Copy, and divide by inverting the divisor and multiplying. Check your work as in the box.

	a	b	c	d	e
2.	$\frac{2}{3} \div \frac{1}{6}$ 4	$\frac{3}{4} \div \frac{1}{8}$ 6	$\frac{7}{8} \div \frac{1}{4}$ $3\frac{1}{2}$	$\frac{5}{12} \div \frac{1}{3}$ $1\frac{1}{4}$	$\frac{5}{6} \div \frac{1}{4}$ 3
3.	$\frac{11}{12} \div \frac{3}{4}$ $1\frac{2}{9}$	$\frac{5}{16} \div \frac{1}{4}$ $1\frac{1}{4}$	$\frac{5}{6} \div \frac{2}{3}$ $1\frac{1}{4}$	$\frac{3}{4} \div \frac{3}{16}$ 4	$\frac{4}{5} \div \frac{1}{10}$
4.	$\frac{7}{8} \div \frac{2}{3}$ $1\frac{5}{16}$	$\frac{3}{4} \div \frac{5}{8}$ $1\frac{1}{5}$	$\frac{2}{3} \div \frac{3}{8}$ $1\frac{7}{9}$	$\frac{13}{16} \div \frac{3}{4}$ $1\frac{1}{12}$	$\frac{8}{9} \div \frac{2}{5}$ 2

194

Book Lesson (pages 194 and 195). *Ex. 1:* Oral work. *Rows 2–4:* Written work. *Ex. 5:* Oral work. *Ex. 6–11:* Written work.

1. At the end of the oral work on page 194, have a few pupils go to the board and show how the common-denominator method could be used to show the written work for solving the problem at the top of the page.

2. To make sure pupils know how to proceed for the written work, ask selected pupils to solve row 2 on the chalkboard and to check them by multiplication.

5. The camping area in which the boys stayed overnight covers about $\frac{3}{4}$ of an acre divided into $\frac{1}{8}$-acre tent sites. What part of the camping area was the boys' tent site?

The red lines of the diagram show the camping area. Why is the tent site, which is $\frac{1}{8}$ of the whole acre, not $\frac{1}{8}$ of the camping area? Count to find what part the $\frac{1}{8}$ A. is of the $\frac{3}{4}$ A. $\frac{1}{6}$

Working with figures, to compare $\frac{1}{8}$ with $\frac{3}{4}$, we can write

$$\frac{1}{8} \div \frac{3}{4} = ?$$

Why will the quotient be a what-part-of number? dividend, $\frac{1}{8}$, is less than divisor, $\frac{3}{4}$.

Explain the division in the box.

Explain the check.

$$\frac{1}{8} \div \frac{3}{4} = \frac{1}{\cancel{8}_{2}} \times \frac{1}{\cancel{4}_{3}} = \frac{1}{6}$$

Check

$$\frac{1}{6} \times \frac{3}{4} = \frac{3}{24} = \frac{1}{8}$$

[W]

Copy and divide. Check as in the box.

	a	b	c	d	e
6.	$\frac{1}{3} \div \frac{1}{2}$ $\frac{2}{3}$	$\frac{1}{2} \div \frac{5}{6}$ $\frac{3}{5}$	$\frac{3}{8} \div \frac{5}{6}$ $\frac{9}{20}$	$\frac{2}{3} \div \frac{7}{8}$ $\frac{16}{21}$	$\frac{1}{4} \div \frac{7}{10}$ $\frac{5}{14}$
7.	$\frac{1}{5} \div \frac{1}{4}$ $\frac{4}{5}$	$\frac{2}{5} \div \frac{7}{10}$ $\frac{4}{7}$	$\frac{1}{8} \div \frac{3}{4}$ $\frac{1}{6}$	$\frac{1}{2} \div \frac{2}{3}$ $\frac{3}{4}$	$\frac{3}{5} \div \frac{9}{10}$ $\frac{2}{3}$
8.	$\frac{1}{4} \div \frac{7}{12}$ $\frac{3}{7}$	$\frac{1}{5} \div \frac{7}{15}$ $\frac{3}{7}$	$\frac{5}{12} \div \frac{5}{6}$ $\frac{1}{2}$	$\frac{1}{6} \div \frac{9}{10}$ $\frac{5}{27}$	$\frac{1}{8} \div \frac{9}{16}$ $\frac{2}{9}$
9.	$\frac{1}{4} \div \frac{4}{5}$ $\frac{5}{16}$	$\frac{5}{16} \div \frac{3}{8}$ $\frac{5}{6}$	$\frac{5}{6} \div \frac{11}{12}$ $\frac{10}{11}$	$\frac{5}{8} \div \frac{5}{6}$ $\frac{3}{4}$	$\frac{3}{5} \div \frac{2}{3}$ $\frac{9}{10}$

Write your work for problems 10 and 11.

10. The boys thought it would take them $\frac{3}{4}$ hr. to clean up the tent site and pack their equipment. They really spent only $\frac{3}{5}$ hr. What part of the estimated time did they use? $\frac{4}{5}$

11. A sign on the mountain trail said, "$\frac{7}{8}$ mi. to the top." The boys stopped to rest after they had climbed for $\frac{1}{4}$ mi. What part of the total distance had they climbed? $\frac{2}{7}$

Extra Practice. For more practice, work Sets 81–83.

3. If you think a more concrete experience is necessary for *slower learners* to understand division of unlike-fractions when the quotient is less than 1, use 8 squares and repeat with them the procedure given for Ex. 5 on page 195 in the text. Whether you use this method or the presentation in the text, be sure these pupils realize that when they are finding how many $\frac{1}{8}$'s in $\frac{3}{4}$ they have really changed $\frac{3}{4}$ to $\frac{6}{8}$ and are, therefore, using the common-denominator method of dividing.

4. You may want to assign row 6 as board work before pupils attempt to work independently.

Differentiations and Extensions

1. Have *all pupils* attempt to tell whether the quotients for rows 2 to 4 and rows 6 to 9 will be more than 1 or less than 1. As they do this, see if they can give exact answers without using pencil and paper. Since this will mean changing fractions to a common denominator, this activity probably ought to be limited to examples in which one of the fractions contains the common denominator (such as Ex. 2**a** to 2**d**, 6**b**, 7**b**, etc.).

2. *More capable children* may:

a. Attempt to solve problems, such as the following, derived from classroom activities:

a. The girls made candy for the school fair. They put $\frac{3}{8}$ pound of candy in each bag. After they had filled 20 bags, they had $\frac{3}{4}$ pounds of candy left. How many more bags could they fill?

b. The boys inflated balloons and tied a string on each one. Each balloon took $\frac{1}{2}$ yard of string. The boys had $5\frac{1}{2}$ yards of string. How many balloons could they tie strings to?

b. Write a series of problems, using the examples in rows 6–9 on page 195, which can be presented to the class as a whole for discussion or used for additional practice for *slower learners.*

3. Assign **Extra Practice Sets 81, 82, and 83** as needed.

Set 81. Dividing a fraction by a fraction; common-denominator method

Divide by using common denominators.

1. a. $\frac{3}{4} \div \frac{1}{4}$ 3 **b.** $\frac{1}{3} \div \frac{2}{3}$ $\frac{1}{2}$ **c.** $\frac{5}{8} \div \frac{3}{8}$ $1\frac{2}{3}$

d. $\frac{15}{16} \div \frac{5}{16}$ 3 **e.** $\frac{6}{7} \div \frac{2}{7}$ 3 **f.** $\frac{7}{8} \div \frac{5}{8}$ $1\frac{2}{5}$

2. a. $\frac{3}{8} \div \frac{7}{8}$ $\frac{3}{7}$ **b.** $\frac{3}{7} \div \frac{6}{7}$ $\frac{1}{2}$ **c.** $\frac{5}{12} \div \frac{7}{12}$ $\frac{5}{7}$

d. $\frac{2}{9} \div \frac{8}{9}$ $\frac{1}{4}$ **e.** $\frac{1}{6} \div \frac{5}{6}$ $\frac{1}{5}$ **f.** $\frac{11}{14} \div \frac{5}{14}$ $2\frac{1}{5}$

Set 82. Dividing a fraction by a fraction; quotient more than 1

Divide by inverting the divisor and multiplying.

1. a. $\frac{7}{8} \div \frac{5}{6}$ $1\frac{1}{20}$ **b.** $\frac{3}{4} \div \frac{1}{2}$ $1\frac{1}{2}$ **c.** $\frac{9}{16} \div \frac{1}{8}$ $4\frac{1}{2}$

d. $\frac{5}{6} \div \frac{7}{12}$ $1\frac{3}{7}$ **e.** $\frac{5}{8} \div \frac{3}{10}$ $2\frac{1}{12}$ **f.** $\frac{1}{4} \div \frac{1}{6}$ $1\frac{1}{2}$

2. a. $\frac{7}{12} \div \frac{2}{9}$ $2\frac{5}{8}$ **b.** $\frac{4}{5} \div \frac{2}{3}$ $1\frac{1}{5}$ **c.** $\frac{5}{6} \div \frac{1}{4}$ $3\frac{1}{3}$

d. $\frac{2}{3} \div \frac{3}{16}$ $3\frac{5}{9}$ **e.** $\frac{8}{9} \div \frac{2}{3}$ $1\frac{1}{3}$ **f.** $\frac{7}{16} \div \frac{3}{8}$ $1\frac{1}{6}$

Set 83. Dividing a fraction by a fraction; quotient less than 1

Divide by inverting the divisor and multiplying.

1. a. $\frac{1}{4} \div \frac{1}{2}$ $\frac{1}{2}$ **b.** $\frac{5}{8} \div \frac{5}{6}$ $\frac{3}{4}$ **c.** $\frac{3}{10} \div \frac{4}{5}$ $\frac{3}{8}$

d. $\frac{3}{4} \div \frac{7}{8}$ $\frac{6}{7}$ **e.** $\frac{1}{3} \div \frac{5}{7}$ $\frac{7}{15}$ **f.** $\frac{3}{5} \div \frac{9}{10}$ $\frac{2}{3}$

2. a. $\frac{3}{8} \div \frac{3}{4}$ $\frac{1}{2}$ **b.** $\frac{1}{2} \div \frac{3}{4}$ $\frac{2}{3}$ **c.** $\frac{7}{12} \div \frac{7}{8}$ $\frac{2}{3}$

d. $\frac{2}{9} \div \frac{6}{7}$ $\frac{7}{27}$ **e.** $\frac{3}{10} \div \frac{4}{5}$ $\frac{3}{8}$ **f.** $\frac{2}{3} \div \frac{3}{4}$ $\frac{8}{9}$

Teaching Pages 196 and 197

Pupil's Objective. To learn how to divide a fraction by a mixed number and a mixed number by a mixed number.

Background. Your pupils will now learn how to divide a fraction by a mixed number or a mixed number by another mixed number. This will complete their introduction to all the various types of division summarized in the chart on page 199 in the text.

Pupils will find that since they know how to divide a fraction by a fraction, dividing a mixed number is not difficult. This is brought out in Ex. 16.

By now pupils are used to the inversion method of dividing, and this is the method shown in the boxes on pages 196 and 197. However, you may want first to let children tell how they could use the common-denominator method in dividing in these types of examples, for this sometimes helps pupils see more clearly why the answer will be a how-many-times or what-part-of number.

Book Lesson. *Ex. 1:* Oral work. *Ex. 2–13:* Written work. *Ex. 14–16:* Oral work. *Rows 17–21:* Written work.

1. After discussing each step of Ex. 1, have pupils check the work by multiplying. Show the written work on the board so that *slower learners* can see each step as other pupils explain it.

2. For Ex. 2–13, have pupils tell which is the smaller number in each pair. If necessary, have one or two examples worked on the board.

3. Assign rows 17–21 for written work. This period will provide an excellent opportunity for the teacher to give individual help wherever it is needed.

Dividing by a Mixed Number

Dividend a fraction [O

1. The Imperial gallon, used in Canada, is larger than the United States gallon. 1 Imperial gallon = $1\frac{1}{5}$ U. S. gallons. Mrs. Day of Toronto, while visiting in Buffalo, bought $\frac{3}{4}$ gal. of maple syrup. What part of an Imperial gallon did she get?

$\frac{3}{4} \div 1\frac{1}{5} = \frac{5}{8}$

Which of these two numbers, $\frac{3}{4}$ or $1\frac{1}{5}$, is the smaller? Will the quotient be a what-part-of number or a how-many-times number?

Explain the work in box A.

On the board check the quotient by multiplying.

A

$\frac{3}{4} \div 1\frac{1}{5} = ?$ Side work $1\frac{1}{5} = \frac{6}{5}$

$\frac{3}{4} \div \frac{6}{5} = \frac{1}{\cancel{3}} \times \frac{5}{\cancel{6}_{2}} = \frac{5}{8}$

[W]

For each of the following pairs of numbers, find what part the smaller number is of the larger and check your answer.

2. $\frac{2}{3}$, $3\frac{1}{3}$ $\frac{1}{5}$ **3.** $7\frac{1}{2}$, $\frac{1}{2}$ $\frac{1}{15}$ **4.** $1\frac{1}{2}$, $\frac{9}{10}$ $\frac{3}{5}$ **5.** $\frac{1}{3}$, $1\frac{1}{4}$

6. $\frac{7}{12}$, $2\frac{1}{3}$ $\frac{1}{4}$ **7.** $3\frac{3}{5}$, $\frac{8}{15}$ $\frac{4}{27}$ **8.** $\frac{7}{9}$, $6\frac{5}{6}$ $\frac{14}{23}$ **9.** $1\frac{1}{2}$, $\frac{1}{4}$

10. $\frac{5}{8}$, $1\frac{7}{8}$ $\frac{1}{3}$ **11.** $2\frac{1}{2}$, $\frac{5}{8}$ $\frac{4}{7}$ **12.** $3\frac{1}{3}$, $\frac{5}{6}$ $\frac{4}{7}$ **13.** $\frac{3}{8}$, $4\frac{1}{2}$

Extra Practice. For more practice, work Sets 84 and 85.

Differentiations and Extensions

1. Pupils may estimate answers in rows 17–21 at the bottom of page 197 in the text. This may be done by rounding mixed numbers to the nearest whole number and dividing. Another and perhaps easier way is to neglect the fraction part of each mixed number and use only the whole numbers in estimating. Neither method will be very helpful in discovering ways of working without paper and pencil. Nor will either method produce very accurate estimates, but a good check on the reason-

B	C
How Many Times?	**What Part Of?**
Don walked $12\frac{3}{4}$ mi., and Larry walked $4\frac{1}{2}$ mi. Don walked how many times as far as Larry?	Tom started for a camp $10\frac{1}{2}$ mi. away. What part of his hike had he covered after going $1\frac{3}{4}$ mi.?
Side work $12\frac{3}{4} \div 4\frac{1}{2} = ?$ $\quad 12\frac{3}{4} = \frac{51}{4}$ $4\frac{1}{2} = \frac{9}{2}$	Side work $1\frac{3}{4} \div 10\frac{1}{2} = ?$ $\quad 1\frac{3}{4} = \frac{7}{4}$ $10\frac{1}{2} = \frac{21}{2}$
$\frac{51}{4} \div \frac{9}{2} = \frac{\overset{17}{\cancel{51}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} = \frac{17}{6}$, or $2\frac{5}{6}$	$\frac{7}{4} \div \frac{21}{2} = \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{21}}} = \frac{1}{6}$

14. In problems B and C, we have to divide a mixed number by a mixed number. What do we do first with the mixed numbers? *change them to improper fractions* What kind of examples do we have then? *fraction ÷ fraction*

15. In problem B, why is the quotient a how-many-times number? *dividend, $12\frac{3}{4}$, larger than divisor, $4\frac{1}{2}$.* In problem C, why is the quotient a what-part-of number? *Dividend, $1\frac{3}{4}$, smaller than divisor, $10\frac{1}{2}$.*

16. If you know how to divide a fraction by a fraction, why is it easy to divide a mixed number by a mixed number? *Both mixed numbers may be changed to improper fractions.*

For rows 17 to 21, tell whether each quotient will be a what-part-of number (P) or a how-many-times (T) number.

[W]

Divide by the inversion method. Check by multiplying.

	a	b	c	d	e
17.	T $4\frac{1}{2} \div 1\frac{1}{2}$ 3	P $2\frac{3}{4} \div 4\frac{1}{2}$ $\frac{11}{18}$	P $1\frac{1}{3} \div 2\frac{1}{2}$ $\frac{2}{3}$	T $6\frac{2}{9} \div 1\frac{1}{3}$ $4\frac{2}{3}$	T $3\frac{1}{4} \div 1\frac{1}{2}$ $2\frac{1}{6}$
18.	T $6\frac{1}{4} \div 2\frac{1}{2}$ $2\frac{1}{2}$	P $4\frac{1}{2} \div 7\frac{1}{2}$ $\frac{3}{5}$	T $7\frac{1}{3} \div 2\frac{1}{3}$ $2\frac{7}{21}$	T $1\frac{1}{2} \div 1\frac{1}{3}$ $\frac{1}{8}$	P $2\frac{1}{2} \div 3\frac{1}{4}$ $\frac{10}{13}$
19.	T $2\frac{1}{10} \div 1\frac{2}{5}$ $\frac{1}{2}$	T $3\frac{3}{4} \div 1\frac{2}{3}$ $2\frac{1}{4}$	P $1\frac{1}{2} \div 4\frac{1}{2}$ $\frac{1}{3}$	P $2\frac{1}{2} \div 4\frac{1}{6}$ $\frac{3}{5}$	T $4\frac{1}{2} \div 3\frac{3}{8}$ $1\frac{1}{3}$
20.	P $2\frac{1}{4} \div 4\frac{1}{2}$ $\frac{1}{2}$	T $2\frac{5}{8} \div 1\frac{3}{4}$ $\frac{1}{2}$	T $2\frac{1}{2} \div 1\frac{3}{4}$ $\frac{3}{7}$	P $1\frac{1}{8} \div 5\frac{5}{8}$ $\frac{1}{5}$	T $1\frac{5}{9} \div 1\frac{2}{3}$ $\frac{1}{9}$
21.	P $2\frac{1}{4} \div 2\frac{9}{10}$	T $3\frac{1}{8} \div 1\frac{2}{3}$ $\frac{7}{8}$	T $12\frac{3}{5} \div 4\frac{1}{2}$ $2\frac{4}{5}$	P $2\frac{1}{10} \div 3\frac{7}{12}$	T $5\frac{1}{3} \div 3\frac{1}{9}$ $\frac{3}{7}$

Extra Practice. For more practice, work Sets 86 and 87.

ableness of the exact answers obtained can be made in this way.

2. Assign **Extra Practice Sets 84, 85, 86, and 87** as needed.

Set 84. Dividing a fraction by a mixed number

Divide by inverting the divisor and multiplying.

1. a. $\frac{1}{2} \div 2\frac{1}{2}$ $\frac{1}{5}$ **b.** $\frac{7}{8} \div 3\frac{3}{4}$ $\frac{7}{30}$ **c.** $\frac{2}{15} \div 6\frac{2}{5}$ $\frac{1}{48}$

d. $\frac{7}{10} \div 5\frac{3}{5}$ $\frac{1}{8}$ **e.** $\frac{5}{16} \div 1\frac{3}{5}$ $\frac{25}{128}$

2. a. $\frac{3}{16} \div 2\frac{2}{3}$ $\frac{9}{128}$ **b.** $\frac{2}{3} \div 1\frac{5}{6}$ $\frac{4}{11}$ **c.** $\frac{3}{8} \div 1\frac{1}{4}$ $\frac{3}{10}$

d. $\frac{5}{6} \div 3\frac{1}{3}$ $\frac{1}{4}$ **e.** $\frac{1}{7} \div 5\frac{5}{7}$ $\frac{1}{40}$

3. a. $\frac{5}{8} \div 8\frac{1}{3}$ $\frac{3}{40}$ **b.** $\frac{7}{12} \div 4\frac{2}{3}$ $\frac{1}{8}$ **c.** $\frac{3}{4} \div 4\frac{1}{2}$ $\frac{1}{6}$

d. $\frac{7}{9} \div 2\frac{2}{9}$ $\frac{7}{20}$ **e.** $\frac{4}{5} \div 4\frac{1}{2}$ $\frac{8}{45}$

Set 85. Dividing a fraction by a mixed number

Divide by inverting the divisor and multiplying.

1. a. $\frac{7}{8} \div 8\frac{3}{4}$ $\frac{1}{10}$ **b.** $\frac{3}{5} \div 2\frac{7}{10}$ $\frac{2}{9}$ **c.** $\frac{9}{16} \div 6\frac{3}{16}$ $\frac{1}{11}$

d. $\frac{1}{10} \div 1\frac{4}{5}$ $\frac{1}{18}$ **e.** $\frac{5}{6} \div 5\frac{5}{12}$ $\frac{2}{13}$

2. a. $\frac{2}{3} \div 1\frac{4}{15}$ $\frac{10}{19}$ **b.** $\frac{5}{12} \div 3\frac{3}{4}$ $\frac{1}{9}$ **c.** $\frac{5}{18} \div 5\frac{5}{6}$ $\frac{1}{21}$

d. $\frac{4}{9} \div 7\frac{1}{5}$ $\frac{5}{81}$ **e.** $\frac{3}{8} \div 6\frac{1}{4}$ $\frac{3}{50}$

3. a. $\frac{3}{16} \div 6\frac{3}{4}$ $\frac{1}{36}$ **b.** $\frac{3}{7} \div 2\frac{2}{5}$ $\frac{5}{28}$ **c.** $\frac{5}{8} \div 2\frac{13}{16}$ $\frac{2}{9}$

d. $\frac{1}{2} \div 1\frac{4}{5}$ $\frac{5}{18}$ **e.** $\frac{3}{4} \div 3\frac{3}{8}$ $\frac{2}{9}$

Set 86. Dividing a mixed number by a mixed number

Divide by inverting the divisor and multiplying.

1. a. $2\frac{1}{2} \div 1\frac{1}{4}$ 2 **b.** $4\frac{1}{2} \div 3\frac{3}{8}$ $1\frac{1}{3}$ **c.** $5\frac{3}{7} \div 5\frac{3}{7}$ 1

d. $1\frac{3}{4} \div 2\frac{4}{5}$ $\frac{5}{8}$ **e.** $2\frac{2}{9} \div 3\frac{1}{3}$ $\frac{2}{3}$

2. a. $1\frac{7}{8} \div 3\frac{1}{3}$ $\frac{9}{16}$ **b.** $5\frac{5}{6} \div 1\frac{2}{3}$ $3\frac{1}{2}$ **c.** $6\frac{1}{4} \div 7\frac{1}{2}$ $\frac{5}{6}$

d. $9\frac{3}{8} \div 4\frac{1}{6}$ $2\frac{1}{4}$ **e.** $3\frac{1}{4} \div 1\frac{1}{2}$ $2\frac{1}{6}$

3. a. $1\frac{1}{2} \div 2\frac{1}{4}$ $\frac{2}{3}$ **b.** $1\frac{1}{8} \div 6\frac{3}{4}$ $\frac{1}{6}$ **c.** $2\frac{2}{3} \div 5\frac{1}{3}$ $\frac{1}{2}$

d. $1\frac{7}{8} \div 6\frac{1}{4}$ $\frac{3}{10}$ **e.** $3\frac{2}{3} \div 1\frac{4}{5}$ $2\frac{1}{27}$

Set 87. Dividing a mixed number by a mixed number

Divide by inverting the divisor and multiplying.

1. a. $2\frac{5}{8} \div 3\frac{1}{2}$ $\frac{3}{4}$ **b.** $10\frac{2}{3} \div 2\frac{2}{3}$ 4 **c.** $1\frac{3}{4} \div 2\frac{1}{2}$ $\frac{7}{10}$

d. $13\frac{1}{2} \div 3\frac{3}{5}$ $3\frac{3}{4}$ **e.** $1\frac{7}{15} \div 1\frac{2}{9}$ $1\frac{1}{5}$

2. a. $6\frac{3}{4} \div 10\frac{1}{2}$ $\frac{9}{14}$ **b.** $6\frac{7}{8} \div 11\frac{1}{4}$ $\frac{11}{18}$ **c.** $2\frac{1}{4} \div 5\frac{1}{4}$ $\frac{3}{7}$

d. $1\frac{7}{10} \div 1\frac{3}{5}$ $1\frac{1}{16}$ **e.** $4\frac{1}{2} \div 7\frac{1}{2}$ $\frac{3}{5}$

3. a. $11\frac{2}{3} \div 4\frac{3}{8}$ $2\frac{2}{3}$ **b.** $4\frac{7}{12} \div 2\frac{3}{4}$ $1\frac{2}{3}$ **c.** $3\frac{1}{8} \div 2\frac{3}{16}$ $1\frac{3}{7}$

d. $16\frac{2}{3} \div 3\frac{1}{5}$ $5\frac{5}{24}$ **e.** $5\frac{3}{5} \div 4\frac{4}{5}$ $1\frac{1}{6}$

Making Up Your Own Problems

D. of fractions and mixed numbers [W]

One day Miss Pike suggested that the pupils in her sixth-grade arithmetic class make up their own problems in division with fractions and mixed numbers. Some of the problems that her pupils wrote are given below.

Solve each problem, and then make another problem like it.

1. Tina: Mother gave me $4\frac{1}{2}$ yards of ribbon to cut into $\frac{3}{4}$-yard pieces for apron ties. How many ties could I make? 6

2. Bill: For our picnic Dad got $6\frac{1}{4}$ pounds of lobsters. They averaged $1\frac{1}{4}$ pounds apiece. How many did we have? 5

3. Anne: The dessert recipe called for $1\frac{1}{2}$ cups of milk. Mother found that she had only $\frac{3}{4}$ cup. How could she change the amounts in the recipe so that she could use only $\frac{3}{4}$ cup of milk? (That is, $\frac{3}{4}$ is what part of $1\frac{1}{2}$?) $\frac{1}{2}$

4. Art: In science class I learned that 1 gallon of water weighs about $8\frac{1}{3}$ pounds. How much would $\frac{1}{2}$ gallon weigh? $4\frac{1}{6}$ lb.

5. Pete: To give good measure, the man at the fruit stand puts $3\frac{1}{16}$ pounds of apples in the bags he sells as 3 pounds. How many bags could he fill from $24\frac{1}{2}$ pounds of apples? 8

6. Nell: How many $\frac{1}{4}$-hour radio programs could be planned for a $1\frac{1}{2}$-hour period? 6

Dividing with Fractions

A summary [O]

The chart below shows the kinds of division examples you have had in this chapter. Where two examples are in a box, one dividend is larger than the divisor and the other is smaller than the divisor. Tell for each example in the chart whether the quotient will be more (M) or less (L) than 1.

Dividend	Divisor		
	Whole number	Fraction	Mixed number
Whole number	M 1. $42 \div 12\frac{3}{2}$ L 2. $8 \div 30\frac{4}{15}$	M 3. $5 \div \frac{5}{6}$ 6	M 4. $8 \div 1\frac{1}{3}$ 6 L 5. $2 \div 4\frac{1}{2}\frac{4}{9}$
Fraction	L 6. $\frac{3}{4} \div 6\frac{1}{8}$	M 7. $\frac{2}{3} \div \frac{1}{6}$ 4 L 8. $\frac{1}{4} \div \frac{3}{8}\frac{2}{3}$	L 9. $\frac{5}{8} \div 2\frac{1}{4}\frac{5}{18}$
Mixed number	M 10. $8\frac{3}{8} \div 6\frac{19}{48}$ L 11. $1\frac{1}{2} \div 3\frac{1}{2}$	M 12. $4\frac{2}{3} \div \frac{5}{6}\frac{3}{5}$	M 13. $6\frac{2}{3} \div 3\frac{1}{8}$ 2 L 14. $4\frac{1}{2} \div 6\frac{1}{2}\frac{9}{13}$

[W]

1–14. Work all the examples in the chart by inversion.

Copy, divide by the inversion method, and check.

	a	b	c	d	e
15.	$5\frac{1}{2} \div 1\frac{3}{4}3\frac{1}{7}$	$14 \div \frac{7}{8}16$	$12 \div 42\frac{2}{7}$	$7 \div \frac{1}{5}35$	$4\frac{1}{5} \div 7\frac{3}{5}$
16.	$2\frac{1}{12} \div \frac{5}{6}2\frac{1}{2}$	$8\frac{1}{4} \div 3\frac{1}{4}2\frac{7}{13}$	$\frac{3}{8} \div 6\frac{1}{16}$	$1\frac{2}{3} \div 3\frac{1}{3}\frac{1}{2}$	$\frac{5}{8} \div 1\frac{5}{16}\frac{2}{3}$
17.	$7\frac{7}{8} \div \frac{3}{4}10\frac{1}{2}$	$\frac{7}{8} \div 2\frac{11}{12}\frac{3}{10}$	$68 \div 6\frac{1}{13}$	$7\frac{2}{10} \div \frac{4}{5}9$	$2\frac{2}{3} \div 8\frac{1}{3}$
18.	$18\frac{3}{4} \div 2\frac{1}{2}7\frac{1}{2}$	$2\frac{1}{7} \div 5\frac{3}{7}$	$\frac{4}{5} \div 12\frac{1}{15}$	$24 \div 60\frac{2}{5}$	$6 \div 1\frac{4}{5}3$
19.	$12 \div \frac{4}{5}15$	$6 \div 4\frac{1}{2}\frac{1}{3}$	$3\frac{3}{8} \div 6\frac{3}{4}\frac{1}{2}$	$\frac{3}{8} \div \frac{1}{4}\frac{1}{2}$	$\frac{5}{6} \div 3\frac{3}{4}\frac{2}{9}$
20.	$10 \div \frac{2}{3}15$	$3 \div 7\frac{4}{5}\frac{5}{13}$	$\frac{5}{12} \div \frac{1}{3}\frac{1}{4}$	$9\frac{1}{2} \div 4\frac{3}{4}2$	$\frac{3}{8} \div \frac{5}{12}\frac{9}{10}$

Extra Practice. For more practice, work Sets 88 and 89.

To divide with fractions, first change whole numbers and mixed numbers to fractions. Then you can invert the divisor and multiply. Cancel if you can.

Teaching Pages 198 and 199

Pupil's Objectives. (*a*) To solve, and also write, problems involving division of fractions and mixed numbers; (*b*) to see summarized all the different kinds of division examples containing fractions and mixed numbers; (*c*) to obtain mixed practice in dividing fractions.

Background. Now that pupils have had all the different types of division examples, they are asked to solve problems and make up other problems that use these types of examples. This procedure helps children realize the importance of learning how to divide fractions. Creating problems that use fractions in division is not always easy, and for this reason you may want to omit this part of the lesson for *slower learners*.

The summary of the types of division examples on page 199 shows pupils how much progress they have made and allows the teacher to discover the kinds of division examples needing reteaching.

Book Lesson (page 198). *Ex. 1–6:* Written work. Have each problem read aloud and have the

procedure explained. In some cases *slower learners* may be helped by dramatizing a few of the problems. For Ex. 1, these pupils may measure and cut a piece of string, ribbon, or tape $4\frac{1}{2}$ yd. long into pieces $\frac{3}{4}$ yd. in length. In Ex. 4, first a gallon and then a half gallon of water can be weighed.

Book Lesson (page 199). *Top of the page:* Oral work. *Ex. 1–20:* Written work.

1. Discuss the examples in the chart and have pupils determine whether the quotients will be more than or less than 1.

2. Try to lead pupils to state in their own words the generalization at the bottom of the page in the text.

Differentiations and Extensions

1. The examples in rows 15 to 20 on page 199 offer a good opportunity for pupils to test their ability to estimate quotients. If you give an oral lesson in estimating quotients, let pupils tell how each estimate was obtained and about how accurate it will be. In some cases the bracketing method should be used, in which case pupils would give an upper and lower limit, or a range, within which the exact quotient would fall. See if some pupils can refine their estimates in such a way that exact answers are obtained without using pencil or paper.

2. Assign **Extra Practice Sets 88 and 89** as needed.

Set 88. Mixed practice in dividing fractions

Divide by the inversion method.

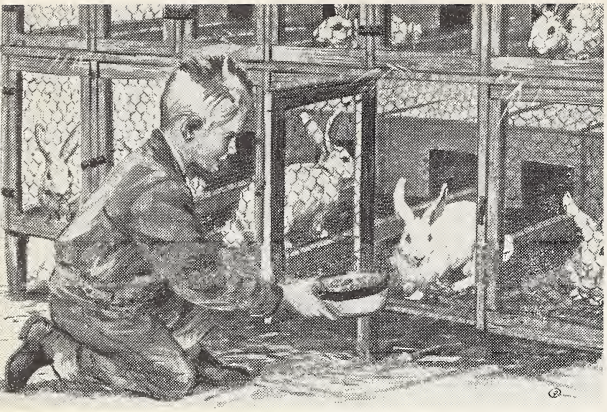
	a	b	c	d	e
1.	$12 \div \frac{3}{5}$ **20**	$\frac{3}{8} \div 6$ $\frac{1}{16}$	$3\frac{1}{2} \div 7$ $\frac{1}{2}$	$\frac{1}{8} \div \frac{5}{8}$ $\frac{1}{5}$	$\frac{3}{5} \div \frac{7}{10}$ $\frac{6}{7}$
2.	$9\frac{1}{3} \div \frac{5}{6}$ $11\frac{1}{5}$	$7\frac{1}{5} \div \frac{3}{5}$ **12**	$4\frac{1}{6} \div \frac{7}{8}$ $4\frac{16}{21}$	$12 \div 5\frac{1}{3}$ $2\frac{1}{4}$	$2 \div 10\frac{1}{2}$ $\frac{4}{21}$
3.	$\frac{7}{8} \div 2\frac{4}{5}$ $\frac{5}{16}$	$\frac{5}{6} \div 1\frac{1}{12}$ $\frac{10}{13}$	$7 \div \frac{4}{7}$ $12\frac{1}{4}$	$\frac{3}{4} \div 12$ $\frac{1}{16}$	$8\frac{3}{4} \div 12\frac{1}{2}$ $\frac{7}{10}$
4.	$8\frac{1}{8} \div 2\frac{3}{5}$ $3\frac{1}{8}$	$2\frac{7}{10} \div 3$ $\frac{9}{10}$	$4\frac{1}{2} \div 6\frac{3}{7}$ $\frac{7}{10}$	$9\frac{3}{5} \div 16$ $\frac{3}{5}$	$1\frac{5}{16} \div 2\frac{5}{8}$ $\frac{1}{2}$
5.	$16 \div \frac{3}{4}$ $21\frac{1}{3}$	$\frac{1}{6} \div \frac{5}{6}$ $\frac{1}{5}$	$\frac{1}{2} \div 5$ $\frac{1}{10}$	$\frac{7}{16} \div \frac{7}{16}$ **1**	$\frac{7}{12} \div \frac{3}{5}$ $\frac{35}{36}$

Set 89. Mixed practice in dividing fractions

Divide, using inversion.

1.	$\frac{3}{4} \div 6$ $\frac{1}{8}$	$8 \div \frac{2}{3}$ **12**	$4\frac{1}{2} \div 2\frac{1}{2}$ $1\frac{4}{5}$	$\frac{5}{12} \div 2$ $\frac{5}{24}$	$15\frac{1}{2} \div \frac{2}{3}$ $23\frac{1}{4}$
2.	$\frac{2}{7} \div \frac{4}{5}$ $\frac{5}{14}$	$\frac{3}{16} \div 9$ $\frac{1}{48}$	$9\frac{1}{3} \div \frac{5}{7}$ $13\frac{1}{15}$	$5\frac{5}{16} \div \frac{5}{8}$ $8\frac{1}{2}$	$\frac{7}{8} \div \frac{3}{16}$ $4\frac{2}{3}$
3.	$7\frac{1}{2} \div 5$ $1\frac{1}{2}$	$3\frac{1}{2} \div 10$ $\frac{7}{20}$	$\frac{5}{8} \div \frac{1}{2}$ $1\frac{1}{4}$	$12\frac{1}{2} \div 4$ $3\frac{1}{8}$	$6\frac{2}{3} \div \frac{7}{10}$ $9\frac{11}{21}$
4.	$15 \div 1\frac{2}{5}$ $10\frac{5}{7}$	$6 \div 6\frac{3}{8}$ $\frac{16}{17}$	$4\frac{3}{16} \div 5$ $\frac{67}{80}$	$\frac{3}{8} \div 1\frac{7}{8}$ $\frac{1}{5}$	$\frac{7}{16} \div 2\frac{1}{2}$ $\frac{7}{40}$
5.	$\frac{7}{8} \div 1\frac{5}{16}$ $\frac{2}{3}$	$1\frac{1}{8} \div 5\frac{1}{3}$ $\frac{27}{128}$	$9 \div 15\frac{3}{4}$ $\frac{4}{7}$	$13\frac{1}{3} \div 1\frac{1}{4}$ $10\frac{2}{3}$	$2\frac{2}{9} \div 14\frac{2}{3}$ $\frac{5}{33}$

NOTES



Bill's Rabbits

M. and D. problems with fractions [O]

In some of these problems you multiply, in others you divide. Tell what to do to find each answer.

1. Each day Bill feeds his rabbits $4\frac{1}{2}$ lb. of feed. How long will 100 lb. of feed last? *D.; $22\frac{2}{9}$ days*

2. The feed costs Bill about $5\frac{3}{4}$¢ a pound. If he uses $4\frac{1}{2}$ lb. a day, about what is the daily cost of the feed? *M.; 26¢*

3. One day Bill sold rabbits of the following weights: $5\frac{1}{8}$ lb., $3\frac{5}{8}$ lb., $5\frac{1}{4}$ lb., and $5\frac{1}{2}$ lb. What was the average weight of the rabbits sold that day? *A. and D.; $4\frac{7}{8}$ lb.*

4. In July, Bill received 25 dollars for the rabbits he sold. His expenses that month were $18\frac{3}{4}$ dollars. What part of the money that he received did he pay in expenses? *D.; $\frac{3}{4}$*

5. With some of the money he earned, Bill bought $\frac{3}{8}$ lb. of candy which sold for 72¢ a pound. How much did he pay? *M.; 27¢*

6. If Bill's rabbits average $4\frac{7}{8}$ lb. and sell at 40¢ a pound, how much does he get for an average-sized rabbit? *M.; $1.95*

7. One day Bill had an order for about 20 lb. of rabbits. If his rabbits averaged $4\frac{7}{8}$ lb., about how many rabbits would be needed? (Round to the nearest whole number.) *D.; 4*

8. If Bill mixes $\frac{3}{8}$ lb. of insect powder in 6 gal. of water, how much powder is used per gallon? *D.; $\frac{1}{16}$ lb., or 1 oz.*

9. Bill used $2\frac{3}{4}$ yd. of wire for each cage for his rabbits. How many yards of wire did he use for 5 cages? *M.; $13\frac{3}{4}$ yd.*

10. For the cages, how many glass containers holding $\frac{3}{16}$ qt. can be filled with water from a quart bottle which is $\frac{3}{4}$ full? *D.;*

11. If it takes $1\frac{1}{2}$ hr. to drive to the city, what part of the distance will Bill and his father drive in $\frac{3}{4}$ hr.? *D.; $\frac{1}{2}$*

12. Bill's father bought $12\frac{3}{10}$ gal. of gasoline at $27\frac{1}{2}$¢ a gallon. How much did he pay for it? (In the answer count any fraction of a cent as 1¢ more.) *M.; $3.39*

13. Bill and his father left the car at a parking lot and walked $\frac{3}{4}$ mi. to the City Market. If a city block is $\frac{1}{20}$ mi., how many blocks did they walk? *D.; 15 blocks*

[W]

Now go back and solve problems 1–13.

Oral Exercises

1. In division, when is the quotient less than 1? *when the dividend is less than the divisor*
2. $\frac{8}{11} \div 4 = \frac{2}{11}$ How do you find the quotient quickly? *8÷4=2 (elevenths)*
3. Tell the smallest common denominator for $\frac{1}{6}$ and $\frac{1}{8}$. *24*
4. Is $\frac{5}{6} \times \frac{2}{3}$ equal to $\frac{2}{3} \times \frac{5}{6}$? *Yes* Explain. *Order of factors does not affect product.*
5. What is the first step in dividing 6 by $2\frac{2}{3}$? *Change $2\frac{2}{3}$ to improper fraction, $\frac{8}{3}$.*
6. What is the first step in dividing $\frac{2}{3}$ by 4? *Invert divisor, 4.*
7. How many $\frac{1}{2}$'s are there in 7 wholes? *14*
8. To find $\frac{1}{2}$ of 5, can you write "$\frac{1}{2} \div 5 = \frac{1}{2} \times \frac{1}{5}$"? *No*
9. How many $\frac{3}{10}$'s are there in $\frac{9}{10}$? *3*

Teaching Pages 200 and 201

Pupil's Objectives. (*a*) To solve problems involving multiplication and division of fractions; (*b*) to review techniques and understandings related to work with fractions.

Background. Differentiating multiplication and division problems involving fractions helps children to clarify their ideas about the meaning of multiplication and division while, at the same time, offering further practice in working computation of this kind. Also, of course, pupils gain an increased awareness of the usefulness of fractions in everyday affairs.

Book Lesson (page 200 and top of page 201). *Ex. 1–13:* Oral work. *Bottom of lesson:* Written work. Have pupils read the problems orally and discuss what has to be done to find each answer.

Book Lesson (bottom of page 201). *Ex. 1–9:* Oral work. As a reminder of concepts previously learned, you may want pupils to do this oral work before solving the problems in the lesson that begins on page 200.

Differentiations and Extensions. *More capable pupils* may work the following problems:

a. Mary needed $\frac{3}{4}$ cup of sugar for the cookies she was making. She did not have enough white sugar so she decided to use half white and half brown. How much of each did she use? Find the answer first by multiplying and then by dividing. ($\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; $\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$)

b. The girls made $3\frac{1}{2}$ dozen cookies to sell. How many bags with $\frac{1}{4}$ dozen cookies each can they fill? (14)

c. They figure that each cooky had cost $1\frac{1}{2}$¢. How much did the batch of cookies cost? ($3\frac{1}{2} \times 12 = 42$ cookies; $42 \times 1\frac{1}{2}$¢ $= 63$¢)

Teaching Pages 202 and 203

Pupil's Objective. To discover how to find *n*, a missing factor or a missing product, both when the example is written as a multiplication example and when it is written as a division example.

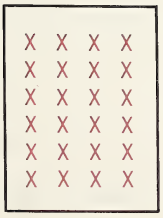
Background. In their work with whole numbers pupils have already learned that multiplication and division are opposite processes. Multiplication involves combining groups, while division involves separating groups. Therefore, since $5 \times 7 = 35$, we know that $35 \div 5 = 7$. It is possible in the latter case to think of the dividend 35 as the product and to think of the divisor and the quotient as the factors of that product. If pupils see these relationships clearly, they may develop independently the generalization given on page 203 in the text. This is true of both whole numbers and fractions.

Pre-book Lesson

1. To discover how much your pupils know about the factor-product relationship when whole numbers are used, present an example such as that shown in box A on page 202 in the text, and see if they can tell how they would find *n* in each case. Do the same thing using division examples similar to those in box B on the same page.

2. If pupils seem confused and do not understand how multiplication and division are related, use objects or present a pattern such as the one at the right so that they will see how 6 rows of 4 objects give these four multiplication and division facts:

$4 \times 6 = 24$ $\quad$ $24 \div 6 = 4$
$6 \times 4 = 24$ $\quad$ $24 \div 4 = 6$



Finding n

Factor-product relationship (O)

A

$5 \times 7 = n$ (35; $5 \times 7 = 35$)	$5 \times n = 35$ (7; $35 \div 5 = 7$)	$n \times 7 = 35$ (5; $35 \div 7 = 5$)

1. In the first example in box A, **n** stands for a missing product. In the other two examples, **n** stands for a missing factor. Explain how to find **n** in each example. (See box.)

2. If you know all the factors, explain how to find the product. Multiply the factors.

3. If you know a product and one of its two factors, how do you find the missing factor? Divide product by known factor.

The relationship between a product and its factors is the same for whole numbers, fractions, and mixed numbers.

4. $\frac{1}{7} \times 14 = n$. In this example, **n** is the product of $\frac{1}{7}$ and 14. Explain how to find the value of **n**. $\frac{1}{7} \times 14 = \frac{14}{7} = 2$

5. In the example $\frac{1}{7} \times n = 2$, a product and one of its factors are given. To find **n**, the missing factor, why can you divide the product by the known factor? On the board, divide 2 by $\frac{1}{7}$ and see if you get the missing factor. Then explain how to find **n** in $n \times 14 = 2$. product ÷ one factor = other factor. $n = 2 \div 14 = \frac{1}{7}$

6. Explain how to find **n** in each of the following:

a. $\frac{3}{5} \times \frac{5}{7} = n$ $(\frac{3}{7})$ $\quad$ **b.** $4\frac{1}{2} \times n = 3$ $(\frac{2}{3})$ $\quad$ **c.** $n \times \frac{3}{10} = \frac{1}{4}$ $(\frac{5}{6})$

d. $\frac{7}{16} \times n = \frac{7}{12}$ $(\frac{1}{3})$ $\quad$ **e.** $8 \times 5\frac{3}{4} = n$ (46) $\quad$ **f.** $n \times 3\frac{1}{7} = 66$ (21)

7. In $35 \div 5 = 7$, which number is the dividend? 35 the divisor? 5 the quotient? 7 Can we think of the dividend as the product of the divisor and the quotient? yes Explain. ★ Using what you know about a product and its factors, explain how to find **n** in each of the examples in box B.

B

$35 \div n = 5$ (7)
$35 \div 7 = n$ (5)
$35 \div 5 = n$ (7)
$35 \div n = 7$ (5)

202

For the multiplication facts, pupils must think of rows and columns of equal groups. For the division facts, they must think of the total separated into rows and columns of a certain number each.

3. Finally, present examples like Ex. 6 and 13 in the text to see if pupils can tell without further

★Answers Not on Reproduced Page 202

7. Dividend ÷ divisor = quotient; i.e., the quotient is the number of times that the divisor is contained in the dividend, or, quotient × divisor = dividend.

factors — product

8. In $2 \div \frac{1}{7} = 14$, which number is the dividend? 2 the divisor? $\frac{1}{7}$ the quotient? 14 Which number can we think of as a product? 2 Which numbers are its factors? $\frac{1}{7}$ and 14

9. Explain how we make use of the relationship between a product and its factors when we check our answer to a division example. quotient × divisor = dividend

10. Explain how to find **n** in $2 \div \frac{1}{7} = \text{n}$. $n = 2 \div \frac{1}{7} = 14$

11. To find **n** in $2 \div \overset{\frac{1}{7}}{\text{n}} = 14$, why can we divide 2 by 14? product ÷ denom. factor = n Prove your answer by working the example on the board.

12. Explain the work in boxes C and D.

C

$$\frac{1}{2} \div \text{n} = \frac{2}{3};\ \text{n} = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

D

$$\frac{1}{2} \div \frac{3}{4} = \text{n};\ \text{n} = \frac{1}{2} \div \frac{3}{4} = \frac{1}{\cancel{2}_{1}} \times \frac{\overset{2}{\cancel{4}}}{3} = \frac{2}{3}$$

13. Tell what you would do to find **n** in each of Ex. a–f.

a. $\frac{3}{5} \div \text{n} = 4\frac{1}{2}$ ($\frac{2}{15}$) b. $\frac{7}{8} \div \frac{1}{2} = \text{n}$ ($1\frac{3}{4}$) c. $8 \div 3\frac{3}{7} = \text{n}$ ($2\frac{1}{3}$)
d. $33 \div \text{n} = 5\frac{1}{2}$ (6) e. $\frac{9}{10} \div \text{n} = \frac{2}{5}$ ($2\frac{1}{4}$) f. $2\frac{4}{5} \div 9\frac{1}{3} = \text{n}$ ($\frac{3}{10}$)

Whenever we know a product and one of its two factors, we can find the missing factor by dividing the product by the known factor.

[W]

Find **n** in each of Ex. 14–25. Write all your work.

14. $\overset{9}{\text{n}} \times \frac{5}{12} = 3\frac{3}{4}$	18. $\overset{5}{\text{n}} \times 8\frac{2}{5} = 42$	22. $\overset{2\frac{1}{2}}{\text{n}} \times 3\frac{3}{5} = 9$
15. $\frac{7}{8} \times \overset{32}{\text{n}} = 28$	19. $9 \div 3\frac{3}{5} = \overset{2\frac{1}{2}}{\text{n}}$	23. $5 \div 8\frac{3}{4} = \overset{\frac{4}{7}}{\text{n}}$
16. $1\frac{7}{8} \div \overset{2\frac{1}{4}}{\text{n}} = \frac{5}{6}$	20. $\frac{11}{12} \div \overset{\frac{5}{8}}{\text{n}} = 7\frac{1}{3}$	24. $3\frac{1}{2} \div \text{n} = \frac{5}{12}$ ($\frac{8\frac{2}{3}}{\frac{4}{7}}$)
17. $7\frac{3}{4} \div 2 = \overset{3\frac{7}{9}}{\text{n}}$	21. $3\frac{7}{8} \times \overset{2}{\text{n}} = 7\frac{3}{4}$	25. $8\frac{3}{4} \times \overset{\frac{4}{7}}{\text{n}} = 5$

explanation how to find *n* in multiplication and division examples involving fractions.

Book Lesson. *Ex. 1–13:* Oral work. *Ex. 14–25:* Written work.

Differentiations and Extensions

1. *Slower learners* may need much work in finding *n* in multiplication and division examples involving whole numbers as background for work with fractions. If so, use the following examples:

$8 \times 31 = n$	$8 \times n = 248$	$n \times 31 = 248$
$45 \times n = 270$	$n \times 13 = 169$	$3 \times n = 441$
$248 \div 8 = n$	$248 \div n = 31$	$n \div 8 = 31$
$270 \div n = 45$	$169 \div n = 13$	$n \div 3 = 147$

2. For Ex. 14 to 25 at the bottom of page 203, it may be advantageous to have *slower learners* tell just what they are going to do to find *n* in each example. In any case, be sure pupils show all the written work for these examples so that you will be able to tell whether they are having difficulty in multiplying and dividing fractions or in knowing how to proceed to find *n*.

3. There may be a few pupils whom you will want to instruct in the algebraic method for solving examples 14 to 25. They may be capable, for example, of writing Ex. 14 as at the right. The use of the complex fraction is not new to them, and this form of solving for *n* will be exceedingly useful in later work in mathematics. From the complex fraction, *n* can be found using the common-denominator method or the example can be rewritten and inversion used. At this time the important point is the form for writing the complex fraction.

$$n = \frac{3\frac{3}{4}}{1\frac{5}{12}}$$

Teaching Pages 204 and 205

Pupil's Objectives. (*a*) To learn about the three types of problems involving multiplication and division; (*b*) to discover that the three types of problems occur in multiplication and division of fractions as well as with whole numbers.

Background. Now that pupils have reviewed the factor-product relationship, know the meaning of ratio, and can work multiplication and division in problems involving whole numbers or fractions, the three types of problems that can exist in multiplication and division are conveniently summarized in a chart. Notice that the how-many-times and what-part-of concept is important here in finding the ratio factor.

Pupils have solved the three types of problems before, but now they are identified by the terms *Type I*, *Type II*, and *Type III*. These terms will be used continuously in the problem-solving program throughout the year for work involving not only whole numbers and fractions, but also decimals and per cents.

Pre-book Lesson. Write on the board a quantitative situation similar to the one in the first column on the next page. Beneath it write the three kinds

Three Kinds of Problems in M. and D.

With whole numbers [O]

When you understand how a product and its factors are related, you have mastered one of the most important ideas of arithmetic. You have often used this idea in solving problems with whole numbers. Sometimes you have multiplied two factors together to find a product; sometimes you have divided a product by one factor to find the other factor.

You know that a divisor or a quotient can be thought of as a factor of the dividend. For this reason, we can state any problem of this kind as a multiplication example, using **n** to stand for the missing number. Study Ex. 1–3 to see how this works.

1. Rae has 6 tropical fish. Nancy has 3 times as many. How many fish does Nancy have? (Is $3 \times 6 = n$ a correct statement of the example? Yes Tell how to find **n**.) $n = 3 \times 6 = 18$

2. Nancy has 18 tropical fish, and Rae has 6. Nancy has _?_ times as many fish as Rae. (Is $n \times 6 = 18$ a correct statement of the example? Yes Tell how to find **n**.) $n = 18 \div 6 = 3$

3. Nancy has 18 tropical fish. This is 3 times as many fish as Rae has. How many fish has Rae? (Is $3 \times n = 18$ a correct statement of the example? Yes Tell how to find **n**.) $n = 18 \div 3 = 6$

We work just the same way when the product and its factors are fractions or mixed numbers. It helps us to understand the problem question if we think of each problem as belonging to one of these three kinds, or types:

Type I. We know two factors. We want to find their product.

Type II. We know the product and one of its factors. We want to find the *ratio* factor—that is, the factor that shows how the product and the known factor are related. The ratio factor is either a how-many-times number or a what-part-of number.

Type III. We know the product and the *ratio* factor. We want to find the other factor.

With fractions

The boxes below show how to work each type of problem. Study the work and see if you understand every step.

Finding the product. Ralph swam 75 yd. Joe swam $\frac{3}{5}$ as far. How far did Joe swim? $\frac{3}{5} \times 75 = n$ **You know the two factors.** $n = 45$ **You multiply to find the product.**	Type I $\frac{3}{5} \times 75 = n$ Multiply
Finding the factor which shows a relationship, or ratio. Ralph swam 75 yd., and Joe swam 45 yd. Joe's distance was what part of Ralph's? $n \times 75 = 45$ **You know the product and one factor.** $n = 45 \div 75 = \frac{45}{75} = \frac{3}{5}$ **You divide to find the missing factor.** The answer, $\frac{3}{5}$, tells us that Joe's distance was $\frac{3}{5}$ of Ralph's. Is it a how-many-times number or a what-part-of number? The answer to a Type II problem shows the relation between the product and the known factor. It may be a what-part-of number or a how-many-times number.	Type II $n \times 75 = 45$ Divide
Finding the other factor when the given factor is the ratio. Joe swam 45 yd. This was $\frac{3}{5}$ as far as Ralph swam. How far did Ralph swim? $\frac{3}{5} \times n = 45$ **You know the product and one factor.** $n = 45 \div \frac{3}{5} = 45 \times \frac{5}{3} = 75$ **You divide to find the missing factor.**	Type III $\frac{3}{5} \times n = 45$ Divide

Tell which type of fraction problem each of these is:

4. Sam's father planted $\frac{3}{4}$ of a 12-acre lot in corn. How many acres did he plant in corn? ($\frac{3}{4} \times 12 = n$) I ($n = 9$)

5. The meat of a pecan is $\frac{2}{5}$ of the weight of the nut. How many pounds of whole pecans are needed to make 10 lb. of nut meats? ($\frac{2}{5} \times n = 10$) III ($n = 25$)

6. Of the 36 books on a shelf, 20 books are histories. What part of the books are histories? ($n \times 36 = 20$) II ($n = \frac{5}{9}$)

of problems that might result if a different piece of information were missing each time.

Glenn had 12 hits in last week's baseball games, or 3 times as many as the 4 hits made by Vin.

a. Glenn had 3 times as many hits as the 4 hits made by Vin. How many hits did Glenn make?

b. Glenn had 12 hits and Vin had 4 hits. The number of hits made by Glenn was how many times as great as the number made by Vin?

c. Glenn had 12 hits, or 3 times as many hits as Vin. How many hits did Vin make?

Let pupils set up on an n-stand (see page 314 in the Manual) the example that describes the quantitative situation. As they cover with an n-card the information that is missing in each problem, have them tell whether it is the product, the ratio factor, or the other factor that must be found. During the discussion, make a record on the board similar to the following:

Glenn's hits
Vin's hits
Ratio of Glenn's to Vin's hits

	3	×	4	=	12
Type I. Finding the product	3	×	4	=	$\boxed{n}$
Type II. Finding the ratio factor	$\boxed{n}$	×	4	=	12
Type III. Finding the other factor	3	×	$\boxed{n}$	=	12

Book Lesson. *Ex. 1–6:* Oral work. Have pupils read and discuss each step on page 204. After the explanation at the bottom of the text page, have

pupils use one of the problems in Ex. 1–3 to illustrate each type of problem. Follow the same procedure on page 205, letting pupils fit the problems in Ex. 4, 5, and 6 to the three types of problems illustrated in the table on the same page. Pupils will find that this table will serve as a useful reference as they continue in the problem-solving program.

Differentiations and Extensions. *All pupils* may work the following problems:

a. Tom earned 28¢. Ben earned 14¢. Write and solve a multiplication or division problem that tells how Ben's money can be compared with Tom's money.

b. Mary sold 42 tickets for the concert. Edith sold $\frac{2}{3}$ as many? How many tickets did Edith sell?

c. Bill has 25 marbles or $\frac{1}{5}$ the number John has. How many marbles has John?

Before solving the problems suggested above, it would be helpful for the class to name the type for each and to write the example for each problem on the board.

Teaching Pages 206 and 207

Pupil's Objectives. (*a*) To obtain practice in recognizing and solving the three types of problems in multiplication and division; (*b*) to review meanings, processes, and techniques which have been previously learned.

Background. After learning about the three types of multiplication and division problems, pupils are asked to put the concept to work by writing the example and telling the type of problem given in Ex. 1 to 9 on page 206 in the text. They then complete the process by solving the examples they have written and others (rows 10 to 12) written with *n*.

Keeping alive understandings of concepts is just as important as maintaining skills and abilities in computation. This goal is partially achieved by the work on page 207 in the text. Notice, also, that Ex. 1 serves as review for what pupils have learned about scales and graphs and also as preparation for the work with graphs that follows on the next four pages in the text.

Book Lesson (page 206). *Ex. 1–9:* Oral work. *Rows 10–12:* Written work.

1. After pupils have read a problem and told its type, the example to be used in solving might be written on the board. *Slower learners* may need to do this for every problem.

2. Assign problems 1–9 and rows 10–12 for written work. If the examples have not been put on the board, pupils' papers should be checked to see that they are using the correct equation.

Book Lesson (page 207). *Ex. 1–13:* Written work. The results of this page will give a good basis for grouping your class for remedial work. Assign the page for independent written work.

Differentiations and Extensions. As needed, assign **Extra Practice Set 90** (given with answers on the next Manual page.)

Three Kinds of Problems

Give the example you would use for each problem and te
which kind it is (Type I, II, III).

1. Mrs. Wait had saved $20. She spent $\frac{3}{4}$ of it for a jacke
How much did the jacket cost? $n = \frac{3}{4} \times \$20$; I; $n = \$15$

2. A grocer sold 140 oranges out of a box of 196 orange
What part of the box did he sell? $140 = n \times 196$; II; $n = \frac{5}{7}$

3. A pail which was $\frac{2}{3}$ full contained 6 qt. of milk. How man
quarts would the pail hold if full? $6 = \frac{2}{3} \times n$; III; $n = 9$

4. Jim has walked $\frac{3}{4}$ mi. of a $4\frac{1}{2}$-mile hike. What part of th
hike has he walked? $\frac{3}{4} = n \times 4\frac{1}{2}$; II; $n = \frac{1}{6}$

5. Joe was to have $\frac{2}{5}$ of all the cherries he picked. If hi
share was 18 qt., how many quarts did he pick in all? $\frac{2}{5} \times n =$
III; $n = 45$ qt.

6. In a year Miss Reed saved $234 by putting aside $\frac{3}{20}$ o
her salary. What was her salary? $\frac{3}{20} \times n = \$234$; III; $n = \$1,560$

7. $\frac{3}{16}$ of a farm of 300 acres is a wood lot. How many acre
are there in the wood lot? $\frac{3}{16} \times 300 = n$; I; $n = 56\frac{1}{4}$

8. We are trying to raise $500 for playground equipment
We now have $240. What part of the total have we raised? $n \times \$500 = \$240$; II; $n =$

9. When Bob has read 40 pages of a story of 90 pages, wha
part of the whole story has he read? $n \times 90 = 40$; II; $n = \frac{4}{9}$

Now go back and write your work for Ex. 1–9.

Find **n** in each example below. Write all your work.

	a	b	c
10.	$36 = \frac{1}{4}$ of n (144)	$5 = \frac{1}{12}$ n × 60	$\frac{4}{5}$ of 30 =
11.	$6 = \frac{3}{5}$ of n (10)	$\frac{2}{3}$ of $\frac{6}{10} = \frac{2}{2}$ n	$\frac{3}{8}$ of n (192) = 72
12.	$4 \times \frac{2}{2}$ n = 10	$\frac{4}{3}$ of 9 = (12) n	$\frac{1}{4} = \frac{1}{20}$ n × 5

Extra Practice. For more practice, work Set 90.

206

Thinking about What You Have Learned

[W]

Write your answers.

1. On a city map a small park is $\frac{3}{8}$ in. long. The scale of the map is *1 in. = 1,000 ft.* How many feet long is the park? *375 ft.*

2. Show on a number line that $1\frac{1}{2} \div \frac{3}{4} = 2$. *(1) (2)*

3. Copy these multiplication statements and label in each the multiplicand, the multiplier, and the product: *(M) (P)*

a. $73 \times 5 = 365$ *M P*

b. $\frac{2}{3} \times 6 = 4$ *M P*

c. $8 \times \frac{4}{5} = 6\frac{2}{5}$ *M P*

d. Seven $\frac{1}{4}$'s $= 1\frac{3}{4}$ *M P*

4. Copy and show how to use cancellation in $\frac{3}{4} \times 3\frac{1}{3}$. *(2½)*

5. Round to the nearest whole number:

a. $7\frac{2}{3}$ *8* b. $3\frac{2}{5}$ *4* c. $6\frac{7}{8}$ *7* d. $2\frac{4}{9}$ *2*

6. Find the product of $3\frac{1}{4}$ and 28 without changing $3\frac{1}{4}$ to an improper fraction. *(3×28)+(¼×28) = 84+7 = 91*

7. Copy these division statements and label the dividend, the divisor, and the quotient: *(D) (Q)*

a. $3 \div \frac{1}{2} = 6$; *D Q*

b. $\frac{1}{3} \div 5 = \frac{1}{15}$. *D Q*

8. a. Since $3 \div \frac{3}{4} = 4$, then $6 \div \frac{3}{4} =$ *8*

b. Since $12 \div \frac{1}{3} = 36$, then $12 \div \frac{2}{3} =$ *18*

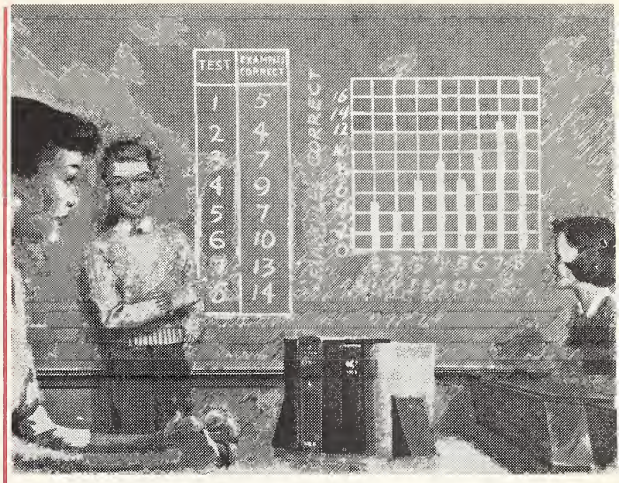
9. Show with an example that if the dividend is more than the divisor, the quotient is more than 1.

10. Show with an example that dividing a smaller number by a larger is finding what part the smaller number is of the larger.

11. Give an example to show that when 1 is divided by a fraction, the answer is the fraction inverted.

12. Write the division statement in which $\frac{3}{10}$ is the divisor, $\frac{3}{5}$ is the dividend, and 2 is the quotient. *$\frac{3}{5} \div \frac{3}{10} = 2$*

13. Write in your own words the rule for division with fractions by the inversion method. *(Similar to that at bottom of text page 199.)*



A Graph Showing Change

Line graph from bar graph [O]

Each month Miss Pike gave her class an arithmetic test with 15 questions. At the end of 8 months, Ken made a table to show the number of correct answers he had had on each test. Then he made a graph of these test scores.

Miss Pike asked Ken to put his work on the board so she could show the class something new about graphs. She asked the following questions. Can you answer them from Ken's work?

1. What two things does the table show? *Test number, and number of correct examples for each.*
2. What two things can you tell from the graph? *(Same as for Ex. 1.)*
3. How can you tell from the graph whether Ken's arithmetic tests became better or worse as the year moved along? *By comparing the height of the bars*
4. On which test did Ken get the best score? *5* the poorest? *2*
5. What kind of graph do we call this one on the board? *bar*

Set 90. Finding n in multiplication examples containing fractions

Find the value of n.

	a	b	c
1.	$\frac{2}{3}$ of $15 = n$ **10**	$n \times 12 = 8$ $\frac{2}{3}$	$\frac{3}{4}$ of $n = 15$ **20**
2.	$32 \times n = 16$ $\frac{1}{2}$	$10 \times n = 6$ $\frac{3}{5}$	$n \times \frac{5}{6} = 20$ **24**
3.	$n \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2}$	$\frac{3}{8}$ of $16 = n$ **6**	$9 \times n = 6$ $\frac{2}{3}$
4.	$6 \times n = \frac{3}{4}$ $\frac{1}{8}$	$5 \times n = \frac{1}{2}$ $\frac{1}{10}$	$n \times 4 = \frac{8}{9}$ $\frac{2}{9}$
5.	$\frac{3}{5}$ of $20 = n$ **12**	$n \times \frac{5}{6} = 10$ **12**	$3 \times \frac{2}{3} = n$ **2**
6.	$n \times \frac{7}{8} = \frac{5}{8}$ $\frac{5}{7}$	$\frac{3}{7}$ of $7 = n$ **3**	$\frac{5}{8}$ of $n = 20$ **32**

Teaching Pages 208 and 209

Pupil's Objectives. (*a*) To review reading bar graphs; (*b*) to learn that line graphs show change clearly and may be made from bar graphs.

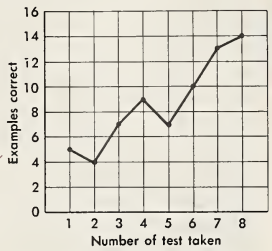
Background. Pupils have already reviewed the use of a table and a bar graph. In the text, both of these forms for presenting information are used to introduce line graphs. The essential point to be made is that a line graph shows to better advantage situations in which the change is the important item of information.

Miss Pike said that there is a kind of graph which shows more clearly than the vertical-bar graph whether a change is taking place for the better or for the worse. She put on the board a graph like the one at the right just below.

6. Instead of making bars, Miss Pike put a dot where the top of each bar would be. Read the number of examples correct for each test.

Then what did she do to make the line? *She connected the dots.*

We call this kind of graph a **line graph.** It is used often to show a change which is taking place.



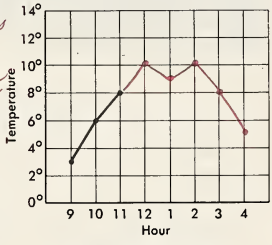
Examples Correct on Monthly Tests

What change would you say has taken place in Ken's work on his tests? *His score has improved.*

What does it mean when the line goes up? *higher score on next test* down? *lower score*

[W]

7. On a very cold day, Ted wrote down the temperature every hour from 9 to 4. Copy the graph at the right and finish it from the table below.



Temperature Every Hour

Time	Temperature	Time	Temperature	Time	Temperature
9 A.M.	3°	12 M.	10°	3 P.M.	8°
10 A.M.	6°	1 P.M.	9°	4 P.M.	5°
11 A.M.	8°	2 P.M.	10°		

The graph at the top right of page 209 in the text corresponds to the bar graph shown in the picture at the top of page 208. By comparing these two forms of presenting material, children see how a line graph emphasizes change.

Since setting up an appropriate grid and deciding on an appropriate scale is not always an easy matter, pupils are not yet asked to construct line graphs independently. Instead, they are asked to use information shown in a table in completing graphs given in the text as models.

Teacher's Preparation. Find the attendance record or temperature record for the past week and arrange the figures on the board in table form.

Pre-book Lesson. As a substitute for the review on page 208 in the text, pupils may use the attendance record or the temperature record for the past week to make a bar graph on the board. Discuss ways of preparing the table for the graph before the actual work is done.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7:* Written work. A discussion of Ex. 6 will enable pupils to understand the transfer of information from a bar graph to a line graph. The graph in Ex. 7 may be copied on the board and worked out co-operatively. Let pupils decide what information the horizontal lines will show and what the vertical lines will show. They must also decide what value each grid space will have.

Differentiations and Extensions

1. The work in the text might be followed by a discussion concerning the advantages of each type of graph. The fact should be brought out that sometimes economy of time is most important and that one should therefore choose the type of graph that can be made and read quickly and easily.

To expand this point, *all pupils* may use the table on the board (see under Pre-book Lesson) to make a bar graph and a line graph. They should then decide the advantages and disadvantages of each type of graph for the information being presented.

2. *All pupils* may collect pictures of graphs for bulletin-board display or for their notebooks.

3. *More capable pupils* may refer to an almanac, or other suitable source, for information concerning normal annual precipitation for a few selected states. They may round these numbers and use them in making either a bar or line graph, whichever they think suitable for the situation.

Teaching Pages 210 and 211

Pupil's Objectives. (*a*) To obtain practice in reading tables involving large numbers; (*b*) to learn how to use rounded numbers in making line graphs.

Background. Information given in table or graph form is often such that it need only represent approximations, exact data either being impossible or unnecessary. Frequently, too, when information is reproduced in either of these forms, it is for convenience in making comparison. Since we often

The Number of Telephones in the World in a Recent Year

Continent	Total telephones	Automatic (or dial)
North America	43,948,000	29,463,000
South America	1,657,000	1,215,000
Europe	20,000,000	12,900,000
Africa	806,000	525,000
Asia	2,367,000	978,000
Oceania	1,522,000	949,000

Reading a Table of Large Numbers

Comparing items [O]

The table above tells the number of telephones in the world by continents in a recent year and also the number of those telephones which are automatic (or dial).

The first line across is read this way:

"North America had in all 43,948,000 telephones. Of these, 29,463,000 were dial telephones."

1. Read the second line; the other lines.

2. Do you think that the numbers given in the table are exact numbers? No Explain your answer. The zeros show that the numbers are rounded to the nearest 1,000.

3. Which continent had the most telephones? North America the fewest? Africa

4. On which continent were fewer than half the telephones automatic? Asia

5. Which two continents had about the same number of dial telephones? Asia and Oceania

6. Which two continents had about the same number of telephones in all? South America and Oceania

7. Using the numbers in the table, find on the board about how many telephones there were in the world that year. 70,300,000

8. In a bar graph made from the table above, for which continent would the bar be longest? North America shortest? Africa

Graphing Large Numbers

Line and bar graphs [O]

1. Why do you think large numbers are more often graphed than small ones? Harder to compare large numbers by computation.

2. We can make large numbers more usable by rounding them. Round 385,160 to ten thousands. 390,000

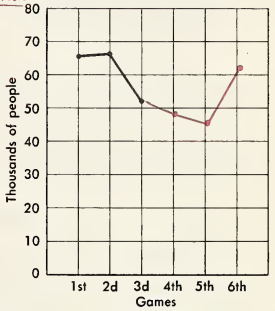
3. When you round 385,160, you can think of it as about 390,000 or as about 39 ten thousands. Which way of rounding would be easier to use in making a graph? Explain. Each division could then stand for ten thousands. one the scale.

[W]

4. In a recent year the paid attendance at the World Series games was as follows: Thousands

1st game,	65,673	66
2d game,	66,804	67
3d game,	52,035	52
4th game,	49,010	49
5th game,	47,530	48
6th game,	61,711	62

Make a table showing each of the numbers above as a number of thousands. (65,673 rounded to the nearest thousand is 66,000; so the first number in your table will be 66.)



Attendance at a Recent World Series

5. Copy and complete the line graph at the right above. Use the table you made for Ex. 4. Make the vertical scale: $\frac{1}{2}$ *in.* = *10* (*thousand people*).

6. On one round-trip voyage a large ocean liner carried 70,000 lb. of meat, 25,000 lb. of poultry, 12,000 lb. of fish, 60,000 lb. of potatoes, 30,000 lb. of fresh vegetables, 10,000 lb. of frozen vegetables, and 12,000 lb. of sugar. Show these facts by a horizontal-bar graph. You can use the following scale: $\frac{3}{8}$ *in.* = *10* (*thousand pounds*).

have difficulty comparing large numbers, we usually round numbers that are to be presented in tables and graphs.

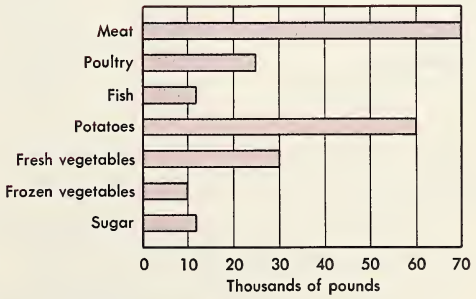
Teacher's Preparation. Put on the board the populations and areas of the Canadian Provinces and Territories as shown in the table under Differentiations and Extensions.

Book Lesson (page 210). *Ex. 1–8:* Oral work. From the oral exercises on this page, the class should develop a technique for interpreting tables. It is important that they understand that in tables of this sort the figures cannot be exact because they are changing from day to day. In this case,

(*Continued on page 224*)

★Answers Not on Reproduced Page 211

6.



Food Carried on One Ocean Liner

rounded numbers are sufficient to give the required comparison and are easier to handle than exact numbers.

Book Lesson (page 211). *Ex. 1–3:* Oral work. *Ex. 4–6:* Written work.

1. Guide the class in discussing Ex. 1–3 so that the following facts will be brought out:

a. Small numbers are easy to understand and compare. Thus they are not often graphed.

b. Large numbers are difficult to work with and, therefore, are often graphed to help the reader understand and use them.

c. In graphs or tables, large numbers are often expressed by small numbers which represent only the number of thousands, millions, or the like. The place value of the numbers used is usually written in the title or as a part of the scale explanation (the scale on the graph in the text reads "thousands of people").

2. For Ex. 4, the table with rounded numbers might be put on the board.

3. Ex. 5 and 6 can be worked out co-operatively at the board, or *more capable pupils* may do them individually.

Differentiations and Extensions

1. The following data can be used with *all pupils* to give more experience in interpreting tables:

Area and Population of Canadian Provinces and Territories

Province or Territory	*Land Area* (Sq. mi.)	*Population* (1951)
Alberta	255,285	939,501
British Columbia	366,255	1,165,210
Manitoba	246,512	776,541
New Brunswick	27,985	515,697
Newfoundland	155,364	361,416
Nova Scotia	21,068	642,584
Ontario	412,582	4,597,542
Prince Edward Island	2,184	98,429
Quebec	594,860	4,055,681
Saskatchewan	251,700	831,728
Northwest Territories	1,304,903	16,004
Yukon Territory	207,076	9,096

Have each line read as follows: Alberta covers an area of 255,285 square miles and has a population of 939,501.

a. All pupils may use the table in answering questions such as these:

Which province or territory covers the largest area?
Which province or territory has the largest population? the smallest population?
Which province is second in area?
Which province is second in population?

b. Slower learners may make a new table rounding each figure to the nearest thousand.

c. More capable pupils might make two line graphs, one to show land area, and the other to show population for the Canadian Provinces and Territories. These pupils might also be shown how to put the two sets of information on one graph by using two colors and a key.

2. Pupils may collect tables of large numbers for their notebooks or the bulletin board. They may also continue collecting graphs for the same purpose.

3. For interested pupils, the work on these pages provides numerous opportunities for correlating arithmetic with other subjects. For example, pupils who have a special interest in history or geography may look up information in encyclopedias and other reference books on a subject, such as the area and cost of various land purchases made by the United States in the 19th century. When they have collected these data, they may list them in table form, showing exact measurements and amounts. Then they may round these numbers and use the rounded numbers in showing the data in graph form.

Other items which might be used for correlating arithmetic and other studies include: statistical tables and the like in science; production and efficiency records for industry; tables of athletic achievements, such as the showing made by members of various teams in a series of Olympic contests; incidence of certain communicable diseases over a period of years; and many others. All of these topics will broaden the scope of the pupils' knowledge and at the same time will help them perfect their skill in collecting and displaying various kinds of data.

Teaching Page 212

Pupil's Objective. To review orally that part of the content of Chapter 5 devoted to work with fractions.

Background. Just prior to the end-of-chapter tests, pupils have an opportunity to discover how much they have learned about fractions in Chapter 5. As a result of this review you may find that

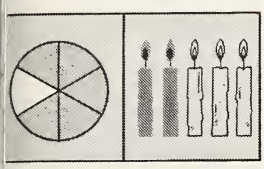
Oral Exercises

Fractions

1. In $\frac{5}{16}$, what tells the number of parts in the fraction? the numerator, 5

2. Use $\frac{3}{4}$ to explain what "fractional unit" means. ★

3. In $\frac{2}{3}$ c., what is the fractional unit? $\frac{1}{3}$ How many equal parts are there in this fraction? 2



4. What part of the circle is colored? $\frac{5}{6}$ What part of the group of candles? $\frac{2}{5}$

5. Joe has 5¢ and Ted 8¢. Tell the ratio of Joe's money to Ted's. $\frac{5}{8}$

6. In the fraction $\frac{7}{16}$, which number is the divisor? 16

7. If you multiply both terms of $\frac{4}{5}$ by 3, what change do you make in the fraction? Terms become larger. What change do you not make? Value is not altered.

8. If you divide both terms of $\frac{9}{12}$ by 3, what is changed? Terms become smaller. What is not changed? Value of fraction ($\frac{3}{4}$) not changed.

9. Give the missing numbers: a. $\frac{2}{3} = \frac{6}{9}$ b. $2\frac{5}{10} = 2\frac{1}{2}$ c. $\frac{20}{5} = 4$ d. $\frac{10}{8} = 2\frac{3}{8}$ e. $2\frac{3}{5} = \frac{13}{5}$ f. $\frac{15}{24} = \frac{5}{8}$

10. $\frac{4}{9}$ means the ratio of 4 to 9.

11. What is the ratio of a foot to an inch? 12 to 1 of an inch to a foot? 1 to 12, or $\frac{1}{12}$ of a quart to a gallon? $\frac{1}{4}$ of a mile to a foot? 5,280 / 1

12. In one school 4 out of every 5 of the pupils are right-handed. What part of the pupils are right-handed? $\frac{4}{5}$

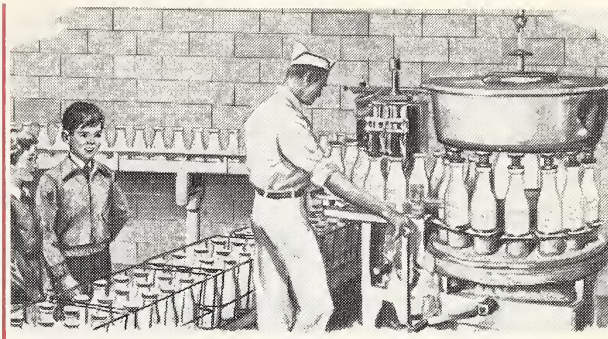
13. Read as division: $\frac{2}{3}$; 2÷3 $\frac{86}{17}$; 86÷17 $\frac{3}{5}$; 3÷5 $\frac{2}{1}$. 2÷1

14. Change to best form: $\frac{10}{3}$; $5\frac{1}{3}$ $\frac{9}{5}$; $1\frac{4}{5}$ $\frac{11}{11}$; 1 $\frac{10}{2}$; 5 $\frac{21}{9}$; $2\frac{1}{3}$ $\frac{32}{6}$; $5\frac{1}{3}$

15. What is the smallest common denominator for each pair of fractions? a. $\frac{1}{2}$, $\frac{3}{4}$; 4 b. $\frac{5}{6}$, $\frac{2}{3}$; 6 c. $\frac{1}{3}$, $\frac{1}{2}$; 6 d. $\frac{1}{4}$, $\frac{1}{6}$; 12

16. Add: a. $\frac{1}{2} + \frac{7}{2}$; 4 b. $\frac{3}{10} + \frac{2}{5}$; $\frac{7}{10}$ c. $2\frac{5}{8} + \frac{3}{4}$; $3\frac{3}{8}$ d. $1 + \frac{2}{3} + 4\frac{1}{3}$. 6

17. Subtract each number from 3: $\frac{1}{4}$; $2\frac{3}{4}$ $1\frac{3}{5}$; $1\frac{2}{5}$ $\frac{7}{10}$; $2\frac{3}{10}$ $2\frac{3}{8}$; $\frac{7}{8}$ $1\frac{4}{7}$. 1



Can You Solve Problems?

Problem Test 5

1. Mr. Henry says that his machine can fill bottles at the rate of 1 every $\frac{4}{5}$ sec. How many can it fill in 1 minute? 75

2. Dick's notebook has 100 pages. If Dick has used 60 pages of his notebook, what part of it is used up? $\frac{3}{5}$

3. Mary has 5 apples. If she needs $\frac{1}{3}$ of an apple for each portion of a salad, how many portions can she make? 15

4. A telephone crew averages $1\frac{1}{2}$ mi. a day in laying cable. How many days will it take this crew to lay 20 mi. of cable? $13\frac{1}{3}$ da.

5. A meat cutter wrapped $1\frac{1}{8}$ lb. of hamburger in each package for the freezer. How many packages could he get from $15\frac{3}{4}$ lb. of hamburger? 14

6. Mr. James owns $26\frac{1}{2}$ A. of cotton. He picked $2\frac{2}{3}$ A. to get a bale of cotton. At that rate, how many bales can he make? $9\frac{15}{16}$, or almost 10 bales.

7. How many notebooks $\frac{3}{8}$ in. thick are there in a stack 9 in. high? 24

8. John says that he can walk $3\frac{1}{2}$ mi. an hour. At that rate, how long would it take him to walk $\frac{7}{8}$ mi.? 15 min.

9. Mrs. Hall bought $\frac{3}{4}$ yd. of tapestry. She used $\frac{1}{2}$ yd. to cover a cushion. What part of the piece did she use? $\frac{2}{3}$

some reteaching is in order for some pupils. Therefore, this may be considered as a kind of diagnostic test. Other pupils may have no difficulty with the oral review and so it may be considered a part of the maintenance program in work with fractions.

Book Lesson. *Ex. 1–17:* Oral work.

★Answers Not on Reproduced Page 212

2. The fraction $\frac{3}{4}$ means 3 of the 4 equal parts into which something is divided, so the whole is measured in fourths ($\frac{1}{4}$'s); that is, $\frac{1}{4}$ is the fractional unit for the fraction $\frac{3}{4}$.

Teaching Pages 213, 214, and 215

Pupil's Objective. To take the four regular end-of-chapter tests.

Book Lesson (page 213). *Ex. 1–9:* Written work. Problem Test 5.

(*Continued on page 226, second column*)

Do You Understand?

Test of Information and Meaning 5

1. What division method is used in Ex. A? in Ex. B? common denom. / inversion

2. In Ex. B which number is the divisor? $\frac{2}{3}$

A	$3 \div \frac{2}{3} = \frac{9}{3} \div \frac{2}{3} = 4\frac{1}{2}$
B	$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

3. In division by a fraction, what does "invert" mean? Turn the divisor upside down.

4. The example, $6 \div \frac{3}{4} = ?$ is usually read "six divided by $\frac{3}{4}$ is what?" Ask this question another way. How many $\frac{3}{4}$'s in 6 ?

5. In $5 \div 6\frac{2}{3}$ what would be the common denominator? 3

6. Write the example that you would use to solve Ex. C by the inversion method. $4\frac{1}{2} \times \frac{1}{2}$, or $\frac{9}{2} \times \frac{1}{2} = \frac{9}{4} = 2\frac{1}{4}$

7. In which example (A, B, C, D, E, or F), is a fraction divided by a whole number? E

C	$4\frac{1}{2} \div 2 = 2\frac{1}{4}$
D	$3\frac{1}{5} \div 1\frac{2}{3} = 1\frac{28}{25}$
E	$\frac{5}{6} \div 4 = \frac{5}{24}$
F	$7 \div 1\frac{3}{4} = 4$

8. In which examples is a mixed number the dividend? C, D

9. In which example is a whole number divided by a fraction? A

10. Write the example that you would use to solve Ex. F by the common-denominator method. $\frac{28}{4} \div \frac{7}{4} = \frac{28}{7} = 4$

11. Write for Ex. F the example you would use to solve by inverting the divisor. $7 \div \frac{7}{4} = 7 \times \frac{4}{7} = 4$

12. In order to show large numbers on a graph, what can you do to make the numbers easy to use? round them

13. How is a line graph different from a bar graph? You read points on a line instead of bar lengths.

14. In a recent year, Mr. Haspel sold fruit from his orchard as follows: plums, 198 bu.; peaches, 826 bu.; cherries, 162 bu.; apples, 1,432 bu. Round these numbers to hundreds and make a horizontal-bar graph to show these sales. ★

2 hundreds / 8 hundreds / 2 hundreds / 14 hundreds

Do You Make Mistakes?

Diagnostic Test 5

	Use the common-denominator method in rows 1 and 2.	Study Pages	Practice Sets
	a b c d		
1.	$8 \div \frac{3}{4}$ 10 $\frac{2}{3}$ $5 \div \frac{8}{3}$ 7 $\frac{1}{2}$ $6 \div \frac{5}{8}$ 9 $\frac{3}{5}$ $12 \div \frac{4}{5}$ 15	175–178	72–73
2.	$6 \div 2\frac{1}{2}$ 2 $\frac{3}{5}$ $10 \div 3\frac{1}{3}$ 3 $4 \div 1\frac{3}{8}$ 2 $\frac{10}{11}$ $7 \div 2\frac{1}{5}$ 3 $\frac{2}{11}$	179	74
	Invert the divisor and multiply in rows 3–6.		
3.	$\frac{7}{8} \div 6\frac{7}{48}$ $\frac{3}{4} \div 5\frac{3}{20}$ $\frac{2}{3} \div 7\frac{2}{21}$ $\frac{4}{5} \div 8\frac{1}{10}$	186–187	75–77
4.	$1\frac{2}{3} \div 6\frac{5}{9}$ $4\frac{1}{2} \div 2\frac{1}{4}$ $3\frac{3}{4} \div 8\frac{15}{32}$ $2\frac{5}{8} \div 4\frac{21}{32}$	188–189	78–80
5.	$\frac{2}{3} \div \frac{1}{8}$ 5 $\frac{1}{3}$ $\frac{7}{8} \div \frac{2}{5}$ 2 $\frac{3}{16}$ $\frac{1}{6} \div \frac{4}{5}$ $\frac{5}{24}$ $\frac{3}{4} \div \frac{5}{6}$ $\frac{9}{10}$	193–195	81–83
6.	$\frac{1}{2} \div 2\frac{1}{2}$ $\frac{1}{5}$ $\frac{2}{3} \div 1\frac{1}{2}$ $\frac{4}{9}$ $4\frac{2}{3} \div 1\frac{1}{4}$ 3 $\frac{11}{15}$ $1\frac{5}{8} \div 4\frac{2}{3}$ $\frac{3}{12}$	196–197	84–87

How Well Can You Figure?

Computation Test 5

Work the examples in rows 1 and 2 by using common denominators.

	a	b	c	d	e
1.	$3 \div \frac{2}{5}$ 7 $\frac{1}{2}$	$2 \div 1\frac{1}{2}$ 1 $\frac{1}{3}$	$4 \div 5\frac{1}{2}$ $\frac{8}{11}$	$5 \div \frac{3}{4}$ 6 $\frac{2}{3}$	$6 \div 2\frac{2}{4}$ 2 $\frac{3}{11}$
2.	$\frac{3}{4} \div \frac{5}{16}$ 2 $\frac{2}{5}$	$\frac{9}{10} \div \frac{4}{15}$ 3 $\frac{3}{8}$	$3\frac{1}{7} \div \frac{1}{2}$ 6 $\frac{2}{7}$	$\frac{11}{12} \div \frac{3}{8}$ 2 $\frac{4}{9}$	$1\frac{1}{8} \div \frac{3}{4}$ 1 $\frac{5}{7}$

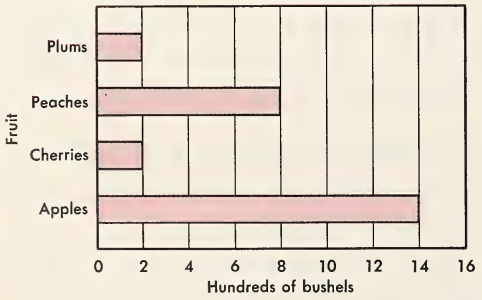
For the examples below, divide by inverting the divisor and multiplying.

3.	$\frac{2}{3} \div \frac{3}{4}$ $\frac{8}{9}$	$\frac{7}{8} \div \frac{3}{4}$ 1 $\frac{1}{6}$	$\frac{5}{6} \div \frac{5}{8}$ 1 $\frac{1}{3}$	$\frac{9}{16} \div 1\frac{1}{2}$ $\frac{3}{8}$	$\frac{7}{8} \div 2\frac{1}{2}$ $\frac{7}{20}$
4.	$7\frac{1}{2} \div \frac{4}{5}$ 9 $\frac{3}{8}$	$5\frac{1}{3} \div \frac{4}{9}$ 12	$8\frac{1}{3} \div 2\frac{1}{2}$ 3 $\frac{1}{3}$	$3\frac{3}{4} \div 5\frac{1}{4}$ $\frac{5}{7}$	$6\frac{2}{3} \div 3\frac{1}{3}$ 2

Check your quotients for row 3 by multiplying the divisor by the quotient.

★Answers Not on Reproduced Page 214

14.



Fruit Sold in One Year by Mr. Haspel

Book Lesson (page 214). *Ex. 1–14:* Written work. Test of Information and Meaning 5.

Book Lesson (top of page 215). *Rows 1–6:* Written work. Diagnostic Test 5. Be sure pupils are using the study pages and practice sets if necessary. For answers to the practice sets refer to the heading *Practice* in the index for this Manual to find where the sets are reproduced on previous pages in this Manual.

Book Lesson (bottom of page 215). *Rows 1–4:* Written work. Computation Test 5. If necessary, this test may be used for still further practice as there is an alternate computation test on page 303 of this Manual.

Differentiations and Extensions

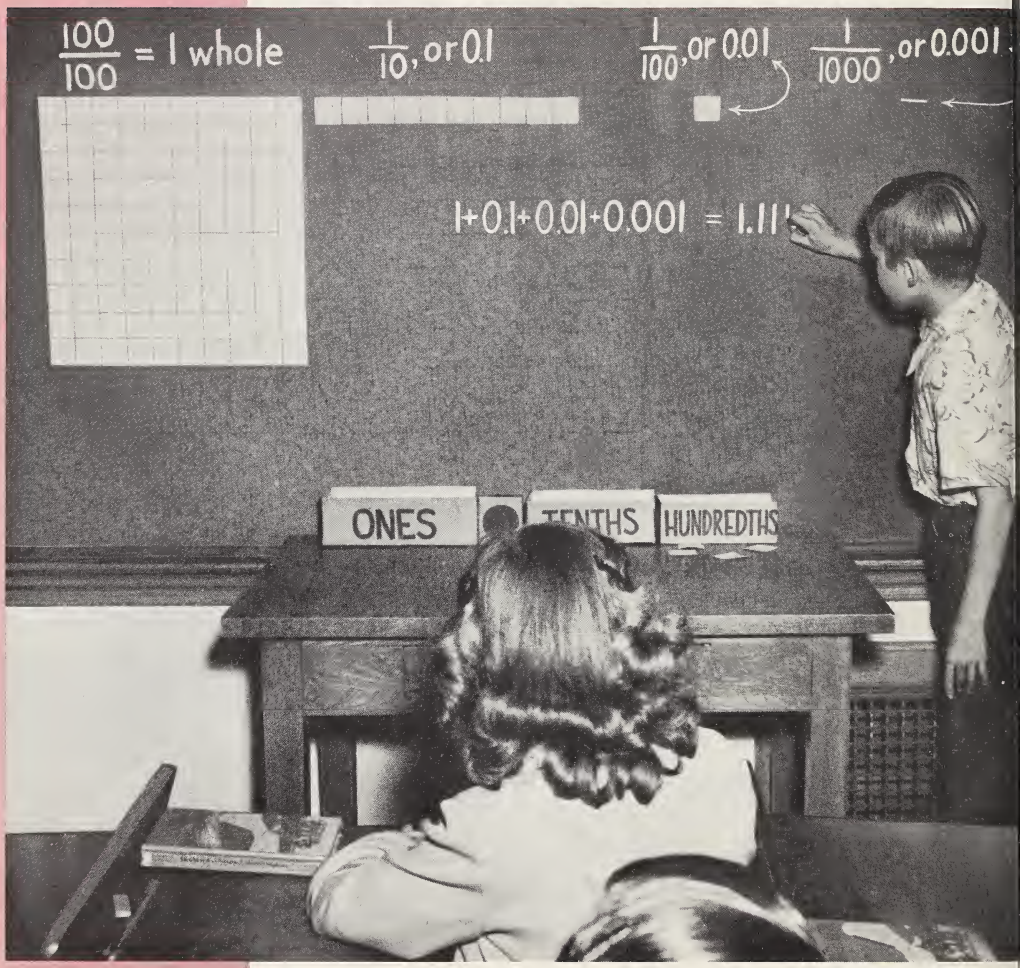
1. On the basis of the test results, plan a program of review and remedial teaching for those pupils who need it. Since the next chapter includes an important section of new work, it is especially important to clear up as many areas of confusion as possible before proceeding. Be on the look-out for weak spots on the part of your *more capable pupils*, since these frequently are not as easily detected as those of the *slower learners*, or, at least, are very often neglected.

2. Use the table at the right as an aid in entering scores on the test record cards.

TABLE OF PER CENTS FOR CHAPTER 5 SCORES

Score	Per Cents for Problem Test 5
1	11
2	22
3	33
4	44
5	56
6	67
7	78
8	89
9	100

Score	Per Cents for Computation Test 5	Score	Per Cents for Computation Test 5
1	5	11	55
2	10	12	60
3	15	13	65
4	20	14	70
5	25	15	75
6	30	16	80
7	35	17	85
8	40	18	90
9	45	19	95
10	50	20	100



Detroit Public Schools

Comparing parts of a hundred-square helps pupils understand the meaning of place-value to the right of the decimal point.

Teaching Chapter 6 of Grade 6

Introduction

The major portion of Chapter 6 is devoted to work with decimals. As readiness for the new work in multiplying decimals, a thorough reteaching of some of the understandings and abilities related to reading and writing decimals and to adding and subtracting decimals is presented.

I. Learning Outcomes in Chapter 6

1. Ability to read, write, and use decimal fractions
2. Ability to add and subtract decimal fractions
3. Understanding and skill in multiplying decimal fractions
4. Ability to solve problems
5. Understanding of various specific concepts, generalizations, relationships, and skills
6. Desirable emotionalized responses (attitudes, appreciations, values)

Ability to Read, Write, and Use Decimal Fractions (pages 217–219, 223, 224, 226, 227, 231, 246, 247)

Some reteaching and some extension of what was learned in Grade 5 regarding this topic occurs in this chapter. For example, pupils review the meaning of, and the reading and writing of tenths and hundredths as decimal fractions. This is then extended to include thousandths and, later, to include the reading of smaller decimals through millionths. Pupils also review the reading and writing of decimal and common-fraction equivalents, extending this skill to include the giving of common-fraction equivalents for decimals in lowest terms.

Completely new in this chapter are the terms *measurement decimals* and *exact decimals*. Except in referring to ratios, decimals are almost always used in some sort of measurement situation. Because of the nature of measuring situations in general, measurement decimals can never be considered completely accurate. Pupils need to realize this so that when they come to the addition and subtraction of measurement decimals (page 229 in the text) they will not annex zeros in trying to make all addends (or the minuend and subtrahend) have the same number of decimal places. The reason zeros cannot always be annexed to measurement decimals in computation is that the degree of preciseness in the answer is increased beyond the precision used in the measuring. The decimal 0.5 inches means that the measuring was done to the nearest $\frac{1}{2}$ inch, whereas the decimal 0.005 inch means the measuring was done to the nearest $\frac{5}{1000}$ of an inch. Of course, when decimals to be added or subtracted are *exact*, but have a different number of decimal places, zeros may be annexed without changing the meaning of the answer.

After measurement and exact decimals have been introduced, pupils learn that exact decimals may be compared by changing to a common denominator, i.e., by making all the decimals being compared have the same number of places.

Ability to Add and Subtract Decimal Fractions (pages 220, 221, 225, 229)

Some reteaching also occurs in this area. Pupils review adding tenths and hundredths both as decimal and common fractions. The skill is then extended to include adding and subtracting thousandths.

Also new in this area is the idea that in adding and subtracting measurement decimals it is necessary to change all of the decimals so that they have the *largest* fractional unit. Pupils will be prepared for this by work showing the difference between measurement and exact decimals.

Understanding and Skill in Multiplying Decimal Fractions (pages 231–235, 238–240, 242, 244)

Multiplication of decimals is wholly new in Grade 6. The sequence of presentation for the various kinds of multiplication examples involving decimals is as follows:

a. Whole number times tenths, then times hundredths and thousandths (includes mixed decimals)

b. Decimal times a whole number, followed by a mixed decimal times a whole number

c. Decimal times a decimal

- Tenths times tenths
- Mixed decimal times a mixed decimal with up to three decimal places in the product
- Mixed decimal times a mixed decimal with more than three decimal places in the product

If pupils really understand the meaning of decimals, they should have very little difficulty in multiplying them, since the computational procedure for decimals is similar to the procedure used with whole numbers. Perhaps the point that will cause the most difficulty is placing the decimal point in the product. The best way to achieve permanent ability in this area is for pupils to discover why the decimal point is placed where it is in the product. They do this by relating the multiplication of decimals to the multiplication of fractions, thereby making available to them some of the skills in estimating products that have been developed in working with fractions. With this as background, they develop appropriate generalizations similar to the following: *Tenths times tenths gives hundredths in the product.* It is then a fairly simple matter to develop the generalization that the product should have as many decimal places as there are in the multiplier and multiplicand together.

Ability to Solve Problems (pages 222, 230, 236, 245, 248)

As new skills are developed in Chapter 6, they are applied to problem-solving situations. In addition to this part of the program, pupils also differentiate addition and subtraction in problems involving decimals, they use information from other problems, and they choose sensible answers for problems involving decimals by using their skills in rounding and estimating products.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:*

common fraction
decimal fraction
decimal point
exact decimals
hundred-thousandths
hundredth
measurement decimals
millionths
mixed decimals
pointing off (decimal places)
ten-thousandths
tenths
thousandths

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 6 will be found on the following pages in the text and on the corresponding pages in this Manual: pages 233, 234, 237, 239, 240, 243.

3. *Skills beyond those already listed:*

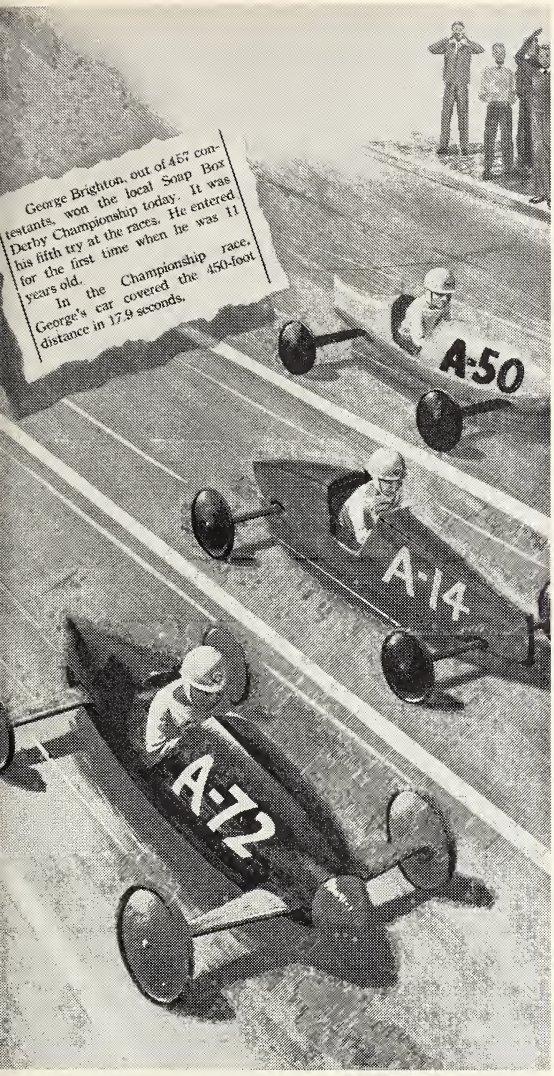
a. Ability to round decimal fractions for use in estimating products (pages 228, 244)

b. Ability to multiply whole numbers by 10, 100, and 1,000 by moving the decimal point to the right (page 237)

c. Ingenuity in discovering other ways to multiply decimal fractions (page 249)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

There is nothing new to be said here except to urge again that desirable attitudes children can form toward arithmetic, appreciations they acquire of it, and the values they can attach to it are outcomes no less important and no less attainable than are the skills they learn.


George Brighton, out of 457 contestants, won the local Soap Box Derby Championship today. It was his fifth try at the races. He entered for the first time when he was 11 years old.
In the Championship race, George's car covered the 450-foot distance in 17.9 seconds.
A-50
A-14
A-72

The Soap Box Derby Champion

Meaning of tenths [O]

1. From the newspaper clipping on the opposite page, read the time George's car took to cover the Derby course.

To help Sue, who was in the fourth grade, read the number 17.9, George covered the .9 and had her say "17." He then told her to say "and" for the point and "nine tenths" for the 9. Sue then read "seventeen and nine-tenths seconds."

George now wrote the **decimal fraction** ".9" and told Sue that this means the same as $\frac{9}{10}$. Why would it have been better to write "0.9" instead of ".9"? So the decimal point wouldn't be overlooked.

2. To time a race, a stop watch like the one shown in the picture can be used. The numbers around the outside of the dial mean seconds.

a. How many spaces are there between 17 and 18? 10 What part of a second does each space represent? $\frac{1}{10}$

b. Read the table at the right and tell where the hand would point for each number in the table.

c. On the stop watch, can you figure out why the red numbers are there? To remind Sue to count around where is the minute hand? In the small dial numbered 1-15

17.0	seventeen
17.1	seventeen and 1 tenth
17.2	seventeen and 2 tenths
17.3	seventeen and 3 tenths
17.4	seventeen and 4 tenths
17.5	seventeen and 5 tenths
17.6	seventeen and 6 tenths
17.7	seventeen and 7 tenths
17.8	seventeen and 8 tenths
17.9	seventeen and 9 tenths
18.0	eighteen

3. Read the following numbers. (Remember that 0 at the left of the point means that there is no whole number.)

a. 25.4 **b.** 0.8 **c.** 245.6 **d.** 8,275.3 **e.** 85.7

Teaching Pages 216, 217, 218, and 219

Pupil's Objectives. (*a*) To review the meaning of tenths expressed as decimal fractions; (*b*) to practice reading and writing tenths that occur as common and decimal fractions; (*c*) to discover that a mixed decimal may have several meanings.

Background. Decimals represent fractions which have a denominator of 10 or some power of 10. Therefore, a review of the meanings of numerators and denominators (when the latter are written as a ten) would make a good introduction to this review of the meaning of tenths written as decimals.

Reading and Writing Tenths

With common and decimal fractions [O]

Read Ex. 1–7 and explain each decimal number.

1. A new local race track for the Soap Box Derby has been built with a racing distance of 975.4 feet. (975 $\frac{4}{10}$)

2. Mr. Ames bought 12.6 gallons of gasoline. (12 $\frac{6}{10}$)

3. The stopping distance for a car going 30 miles an hour is 100.5 feet. The car moves 67.5 feet of that distance while the brake is being pushed down. (100 $\frac{5}{10}$; 67 $\frac{5}{10}$)

4. The stopping distance for a car going 50 miles an hour is 242.5 feet. The car moves 187.5 feet while the brake is being pushed down. (242 $\frac{5}{10}$; 187 $\frac{5}{10}$)

5. When the Blairs got home from their vacation, the mileage meter on their new car showed a total of 1,826.7 miles. (1,826 $\frac{7}{10}$)

6. One of the newest airplanes costs 1.8 million dollars. (1 $\frac{8}{10}$)

7. The rainfall this month was reported as 3.2 inches. (3 $\frac{2}{10}$)

8. In sentence 1, above, why may we write the number as "975 $\frac{4}{10}$" if we wish to? .4 and $\frac{4}{10}$ mean the same

9. At the board, write each of the numbers in Ex. 2–7 with a fraction having the fraction bar. (We call this kind of fraction a common fraction.) Then read each sentence again, using the number you wrote on the board.

10. What do you notice about the number of 0's in the denominator of the common fraction and the number of places at the right of the decimal point in the decimal fraction? same

[W]

11. Write these numbers with common fractions:

a. 0.5 $\frac{5}{10}$ b. 28.3 28 $\frac{3}{10}$ c. 529.4 529 $\frac{4}{10}$ d. 8,240.5 8,240 $\frac{5}{10}$ e. 75,629.8 ✓ 75,629 $\frac{8}{10}$

12. Write these numbers with decimal fractions:

a. 9 $\frac{1}{10}$ 9.1 b. 45 $\frac{3}{10}$ 45.3 c. 785 $\frac{8}{10}$ 785.8 d. 4,872 $\frac{9}{10}$ ✓ 4,872.9 e. 21,572 $\frac{5}{10}$ ✓ 21,572.5

To show meaning [O]

In the number 9,542.7, when you cover the point and the 7 tenths, you see the whole number which means 9 thousands, 5 hundreds, 4 tens, and 2 ones. Then as you continue to the right beyond the point, you have the new tenth's place. The point (usually called **decimal point**) separates the whole number from the decimal fraction.

A

Thousands	Hundreds	Tens	Ones	.	Tenths
9	5	4	2	.	7

We can take a decimal number apart figure by figure just as we take a whole number apart. Study box B.

13. The number 84.6 is called a **mixed decimal.** This number is the sum of how many products? 3 (Use box B for help.)

14. Box C shows three other ways to explain the meaning of the mixed decimal 84.6 Read each line in the box.

B

In 84.6,

the 8 means 8 tens,	or 8×10,	or 80
the 4 means 4 ones,	or 4×1,	or 4
the 6 means 6 tenths,	or $6 \times \frac{1}{10}$, or $\frac{6}{10}$,	or 0.6
		84.6

C

84.6 means
8 tens and 4 ones and 6 tenths;
or 84 ones and 6 tenths;
or 846 tenths.

[W]

15. Copy chart A, above, and write in it the following mixed decimals with each figure in the correct column: ★

a. 25.6 b. 17.4 c. 893.2 d. 6,297.3 e. 478.5

16. As in boxes B and C, write the meaning of 832.7; ★402.6★

In reading decimals, as on page 217 in the text, pupils need to realize that the number to the right of the decimal point stands for the numerator of an equivalent common fraction and that the denominator must be interpreted from the name of the last place to the right of the decimal point. Once pupils understand that decimals are fractions the denominators of which are not visible and are always 10 or a power of 10, they will have developed a real understanding of the meaning of decimals.

One of the big advantages of decimal fractions is that they are an extension of our whole-number notational system. This means that we can compute with decimal fractions just as we do with whole numbers. On page 223 in the text, pupils will see how decimal place values are related to whole-number place values. Because of the similarity of the names of these places, it is important for pupils in reading decimals to emphasize the *th* that indicates a decimal fraction.

In the second paragraph on page 217, pupils learn that the word *and* is used to stand for the decimal point in reading decimals. It is also possible to use the word *point*, and many have adopted this custom to avoid any possible error in reading decimals. Of course, if the word *and* is used,

(★*See top of next column for answers to Ex. 15 and 16*)

15.

	Thousands	Hundreds	Tens	Ones	Tenths
a.			2	5	6
b.			1	7	4
c.		8	9	3	2
d.	6	2	9	7	3
e.		4	7	8	5

16. In 832.7

the 8 means 8 hundreds,	or 8×100,	or 800
the 3 means 3 tens,	or 3×10,	or 30
the 2 means 2 ones,	or 2×1,	or 2
the 7 means 7 tenths,	or $7 \times \frac{1}{10}$, or $\frac{7}{10}$,	or 0.7
		832.7

832.7 means
8 hundreds and 3 tens and 2 ones and 7 tenths;
or 83 tens and 2 ones and 7 tenths;
or 832 ones and 7 tenths;
or 8,327 tenths.

In 402.6

the 4 means 4 hundreds,	or 4×100,	or 400
the 0 means no tens,		
the 2 means 2 ones,	or 2×1,	or 2
the 6 means 6 tenths,	or $6 \times \frac{1}{10}$, or $\frac{6}{10}$,	or 0.6
		402.6

402.6 means
4 hundreds and 0 tens and 2 ones and 6 tenths;
or 40 tens and 2 ones and 6 tenths;
or 402 ones and 6 tenths;
or 4,026 tenths.

---

pupils must be careful not to use it in reading large whole numbers. For example, the first *and* should be omitted in the expression "one hundred and twenty-three and two hundredths."

Note that pupils also learn to place a zero before the decimal point when there is no whole number (third paragraph, page 217), as this helps to avoid errors by emphasizing the existence of the decimal point.

Teacher's Preparation

1. Leaf through Part IV of this Manual noting the various instructional aids you might want to prepare or obtain for the lessons on decimal fractions which follow in the text. Consider especially Calculating Blocks and Strips (for common and decimal fractions), Bead Board, Bead Line, Flannel Board, and Peg Board. Consider also whether you want to use any commercial films, filmstrips, charts, devices, and the like.

2. If possible, obtain some of the following objects to show the usefulness of decimals in everyday affairs: stop watch, pedometer, speedometer, clinical thermometer, railroad timetables, and road maps.

3. Collect various forms and shapes to represent decimals. Number lines, discs (such as pie plates), rectangles, and graph paper can all be used to good advantage for this purpose.

Pre-book Lesson. Demonstrate a stop watch or other suitable device to show a common use for decimal fractions. Suggest other possibilities, such as the gallonage meter on a gasoline pump, the working of which your pupils may observe and discuss.

Book Lesson (pages 216 and 217). *Ex. 1–3:* Oral work.

1. For Ex. 1, ask pupils to try to decide the advantage of saying "and" for the decimal point in reading 17.9.

2. For Ex. 1 or 3, be sure pupils tell *why* the 0 in one's place should be written for decimal fractions less than 1.

3. Additional practice of the kind that occurs in Ex. 3 may be given at the board.

Book Lesson (pages 218 and 219). *Ex. 1–10:* Oral work. *Ex. 11 and 12:* Written work. *Top of page 219 and Ex. 13 and 14:* Oral work. *Ex. 15 and 16:* Written work.

1. As pupils read the decimals in Ex. 1–7 on page 218, try to make them realize that they are reading fractions by using the figure given after the decimal point as the numerator, and interpreting one place after the decimal point to mean tenths.

2. Emphasize the fact that one's place is the focus that establishes the relationship of the numbers, whereas the decimal point only separates the whole number from the decimal fraction.

3. For box C on page 219, elaborate upon the fact that 84.6 may mean 846 tenths. Use calculating strips (see pages 312 and 313 in this Manual) or other suitable devices to show why 1.2 may be thought of as 12 tenths.

Differentiations and Extensions

1. Let *all pupils* use some of the materials mentioned under Teacher's Preparation in depicting decimals, such as 0.6, 1.9, 0.1, etc. As pupils show 0.6 of a paper plate, 1.9 on a number line, 0.1 of a rectangle, etc., they will obtain a better feeling for the relationship of common and decimal fractions.

2. *More capable pupils* may:

a. Assist in assembling devices and leading groups in the understanding and use of them.

b. Use their dictionaries to determine the meanings of the roots of the following words: *decimal, decade, December, Decalogue.*

Teaching Pages 220, 221, and 222

Pupil's Objectives. (*a*) To add and subtract tenths, both as common and as decimal fractions; (*b*) to decide whether it is easier to add and subtract tenths using decimal fractions rather than common fractions; (*c*) to solve problems requiring the addition and subtraction of tenths.

Background. The ease with which decimal fractions may be added and subtracted will make pupils realize immediately one of their important advantages. Both common and decimal fractions are shown in the text so that pupils will realize this advantage. Comparing both methods also helps pupils realize that in both cases they add or subtract the numerators of like-fractions.

With decimals, aligning the decimal points makes each figure in the decimal part of the example fall in the proper place-value column so that decimal fractions with like-denominators can be added. This is exactly analogous to adding or subtracting whole numbers and may be brought out by having pupils use words in adding and subtracting first whole numbers, then decimals. For example, pupils can be asked to add 4 tens and 5 tens, obtaining the answer 9 tens, or 90. They may then be asked to compare this with adding 4 tenths and 5 tenths, obtaining the answer 9 tenths, or $\frac{9}{10}$.

Notice that carrying and borrowing occur in the addition and subtraction of tenths in the text, and that crutches are shown for this in boxes B and D on pages 220 and 221. These crutches help children discover the meaning of carrying and borrowing with decimals but should be eliminated as soon as possible.

Adding and Subtracting Tenths

Meaning [O]

1. It is 1.7 miles from City Park to the Scout Camp. When the boys started from the park, the cyclometer on Jim's bicycle registered 72.6 miles. What did it show when he got to camp?

A	B
Using Common Fractions	Using Decimals
Side work	1
$1\frac{7}{10}$	1.7
$+72\frac{6}{10}$	$+72.6$
$73\frac{13}{10} = 74\frac{3}{10}$	74.3
$\frac{13}{10} = 1\frac{3}{10}$	
$73\frac{13}{10} = 73 + 1\frac{3}{10}$, or $74\frac{3}{10}$	

a. Of course the number of miles Jim rode to camp, 1.7, is to be added to the 72.6 miles already on the meter. Do the decimal fractions in both of these numbers have the same fractional unit? What is it? *tenth*

b. If you had no idea how to add with decimal fractions, you could use common fractions, as in box A. Explain the work.

c. Box B shows how to add with decimal fractions. As with whole numbers, begin to add at the right:

Tenths: $7 + 6 = 13$. Since 10 tenths = 1 whole, then 13 tenths = 1 one and 3 tenths. Write "3" in tenth's place and carry the 1 to one's column. Put the decimal point in the sum to separate the decimal fraction from the whole number.

Ones: 1 (carried) $+ 1 + 2 = 4$. Write "4" in one's place.

Tens: Write "7" in ten's place.

d. Which way is easier, using common or decimal fractions?

Pre-book Lesson. Using a number line, calculating strips, or other devices, have pupils do examples such as the following with common and decimal fractions:

$0.3 + 0.6$ $\quad 1.4 + 5.2$ $\quad 0.9 + 0.8$ $\quad 6.7 + 4.6$

Book Lesson (pages 220 and 221). *Ex. 1–3:* Oral work. *Ex. 4:* Written work.

1. In discussing Ex. 1, explain that a cyclometer

2. When Jim got home, the cyclometer on his bicycle read 85.2 miles. From the time he left City Park, when the cyclometer registered 72.6 miles, how far had Jim's bicycle gone?

C	D
Using Common Fractions	Using Decimals
$85\frac{2}{10} = 84\frac{12}{10}$ $-72\frac{6}{10} = 72\frac{6}{10}$ $12\frac{6}{10} = 12\frac{3}{5}$	$\overset{4}{8}\overset{(12)}{5.2}$ -72.6 12.6

a. To subtract with common fractions (box C), why must we first change $85\frac{2}{10}$ to $84\frac{12}{10}$? *2 from 6 not possible* Explain the work in box C.

b. In box D, we subtract with decimals.

Tenths: We cannot subtract 6 from 2, so we borrow 1 one, change it to 10 tenths, and add 10 tenths to the 2 tenths to make 12 tenths. $12 - 6 = 6$ We put a decimal point in the remainder to separate the decimal fraction from the whole number.

Ones: $4 - 2 = 2$ Tens: $8 - 7 = 1$

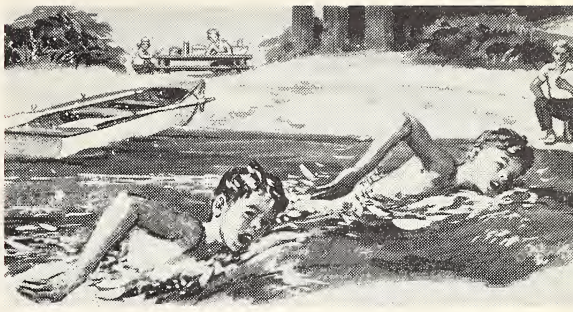
c. Which way of subtracting seems easier, with common fractions or (with decimals)?

3. Explain how carrying and borrowing with decimals is like carrying and borrowing with whole numbers. *In both we carry to or borrow from the next larger place column.* [W]

4. Add or subtract. Check a–e by using common fractions. Check f–o just as you check A. and S. with whole numbers.

a. $4.6 + 25.4$ *30.0*	f. $46.0 - 28.4$ *17.6*	k. $496.5 + 175.6$ *672.1*
b. $19.7 - 2.5$ *17.2*	g. $75.2 - 74.6$ *0.6*	l. $195.9 - 83.9$ *112.0*
c. $28.5 - 0.7$ *27.8*	h. $28.7 + 93.2$ *121.9*	m. $241.3 + 297.0$ *538.3*
d. $8.5 + 72.8$ *81.3*	i. $16.4 + 0.7$ *17.1*	n. $578.2 - 48.7$ *529.5*
e. $0.3 + 7.9$ *8.2*	j. $42.6 + 17.8$ *60.4*	o. $385.4 + 963.5$ *1,348.9*

Extra Practice. For more practice, work Set 92.



Decimals Go to a Beach Picnic

A. and S. of tenths [W]

Add or subtract in these problems, using decimals. Check by using common fractions.

1. When the Fraser family went to the beach for an all-day picnic, they drove 4.9 miles to the store and then 8.7 miles farther to the beach. How far did they drive in all? *13.6 mi.*

2. In getting ready for the picnic Sally filled the two salt shakers from a new 1-pound box of salt. One of the shakers held 1.5 ounces and the other held 0.5 ounce. How much salt was left in the box? *14 oz.*

3. For sandwich filling, Mrs. Fraser used a 7.5-ounce can of tuna fish and a 2.5-ounce can of chopped olives. How many ounces of tuna fish and olives did she use in all? *10 oz.*

4. Paul and Ed asked their father to take his stop watch so he could time their swimming races. Paul swam to Rocky Point in 8.6 seconds. Ed's time was 9.3 seconds. Paul's time was how much less than Ed's? *0.7 sec.*

5. On the way home the Frasers drove 5.3 miles to the post office and then 7.8 miles home from there. Was the return trip longer or (shorter) than the trip to the beach (Ex. 1)? How much longer or shorter? *0.5 mi.* *13.1 mi.*

records distance traveled for a bicycle just as the speedometer (or odometer) records it for an automobile.

2. It may be desirable to have pupils do the examples in boxes A and B at the board during the discussion.

3. Before pupils do written work, make the point that decimals are written in vertical form for adding and subtracting according to place-value columns, just as are whole numbers.

4. For the written work, remind pupils to watch carefully for process signs.

Book Lesson (page 222). *Ex. 1–5:* Written work.

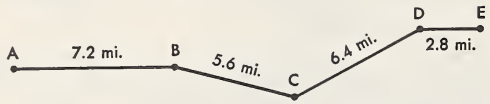
1. Let pupils read the problems silently before starting the written work so that they will have an opportunity to ask questions related to vocabulary and procedure.

2. Help *slower learners* to use diagrams as an aid in solving these problems.

Differentiations and Extensions

1. *All pupils* may make and solve problems in addition and subtraction of decimals. These may be

based on information such as that below written on the board.



2. As *slower learners* use devices in working examples, have them show the following type of written record:

$3.7 + 4.6$	*Side Work*
3 ones and 7 tenths	13 tenths = 1 one
+ 4 ones and 6 tenths	and 3 tenths
7 ones and 13 tenths = 8 ones and 3 tenths = 8.3	

3. *More capable pupils* may record on charts the names of various kinds of measuring instruments, possibly illustrating or defining each. Some that have been mentioned so far in this chapter are the stop watch, speedometer, and cyclometer. Others, such as the micrometer, may be added to the list now or later.

4. Assign **Extra Practice Set 92** as needed.

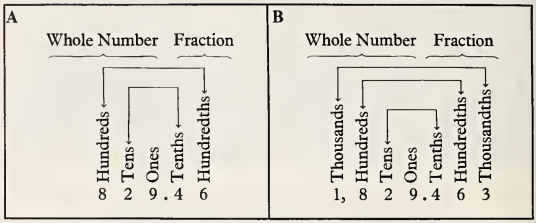
Set 92. Adding and subtracting decimals; tenths

Add or subtract, as indicated.

	a	b	c	d	e	f	g	h
1.	2.5	4.8	3.6	0.9	50.4	48.1	24.0	15.8
	+ 3.5	− 2.7	− 0.9	+ 1.4	− 49.3	− 29.6	+ 0.8	+ 3.9
	6.0	2.1	2.7	2.3	1.1	18.5	24.8	19.7
2.	83.8	0.9	31.1	72.5	1.7	0.3	32.9	45.0
	+ 0.4	+ 0.9	− 1.7	+ 2.9	− 0.8	+ 0.8	− 32.9	− 2.5
	84.2	1.8	29.4	75.4	0.9	1.1	0.0	42.5

Smaller Decimals

Meaning of hundredth; thousandth [O]



1. In box A, $829\frac{46}{100}$ is written as a mixed decimal. What is the name of the place farthest right in this number? hundredths

2. In box B, $1{,}829\frac{463}{1{,}000}$ is written as a mixed decimal. What is the name of the place farthest right in this number? thousandths

3. In our decimal number system, one's place is the center. Then, in relation to one's place, what is true of ten's place and tenth's place? equally distant from one's place of hundred's place and hundredth's place? equally distant of thousand's place and thousandth's place? equally distant

4. In writing $829\frac{6}{100}$ as a decimal, what must we do to keep the 6 in hundredth's place? Write 0 in tenths place.

5. How would we write $1{,}829\frac{3}{1{,}000}$ as a decimal? 1,829.003 $1{,}829\frac{63}{1{,}000}$? 1,829.063

6. Read from box C some meanings of the number 829.46

C

The number 829.46 means
8 hundreds and 2 tens and 9 ones and 4 tenth and 6 hundredths
or 82 tens and 9 ones and 4 tenth and 6 hundredths
or 829 ones and 4 tenth and 6 hundredthy
or 8,294 tenths and 6 hundredths
or 82,946 hundredths,
or 829 and 46 hundredth

(as done in box C for 829.46)

7. Tell the meaning of 653.709 in as many ways as you can.

Teaching Pages 223 and 224

Pupil's Objectives. (*a*) To relearn, or to learn, the meaning of smaller decimals (hundredths and thousandths); (*b*) to obtain practice in writing as decimals common fractions containing hundredths and thousandths.

Background. Pupils have already reviewed the meaning of tenths written as decimals. Now these ideas will be extended to include hundredths and thousandths. Therefore, many of the points made at the right have already been mentioned previously in the Manual, or have been developed in the text, but you will need to keep them in mind as pupils engage in activities involving work with smaller decimals.

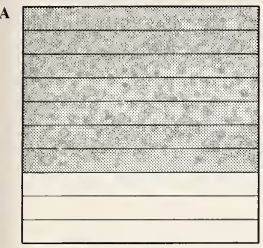
a. Decimal fractions, like common fractions, provide information as to the number and size of the equal parts in a fraction.

b. In a decimal fraction, the number written at the right of the decimal point tells the number of equal parts in the fraction. This number is similar to the numerator of the equivalent common fraction.

c. In a decimal fraction, each position to the right of the decimal point represents a different unit just as each position to the left of a decimal point represents a different unit for whole numbers. For decimals these are fractional units and are related to each other in the same way the values of the whole-number places are related. The fractional unit for the first place at the right of the decimal point is 1 tenth and for the second

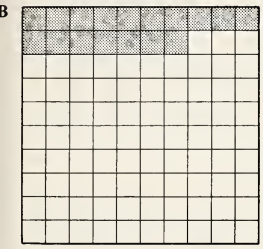
Smaller Decimal Fractions

Meaning of hundredths; thousandths [O]



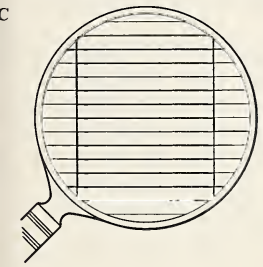
1. Square A is divided into 10 equal parts, so 1 of the equal parts is $\frac{1}{10}$, or 0.1, of the square.

2. In square A, 7 of the 10 equal parts are colored, so $\frac{7}{10}$, or 0.7, is colored.



3. Square B is divided into 100 equal parts. One of the equal parts is $\frac{1}{100}$, or 0.01, of the whole square.

4. How many of the equal parts of square B are colored? 17 Then $\frac{17}{100}$, or 0.17, of the square is colored.



5. If we divided each small square in B into 10 equal parts, as shown by the magnified section in C, how many of these smaller parts would there be in all of square B? 1,000 Then each very small part would be $\frac{1}{1000}$ of square B, or 0.001. 175 of these smaller parts would be $\frac{175}{1,000}$ of square B or, as a decimal, 0.175 of square B.

[W]

6. Write as decimals:

a. $\frac{75}{100}$ 0.75 b. $\frac{295}{1,000}$ 0.295 c. $\frac{4}{100}$ 0.04

d. $\frac{68}{1,000}$ 0.068 e. $\frac{3}{1,000}$ 0.003 f. $\frac{90}{100}$ 0.90

place it is 1 hundredth. As we proceed to the right, each fractional unit has a value 1 tenth of the value of the one on its immediate left.

In 0.5, the fraction has 5 equal parts. Since 5 is written in the first position at the right of the decimal point, the fractional unit is 1 tenth. The fraction 0.5 means 5 × 1 tenth, or 5 tenths.

In 0.05, the fraction has 5 equal parts also, but the 5 is written in the second place at the right of the decimal point, so the fractional unit is 1 hundredth. 0.05 means 5 × 1 hundredth, or 5 hundredths.

The decimal fraction 0.58 means 5 tenths plus 8 hundredths. When changed to like-fractions, we have 50 hundredths + 8 hundredths, or 58 hundredths.

The short way to read a decimal fraction is to read the numerator and obtain the fractional unit from the name of the last place to the right of the decimal point.

By using number lines and parts of wholes, help pupils discover that regardless of the number of places at the right of the decimal point, the value of the decimal fraction is less than one whole.

Teacher's Preparation. Have available bead boards, peg boards, hundred charts, or the like, which may be used to show hundredths, and, if possible, thousandths. Simple devices, such as graph paper or pieces of wrapping paper which have been divided into ten rows of ten squares each on a threadless sewing machine, are useful.

Pre-book Lesson. With one or more of the devices mentioned above, show hundredths, and, perhaps, thousandths, writing the decimals on the board for pupils to see during the discussion. Bring out the relationships of these decimal fractions to tenths and to units as follows:

0.1 = 10 hundredths	1 = 100 hundredths
0.2 = 20 hundredths	1.01 = 101 hundredths
0.5 = 50 hundredths	1.25 = 125 hundredths

Book Lesson (page 223). *Ex. 1–7:* Oral work. For Ex. 4, remind pupils that they have had experience using 0 as a place-holder with whole numbers such as 200 or 406.

Book Lesson (page 224). *Ex. 1–5:* Oral work. *Ex. 6:* Written work. Have your pupils tell, by counting, what parts of boxes A and B are not colored.

Differentiations and Extensions

1. Help the boys especially to understand the batting averages of professional baseball players. They may determine the ranks of players at one time, and the possible changes in rank at another time. You may be able to derive for them the batting averages of their own school team.

2. *More capable pupils*

a. These pupils may illustrate certain decimal fractions, including mixed decimals such as 3.25, on graph paper as a demonstration for *slower learners*, who, in turn, may attempt to illustrate some themselves.

b. Explain to them the common practice of using the word *point* in reading mixed decimals. For example, 98.6 may be read "ninety-eight and six tenths," or "ninety-eight point six."

c. Let these pupils try to discover how the Dewey Decimal System is used in the organization of libraries. If possible, visit a library.

A. and S. of Hundredths and Thousandths

[O]

1. In one part of the country the rainfall was 4.97 in. for June, 3.64 in. for July, and 2.86 in. for August. Find the total for the three months.

The sum of the hundredth's column is 17 hundredths, or _1_ tenth and _7_ hundredths. We write _7_ in hundredth's column and carry 1 *tenth*

The sum of the tenth's column is 24 tenths, or _2_ ones and _4_ tenths. What is carried? *2 (ones)*

The sum of the one's column is _11_.

2. One year the July rainfall was 4.13 in. This was how much more than the 3.64 in. in Ex. 1?

Explain the subtraction in box B. How can you check the remainder? *add subtrahend and remainder.*

3. Mr. Jay's building lot contains 0.329 acre, and Mr. Todd's lot contains 0.404 acre. Which lot is larger? *Mr. Todd's* How much larger?

Explain the work in box C.

4. Both lots in Ex. 3 contain how much land?

Explain the addition in box D.

A

$$\begin{array}{r} {\scriptstyle 2\ 1} \\ 4.97 \\ 3.64 \\ +2.86 \\ \hline 11.47 \end{array}$$

B

$$\begin{array}{r} {\scriptstyle 3\ \textcircled{10}\ \textcircled{13}} \\ 4.\not{1}\not{3} \\ -3.6\ 4 \\ \hline 0.4\ 9 \end{array}$$

C

$$\begin{array}{r} {\scriptstyle 3\ 9\ \textcircled{14}} \\ 0.4\not{0}\not{4} \\ -0.3\ 2\ 9 \\ \hline 0.0\ 7\ 5 \end{array}$$

D

$$\begin{array}{r} {\scriptstyle 1} \\ 0.404 \\ +0.329 \\ \hline 0.733 \end{array}$$

[W]

Add or subtract in rows 5–7. Check your answers.

	a	b	c	d	e
5.	0.37	1.95	$2.05	8.304	4.526
	+0.86	+2.56	−1.36	−2.647	−0.877
	1.23	*4.51*	*$0.69*	*5.657*	*3.649*
6.	7.00	5.28	$2.35	7.265	6.382
	−2.93	−2.09	+2.35	−0.528	−1.876
	4.07	*3.19*	*$4.70*	*6.737*	*4.506*
7.	1.26	4.51	$9.64	6.397	2.876
	0.86	3.09	7.38	0.496	1.359
	+7.42	+0.97	+0.07	+0.587	+0.676
	9.54	*8.57*	*$17.09*	*7.480*	*4.911*

Extra Practice. For more practice, work Sets 91 and 93.

Teaching Page 225

Pupil's Objectives. (*a*) To learn to add and subtract decimals with hundredths and thousandths; (*b*) to obtain practice in adding and subtracting decimals.

Background. In studying whole-number notation, pupils discovered that whole numbers of any size could be formed by filling more and more places to the left of one's place. Pupils also learned that there is a fixed relationship between the positions in a whole number such that each has a value ten times the one at its immediate right or one tenth the value of the one at its immediate left.

Now pupils will find that when a decimal point is written at the right of one's place and positions after the decimal point are filled with digits, a way of representing fractions whose fractional units are related to each other in the same way that whole-number values are related to each other is established. Therefore, adding and subtracting hundredths and thousandths which are written as decimals is no more difficult or involved than adding whole numbers.

The crutches used to show the carrying and the borrowing in the boxes on the page in the text help pupils see the meaning of adding and subtracting decimals but may be discarded by most pupils very quickly. *Slower learners* should be permitted to use them until they have developed confidence in this work.

Pre-book Lesson

1. If pupils have used graph paper with decimals, as has been suggested, let them show how to use it in working a few examples involving addition and subtraction of decimals.

2. It is also possible to let pupils think of a hundred bundle (from the pocket chart used with whole numbers) as representing one whole. Then a ten bundle would represent $\frac{1}{10}$ of the whole, and a single stick $\frac{1}{100}$ of the whole. With this set up, carrying and borrowing in addition and subtraction of tenths and hundredths may be demonstrated in a pocket chart by pupils just as if they were working with whole numbers.

Book Lesson. *Ex. 1–4:* Oral work. *Rows 5–7:* Written work.

1. Call attention of pupils to the fact that Ex. **5c**, **6c**, and **7c** in the written work involve money numbers. Be sure pupils realize that a penny is $\frac{1}{100}$ of a dollar and a dime $\frac{1}{10}$ of a dollar.

2. Carefully examine the written work of *slower learners* as they do row 5. Have them do examples orally and individually with you so that you may uncover weaknesses and provide remedial measures.

Differentiations and Extensions

1. *More capable pupils* may locate weather information pertaining to amount of precipitation. Have them show comparisons and contrasts in respect to time and to distant regions.

2. Further work with decimals is provided in **Extra Practice Sets 91 and 93** (given with answers on the next Manual page).

Write these numbers with decimals.

	a	b	c	d	e	f	g	h
1.	$\frac{2}{10}$ 0.2	$\frac{14}{100}$ 0.14	$\frac{125}{1,000}$ 0.125	$3\frac{7}{10}$ 3.7	$10\frac{21}{1,000}$ 10.021	$12\frac{9}{100}$ 12.09	$4\frac{50}{100}$ 4.50, or 4.5	$6\frac{7}{1,000}$ 6.007

Write these numbers with common fractions.

2. 0.9 $\frac{9}{10}$ 0.007 $\frac{7}{1,000}$ 0.03 $\frac{3}{100}$ 4.329 $4\frac{329}{1,000}$ 21.7 $21\frac{7}{10}$ 47.13 $47\frac{13}{100}$ 0.019 $\frac{19}{1,000}$ 3.107 $3\frac{107}{1,000}$

Write these numbers as common fractions in lowest terms.

3. 0.25 $\frac{1}{4}$ 0.5 $\frac{1}{2}$ 0.10 $\frac{1}{10}$ 0.75 $\frac{3}{4}$ 0.375 $\frac{3}{8}$ 0.8 $\frac{4}{5}$ 0.16 $\frac{4}{25}$ 0.625 $\frac{5}{8}$

4. 0.05 $\frac{1}{20}$ 0.15 $\frac{3}{20}$ 0.6 $\frac{3}{5}$ 0.50 $\frac{1}{2}$ 0.2 $\frac{1}{5}$ 0.120 $\frac{3}{25}$ 0.35 $\frac{7}{20}$ 0.48 $\frac{12}{25}$

Set 93. Adding and subtracting decimals; hundredths and thousandths

Add or subtract, as indicated.

	a	b	c	d	e	f
1.	\$2.25	3.136	4.78	\$6.50	5.470	\$8.96
	+ 3.18	− 0.215	+ 0.92	+ 4.98	− 2.087	− 0.97
	\$5.43	**2.921**	**5.70**	**\$11.48**	**3.383**	**\$7.99**
2.	6.829	\$9.52	7.009	1.125	\$8.92	5.106
	− 0.758	1.60	− 4.775	0.370	− 7.98	0.093
	6.071	+ 0.37	**2.234**	+ 8.625	**\$0.94**	+ 6.008
		\$11.49		**10.120**		**11.207**

NOTES

When Decimals Are Not Like-Fractions

A. and S. [O]

1. On page 224, 0.7 of square A is colored. If you made three copies of a square the size of A and colored 0.7 of one, 0.70 of another, and 0.700 of another, why would the same amount be colored in each square? Then 0.7 and 0.70 and 0.700 are all equivalent and we call them **exact decimals.**

0.7, 0.70, and 0.700 all represent the same part of the whole.

2. Now suppose you found that a line was 0.7 in. long when you measured it with a ruler divided into tenths of an inch. Would you be likely to get *exactly* the same result if you measured it with a ruler divided into hundredths of an inch and then with one divided into thousandths? No **Measurement decimals,** like other measurement numbers, are never exact.

The decimals in the boxes below are exact. Give the equivalents in each row.

	Tenths	Hundredths	Thousandths
3.	0.2	= 0.20	= 0.200
4.	0.5	= 0.50	= 0.500
5.	0.8	= 0.80	= 0.800

	Tenths	Hundredths	Thousandths
6.	0.9	= 0.90	= 0.900
7.	0.1	= 0.10	= 0.100
8.	0.4	= 0.40	= 0.400

9. In the box below, do the exact decimals in the column at the left have the same fractional unit? No

Before we can add or subtract decimal fractions whose fractional units are not the same, we must change them so they will have the same fractional unit.

Explain what is done in the box. Why don't we change all the fractions to tenths? Because all but one have smaller fractional unit than tenths.

0.3	= 0.300
0.45	= 0.450
0.785	= 0.785
+0.82	= 0.820
	2.355

[W]

Add or subtract these exact decimals:

10. $0.8 + 0.35 + 0.7$ 1.85
11. $6.75 + 9.825 + 0.1$ 16.675
12. $1.6 - 0.87$ 0.73
13. $4.8 - 1.275$ 3.525
14. $0.909 + 9.9$ 10.809
15. $5.075 - 4.26$ 0.815

Teaching Page 226

Pupil's Objectives. (*a*) To learn that only decimals with the same fractional unit may be added or subtracted; (*b*) to add and subtract decimals which must first be changed so that the fractional units are the same.

Background. Ex. 1 and 2 in the text explain the difference between exact and measurement decimals. Although exact decimals are not commonly found in everyday affairs, they occur frequently in the form of ratios (and in other forms, too) so that pupils should know how to work with them. A method for adding and subtracting measurement decimals is given on page 229 in the text.

If it should happen that pupils must add "ragged" decimals of the kind shown in the box at the bottom of page 226, they should know how to express each decimal in the same fractional unit to facilitate the process of adding or subtracting. Because zeros can be added to decimals (the effect being only to increase the number of parts while decreasing the size of the parts), ragged decimals in the addition and subtraction of exact decimals can be made to have the same fractional unit by annexing zeros until each decimal has the same number of places.

It must be remembered that changing decimals to smaller fractional units by adding zeros applies to exact decimals only. However, there are cases, as in the division of decimals, when measurement decimals must be changed by adding zeros, and it is well for pupils to be prepared for this.

Pre-book Lesson. Refer to the necessity of having like-numbers in adding and subtracting whole numbers and fractions. See if pupils realize that they cannot add 3 tables and 2 chairs or $\frac{1}{2}$ and $\frac{1}{4}$ without first changing to like-numbers. Apply this understanding to an example with decimals, preferably one that may be illustrated, such as $3.4 + 6.25$. Lead pupils to understand that 0.4 may be changed to hundredths and show the example with both common fractions and decimal fractions, as below.

$$\begin{array}{r} 3\frac{4}{10} = 3\frac{40}{100} \\ +6\frac{25}{100} = 6\frac{25}{100} \\ \hline 9\frac{65}{100} \end{array} \qquad \begin{array}{r} 3.4 = 3.40 \\ +6.25 = 6.25 \\ \hline 9.65 \end{array}$$

Book Lesson. *Ex. 1–9:* Oral work. *Ex. 10–15:* Written work. It may be better for all of your pupils at this time to write out Ex. 10–15 as shown in the box on the text page in order to impress upon them the need for changing the exact decimals in each example to the same fractional unit.

Teaching Pages 227 and 228

Pupil's Objectives. (*a*) To learn how to compare decimal numbers for size; (*b*) to round decimals to whole numbers, tenths, or hundredths.

Background. Pupils have already learned that to compare common fractions it is frequently necessary to change the fractions so that they have the same fractional unit. They will be happy to dis-

Comparing Exact Decimals

[O]

After a discussion about comparing some exact decimals, liss Pike's class agreed on these things:

a. If two decimal fractions have a common denominator (as) or 100 or 1,000), you can quickly tell which fraction is larger. Why do you know that 0.8 is more than 0.4? 8(tenths) larger than 4(tenths)

b. Decimal fractions whose fractional units are unlike can be ade to have a common denominator and can then be compared. Explain why 0.6 is more than 0.58 (0.6 = 0.60) 60(hundredths) larger than 58 (hundredths)

Explain why 0.48 is more than 0.474 480(thousandths) larger than 474(thousandths)

c. Any mixed decimal is more than any decimal fraction. Why is 1.01 more than 0.999? $\frac{999}{1000}$ less than 1

d. If two mixed decimals have different whole numbers, the ie with the larger whole number is greater. 5 is less than 6

Why is 4.01 more than 3.99? 4 is larger than 3 Why is 5.798 less than 6.001?

e. In any two decimal fractions, if the tenth's figure of one larger than the tenth's figure of the other, the fraction with the rger number of tenths is the larger fraction.

Explain why 0.42 is larger than 0.259 4(tenths) larger than 2(tenths)

[W]

In each case which of these exact decimals is larger:

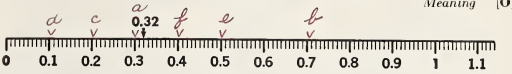
[0.83] or 0.73?	4. [0.61] or 0.061?	7. 0.894 or [1.02]?
2.18 or [3.01]?	5. [1.2] or 0.87?	8. 0.06 or [0.17]?
[0.52] or 0.5?	6. 0.7 or [0.75]?	9. [2.1] or 0.210?

Copy the smallest number in each of these groups:

0. 2.9	[2.41]	2.995		14. 0.57	5.70	[0.057]
1. [0.60]	0.7	0.601		15. 1.95	2.05	[1.85]
2. 0.7	0.701	[0.699]		16. 0.4	[0.38]	0.426
3. [56.2]	57.94	58.603		17. 0.15	0.2	[0.104]

Rounding Exact Decimals

Meaning [O]



You rounded common fractions on page 122. You can round decimal fractions in a similar way.

1. Study the number line above, which shows lines marking off tenths and hundredths. What color are the marks which show tenths? red (at 0, 0.1, 0.2, etc.) What color shows hundredths? black (in between 0 and 0.1; 0.1 and 0.2, etc.)

2. Put the thumb of your right hand on 0 and a finger of that hand on 1. Between those two points show the following: (See number line above.)

a. 0.3 **b.** 0.7 **c.** 0.2 **d.** 0.1 **e.** 0.5 **f.** 0.4

3. Is each of these nearer to 0 or to 1?

a. 0.2 0 **b.** 0.7 1 **c.** 0.9 1 **d.** 0.3 0 **e.** 0.8 1 **f.** 0.6 1

4. Now tell whether to round the tenths in Ex. 3 to 0 or to 1.

5. See if you can round each of these mixed decimals to the nearest whole number:

a. 5.2 5 **b.** 8.7 9 **c.** 4.9 5 **d.** 7.3 7 **e.** 3.5 4 (round 5 upward)

6. Again look at the number line and count the hundredths between 0 and 0.1; between 0.1 and 0.2; between 0.6 and 0.7 10 10 10

7. With your finger on tenth marks, count hundredths 10 at a time: 10 hundredths, 20 hundredths, and so on.

8. Notice the arrow pointing to 32 hundredths (0.32). Is 0.32 nearer to [3 tenths (0.3)] or to 4 tenths (0.4)? To the nearest tenth, would you round 0.32 to [0.3] or to 0.4?

[W]

9. Round to the nearest tenth: **a.** 0.86; **b.** 1.04; **c.** 0.89; **d.** 5.32; **e.** 7.05; **f.** 9.13 0.9 1.0 0.9 5.3 7.1 9.1

10. Use what you have learned about tenths and round to the nearest hundredth: **a.** 0.479; **b.** 6.521; **c.** 2.899 0.48 6.52 2.90

cover that this extra step is not necessary in comparing decimals, since decimals with the same number of places have the same fractional unit. Moreover, since decimal notation is simply an extension of whole-number notation, some of the same principles used in comparing whole numbers can be used in comparing decimals when the decimals being compared have the same number of places. For example, in comparing 0.25 with 0.14, the decimal with the larger figure in tenths place is the larger decimal, just as 25 is larger than 14 because it has more tens. In other cases, as in comparing 0.2 with 0.147, pupils must use one or more of the principles developed in **a** to **e** on page 227 in the text. If examples are used to illustrate each of these points, pupils will discover that they are quite simple.

Again because decimal notation is simply an extension of whole-number notation, decimals are rounded in exactly the same way as are whole numbers. As pupils work with the number line at the top of page 228 in the text, they will realize this, but, of course, they will be working with decimals.

Book Lesson (page 227). *Top of page:* Oral work. *Ex. 1–17:* Written work.

Book Lesson (page 228). *Ex. 1–8:* Oral work. *Ex. 9 and 10:* Written work. For the benefit of

the class, encourage *more capable pupils* to state generalizations similar to the following:

In rounding to the nearest whole number,

a. numbers smaller than 0.5 are rounded downward.
b. numbers larger than 0.5 are rounded upward.
c. numbers equal to 0.5 are rounded upward.

Differentiations and Extensions

1. Prior to the written work on page 227, *slower learners* may attempt exercises such as these:

a. Tell numbers that are smaller than each of these: 0.25; 0.75; 0.346; 0.792

b. Tell numbers that are larger than each of those in *a* above.

c. To compare decimal fractions, make changes so that each has the same fractional unit. To compare 0.4, 0.25, and 0.61, change each decimal so that its fractional unit is _? _.

d. To compare mixed decimals, look at the whole numbers. In the mixed decimals below, _? _ is largest and _? _ is smallest.

5.4 3.68 2.17 7.198 3.006

2. *More capable pupils* may arrange the decimals in the written work on page 227 in descending order of size.

3. After the written work on page 228, *more capable pupils* may show how rounded numbers can be used in estimating, as illustrated below.

4.6 rounds to 5	9.2 rounds to 9
3.2 rounds to 3	− 2.8 rounds to 3
5.7 rounds to 6	6.4 is close to 6
13.5 is close to 14	

Working with Measurement Numbers

A. and S.

1. Mr. Rand was planning to buy a building lot. He looked up the records on two lots that were for sale and found that one was listed as 105.28 ft. across the front and the other as 86.7 ft. The first lot was how much wider than the second?

Why can't we compare the widths by subtracting 86.7 from 105.28? The numbers have different fractional units. What is the fractional unit for the .28? .01 for the .7

In this problem we are working with measurement numbers. We cannot change them as we change exact decimals. If we changed 86.7 to 86.70, we would be saying that the measurement was made more accurately than it really was—to hundredths of a foot instead of to tenths of a foot. When we want to add or subtract measurement decimals, we must first find the decimal with the *largest* fractional unit. Then we must change the others so they will have the same fractional unit as this one does—that is, we must round them.

For 105.28 and 86.7, which decimal has the larger fractional unit?

The box shows how these mixed decimals were changed to get them ready for subtraction. Explain the work.

105.28	=	105.3
−86.7		−86.7
		18.6

Tell how to change the following measurement numbers before adding or subtracting. Do any not need to be changed? (Ex. 1a only)

(See parentheses below.)

a	b	c
2. 4.8 in. (6.8) + 6.82 in. (76.9)	8.7 in. (11 in.) − 3.25 in. (8.2 in.)	0.375 oz. (5.4 in.) + 0.75 oz. (5.6)
3. 76.89 mi. + 5.7 mi. (210.8 lb.)	5.625 in. (9.3) + 9.5 in. (15 in.)	17.75 mi. − 4.008 mi. (7.3)
4. 125.6 lb. (6.0) + 85.2 lb. (10.8 yd.)	9.25 ft. (48 ft.) − 4.5 ft. (3.0)	18.8 ft. (11.5 ft.) − 7.25 ft. (10.88)
5. 5.95 yd. + 4.8 yd.	64.5 ft. (61.5 ft.) + 3.005 ft.	10.875 lb. (13.63 lb.) + 2.75 lb.

[W]

Now write the work for rows 2–5, as in the box.

Teaching Page 229

Pupil's Objectives. (*a*) To learn why decimal numbers used with measures are changed to the largest fractional unit before adding or subtracting; (*b*) to obtain practice in changing measurement decimals to the largest fractional unit before adding or subtracting.

Background. In comparing measurement decimals, or in adding and subtracting them, it is important that the decimals be expressed in the same fractional unit. If the decimals do not have the same fractional unit, the only honest way this can be done with measurement numbers is to round to the largest fractional unit. To round to the smallest fractional unit by annexing zeros (as in changing 2.2 + 5.13 to 2.20 + 5.13) would mean that one of the decimals had been made to look as if a greater degree of accuracy in measuring was used than was actually the case (2.2 means that the measurement was made to the nearest tenth of an inch, whereas 2.20 indicates that the measurement was made to the nearest hundredth of an inch).

Actually, in most practical situations decimals are used in such a way that they have the same fractional unit, the reason being that the measurement is usually done for each situation to the same degree of accuracy. However, pupils should be aware of the fact that there may be times that they must round to the largest fractional unit and that the rounding procedure would be very similar to that used for whole numbers.

Pre-book Lesson

1. Bring out the fact that numbers used in measuring are approximate by having several pupils measure and record the length of the same object. Various measuring devices, such as a ruler, tape, carpenter's rule, etc., should be used.

2. Cite situations such as the following:

a. On an automobile speedometer, 174.3 appears until 174.4 is revealed. (This is also true of other meters based on the same principle.)

b. If an object is found to be 5.6 ft., it may not be written as 5.60 ft., since this would be indicating greater accuracy in measuring than was actually the case.

c. If 3.72 in. is to be added to 5.9 in., 3.72 in. must be thought of as 3.7 in., so that the sum will not show greater accuracy than the least accurate measurement.

Book Lesson. *Ex. 1–5:* Oral work. *Bottom of the page.* Written work.

Differentiations and Extensions. *More capable pupils* may attempt to define and use terms such as the following: *standard of measure, dimensions, quantity, degree, infinitesimal, incalculable.*

Teaching Page 230

Pupil's Objective. To solve problems involving addition and subtraction of decimals.

Background. Pupils have already encountered problems involving addition and subtraction of tenths. On this page they solve problems involving addition and subtraction of smaller decimals. Thus, as is true of the complete problem-solving program, pupils obtain new skills in easy stages.

Book Lesson. *Ex. 1–5:* Written work.

1. *Slower learners* may use dramatization to enrich their understanding of the various problem situations.

2. *All pupils* may first estimate the answer for each problem orally. These may be written on the board and compared later with the exact answers obtained during the written computation.

Differentiations and Extensions

1. *More capable pupils* may write original problems from data collected or about practical situations investigated. For example, they could collect data about the daily requirements for other vitamins and minerals, or they might investigate ways that people in other occupations, such as carpentry, laboratory work, and the like, use decimals.

Using Decimals in a Hospital

A. and S. problems with decimals [W]

Ruth's sister, Jo-Anne, is a student nurse. She told Ruth that nurses have to use decimals in many ways. Here are some of her problems. See if you can solve them.

1. When the patient's temperature is 101.8°, how many degrees above the normal body temperature of 98.6° is it? 3.2°

2. To make a narrow bandage, Jo-Anne cut a strip of muslin 1.5 inches wide from a piece of muslin 6.2 inches wide. How wide was the piece left? 4.7"

3. For dinner, one patient ate 13.5 ounces of meat soup, 5.8 ounces of beefsteak, 1.2 ounces of mashed potato, and a 1.0-ounce roll. How many ounces of food did the patient eat? 21.5oz.

4. One of Jo-Anne's textbooks gave the following table about vitamin C and fruit juices.

Fruit Juice Needed for One Day's Supply of Vitamin C

Juice	Orange	Grapefruit	Pineapple	Tomato
Cups	0.375	0.375	1.750	0.875

a. How much less orange juice than pineapple is needed for 1 day's supply of vitamin C? 1.375 c.

b. How much more tomato juice than grapefruit is needed? 0.500c.

c. Which juice provides the least vitamin C? Pineapple

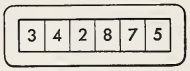
5. Medicine is sometimes weighed in *grams.* A 0.016-gram tablet weighs how much less than a 0.324-gram tablet? 0.308 gr.

2. Some of the problems, based on the use of decimals in various occupations, which were written by the *more capable pupils* may be discussed by the entire class. If interest is sufficiently high, you might have the whole class work on a project to extend their knowledge in this area. The problems just mentioned, and other data collected by pupils, could be used to construct a mural, poster, or scrapbook dealing with the theme, "The Many Ways in which Decimals Work for Us." Do not overlook the possibility of using such a project in correlating arithmetic with other subject-matter areas.

Decimal and Common-Fraction Equivalents

[O]

1. The diagram at the right below shows what Paul saw when he looked at the mileage meter on his father's car one morning. The red 5 means 5 tenths. If Paul reads the mileage correctly, what will he say?★



Box A shows three ways to write the fraction of a mile. Read all three.

Why could Paul change 0.5 to $\frac{5}{10}$ and then to $\frac{1}{2}$?★

A five tenths one half
$0.5 = \frac{5}{10} = \frac{1}{2}$
B
$0.8 = \frac{8}{10} = \frac{4}{5}$
C
$0.625 = \frac{625}{1{,}000}$
$\frac{625 \div 125}{1{,}000 \div 125} = \frac{5}{8}$

2. The meter showed 342958 when Paul's father stopped for gas. Read this number.★

Box B shows the steps in changing 0.8 to a common fraction in best form. Explain each step.

3. The baseball report shows that the Sox have won 0.625 of their games. This is what common fraction of their games? $\frac{5}{8}$ Explain box C.

4. \$0.75 is what fraction of a dollar? $\frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

[W]

Change to common fractions in lowest terms.

5. 0.2 $\frac{1}{5}$ 6. 0.6 $\frac{3}{5}$ 7. 0.25 $\frac{1}{4}$ 8. 0.05 $\frac{1}{20}$ 9. 0.125 $\frac{1}{8}$

10. 0.35 $\frac{7}{20}$ 11. 0.375 $\frac{3}{8}$ 12. 0.55 $\frac{11}{20}$ 13. 0.4 $\frac{2}{5}$ 14. 0.875 $\frac{7}{8}$

15. Copy and finish this table. Then see if you know each decimal and its equivalent common fraction in lowest terms.

$0.5 = \frac{5}{10} = \frac{1}{2}$	$0.2 = \frac{2}{10} = \frac{1}{5}$	$0.125 = \frac{125}{1{,}000} = \frac{1}{8}$
	$0.4 = \frac{4}{10} = \frac{2}{5}$	$0.375 = \frac{375}{1{,}000} = \frac{3}{8}$
$0.25 = \frac{25}{100} = \frac{1}{4}$	$0.6 = \frac{6}{10} = \frac{3}{5}$	$0.625 = \frac{625}{1{,}000} = \frac{5}{8}$
$0.75 = \frac{75}{100} = \frac{3}{4}$	$0.8 = \frac{8}{10} = \frac{4}{5}$	$0.875 = \frac{875}{1{,}000} = \frac{7}{8}$

Teaching Page 231

Pupil's Objective. To determine in lowest terms the common-fraction equivalent for various decimal fractions.

★Answers Not on Reproduced Page 231

1. Thirty-four thousand two hundred eighty-seven and five tenths.

Box A. 0.5 means 5 tenths, which may be written $\frac{5}{10}$. This may be reduced to $\frac{1}{2}$.

2. Thirty-four thousand two hundred ninety-five and eight tenths.

Background. Decimal fractions can be written as common fractions very easily. In fact, if one were to read 0.2 and wanted it written as a decimal, he would have to say so, for the listener could just as easily think of it, and therefore write it, as a common fraction.

Because decimal fractions can be so easily written as common fractions, it is a very easy matter to write common-fraction equivalents for decimals. However, since we usually like to deal with common fractions in lowest terms, it is very often necessary to reduce common-fraction equivalents of decimal fractions to lowest terms. Therefore, ability to write common-fraction equivalents for decimals is somewhat dependent upon the pupil's ability to reduce fractions to lowest terms.

At this stage, it is probably not necessary for your pupils to remember common-fraction equivalents for decimal fractions. However, as they go on in work with decimals, they will want to be able to recognize immediately some of the most used equivalents, such as: $\frac{1}{2}$ for 0.5; $\frac{1}{4}$ for 0.25; $\frac{1}{8}$ for 0.125; etc.

Teacher's Preparation. Use graph paper to prepare a few charts showing $\frac{1}{2}$, $\frac{1}{4}$, and, perhaps, $\frac{1}{8}$, in tenths and hundredths. Color the portion of each whole that represents $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ in such a way that pupils will recognize, without counting tenths or hundredths, how much of the whole has been colored.

Pre-book Lesson. Use the charts mentioned above and ask children to tell how many tenths or hundredths have been colored. Then ask them what common fraction has been represented. Write these as equivalents on the board and see if pupils can discover themselves some ways in which decimal fractions could be changed to common fractions.

Book Lesson. *Ex. 1–4:* Oral work. *Ex. 5–15:* Written work. Pupils may correct their own papers as Ex. 5–15 are worked out on the board after the written work.

Differentiations and Extensions. Have *all pupils* use the solutions worked out on the board to determine which number in each of the following pairs is greater:

a. 0.3 or $\frac{2}{5}$
b. 0.125 or $\frac{1}{10}$
c. 0.25 or $\frac{1}{5}$
d. 0.75 or $\frac{5}{8}$

Teaching Pages 232 and 233

Pupil's Objectives. (*a*) To learn how to multiply tenths by whole numbers; (*b*) to learn to multiply mixed decimals (tenths only) by whole numbers.

Background. So far most of the work in decimals, but not all, has been review or reteaching. This lesson starts the program for multiplication involving decimals, which is entirely new in Grade 6. Before you begin this section with your pupils, it would be helpful for you to read through the explanations given in this Manual for some of the other related lessons in Chapter 6.

Multiplying a whole number times a decimal has the same basic meaning as the multiplication of whole numbers or the multiplication of common fractions, but it is very much easier than the latter.

On page 232 in the text, pupils first count on the number line, then add, then multiply. As the lesson proceeds, the point is developed that when tenths are multiplied by a whole number the product must show tenths. In the written computation, pupils show tenths in a product simply by placing the decimal point in such a way that there is one decimal place.

Pre-book Lesson

1. Use devices, such as a number line or colored squares of graph paper, with which you may illustrate an example such as 4×0.3. Lead pupils to suggest solutions similar to those shown on the number line on page 232 in the text or in boxes A and B.

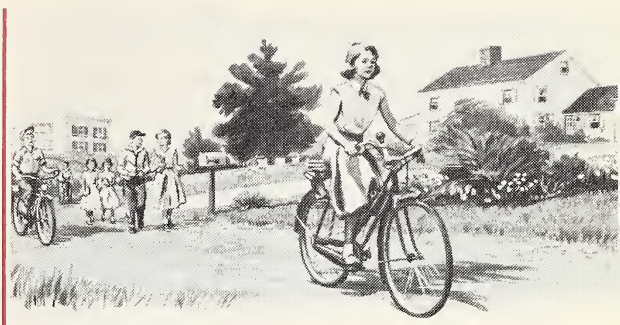
2. Point out that as the multiplicand 0.3 is less than 1, the product will be less than 4 times 1, or less than 4. Some pupils may see that 0.3 is less than $\frac{1}{2}$, and, therefore, be able to give an even closer estimate—less than $\frac{1}{2}$ of 4, or 2. Still others may be able to give the exact answer by thinking "4 times 3 tenths is 12 tenths, or 1 and 2 tenths."

Book Lesson. *Ex. 1:* Oral work. *Ex. 2–7:* Written work. *Ex. 8–9:* Oral work. *Ex. 10–15:* Written work.

1. You may wish to have pupils show on the board how some of Ex. 2–7 should be set down for written work (as in boxes A and B). They should notice that the decimal numbers are addends or multiplicands and that the whole numbers determine the number of addends and are, of course, the multipliers.

2. Prior to the written work on page 233, have pupils determine whether each product for the examples in row 10 will be greater or less than its multiplier. For rows 11 and 12, have them round the mixed decimals and determine approximate answers. Check the results of their estimating before they do the written lesson.

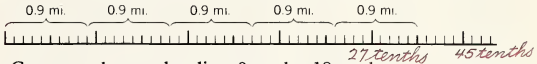
3. The written work for row 10 probably ought to be corrected before pupils proceed.



Multiplying a Decimal

M. of tenths by whole number [O]

1. Gail's cyclometer shows that she rides 0.9 mi. each day in making the round trip to and from school. Gail wants to know how many miles she rides in her 5 round trips a week.



Count on the number line: 9 tenths, 18 tenths, ..., 45 tenths = 4 ones and 5 tenths, or 4.5

Gail added the five 0.9's, as in box A. Read the sum as tenths; then as ones and tenths. 45 tenths, or 4 ones and 5 tenths

The numbers to be added are equal, so Gail could multiply, as in box B. If Gail multiplies tenths, her product must be tenths. Is it? Yes Explain. 4.5 means 45 tenths, or 4 ones and 5 tenths

A	B
0.9	
0.9	0.9
0.9	×5
0.9	4.5
+0.9	
4.5	

[W]

Add to find **n** in Ex. 2–7.

2. $6 \times 0.2 = n$ 1.2
3. $4 \times 0.3 = n$ 1.2
4. $2 \times 0.8 = n$ 1.6
5. $7 \times 0.5 = n$ 3.5
6. $8 \times 0.7 = n$ 5.6
7. $4 \times 0.4 = n$ 1.6

Beside each addition write the multiplication, as in box B.

232

8. In 6 round trips of 1.8 mi. to the store and back, how far will Gail ride?

Box C. Tenths: *Think*, "6 × 8 = 48. 48 tenths equal 4 ones and 8 tenths." Write "8" in tenth's place in the product and carry the 4 ones. Put the decimal point in the product to separate the decimal fraction from the whole number.

C
1.8
×6
10.8

Ones: *Think*, "6 × 1 = 6, and the 4 ones carried make 10 in all." Write "10" at the left of the decimal point.

When you multiply you are finding the total, so when you multiply tenths by a whole number (6 × 8 tenths), must you have tenths in the product? yes Which place at the right of one's place shows tenths? *the first place*

9. Write the addition to show that $6 \times 1\frac{8}{10} = 10.8$ *1.8+1.8+1.8+1.8+1.8+1.8 = 10.8*

[W]

Copy and multiply. Make sure that your products show tenths. Check the answers in row 10 by addition.

	a	b	c	d	e	f	g
10.	0.3	0.8	0.6	0.9	0.5	0.7	0.4
	×2	×4	×7	×4	×3	×5	×2
	0.6	*3.2*	*4.2*	*3.6*	*1.5*	*3.5*	*0.8*
11.	1.7	2.7	6.3	4.6	5.2	3.4	8.9
	×3	×8	×5	×8	×9	×6	×7
	5.1	*21.6*	*31.5*	*36.8*	*46.8*	*20.4*	*62.3*
12.	0.9	6.8	0.4	3.7	0.3	5.6	0.6
	×5	×3	×6	×2	×8	×7	×4
	4.5	*20.4*	*2.4*	*7.4*	*2.4*	*39.2*	*2.4*

13. Does 3 × 0.4 = 12 or [1.2] or 0.012?

14. Does 2 × 6.8 = 136 or 1.36 or [13.6]?

15. Does 4 × 28.4 = 11.36 or [113.6] or 1136?

When a number with the fractional unit tenths is multiplied by a whole number, the product must show tenths.

Differentiations and Extensions

1. Have *all pupils* ascertain the price per gallon for fuel oil or for gasoline. Use this information to obtain multiplication examples with mixed decimals. (*Illustration:* 9 gallons at 27.8¢ per gallon.) Pupils may also study and verify bills or invoices. By the time these forms have been obtained, pupils will have met examples with larger multipliers, as on page 235 of the text.

2. *More capable pupils* ought to be able to do orally row 10 on page 233 in the text.

3. *Slower learners* may need more practice in counting on a number line, as suggested on page 232 in the text, to develop understanding.

Teaching Pages 234 and 235

Pupil's Objective. To learn to multiply decimal numbers containing hundredths and thousandths by whole numbers.

Background. The extension to include multiplying hundredths and thousandths is a simple matter, since multiplying decimals is so similar to multiplying whole numbers.

In Ex. 6–8, and in the written work on page 235, pupils learn how to reduce decimal fractions. This is a simple matter involving omitting unnecessary end zeros. However, pupils should realize that in some cases it is best to retain end zeros (see 2 under Differentiations and Extensions).

Book Lesson. *Ex. 1 and 2:* Oral work. *Rows 3–5:* Written work. *Ex. 6–8:* Oral work. *Rows 9–12:* Written work.

1. Before each written-practice exercise, have pupils decide in which examples the products will be smaller than the multiplier times 1 (4 × 0.13 is less than 4 × 1, or 4). Have them determine also in which examples an approximate answer may be obtained by multiplying a mixed decimal that is rounded to a whole number (for example, 8 × 4.23 will have a product larger than 8 × 4, or 32; and 92 × 2.87 will have a product that is smaller than 92 × 3, or 276).

2. You may want to help pupils interpret the statement at the bottom of page 234 with an exercise similar to the following:

a. In multiplying tenths by a whole number, the product will show _? _. Since tenths take one decimal place, the product will have _? _ decimal place.

b. In multiplying hundredths by a whole number, the product will show _? _. Since hundredths take two decimal places, the product will have _? _ decimal places.

c. Complete statements like those above for multiplying thousandths by a whole number.

Differentiations and Extensions

1. For *slower learners* particularly, examine errors to determine if weaknesses are revealed in knowledge of multiplication facts, in adding carried numbers, or in understanding decimals. Consider to what extent regrouping of pupils will be necessary for reteaching.

2. **Extra Practice Set 94** offers additional work in multiplying with decimals. In this case, ask pupils *not* to reduce decimal fractions in the prod-

Multiplying Hundredths and Thousandths

Multiplier a whole number [O]

1. For joining two pipes, Mr. Best, the plumber, uses an on elbow listed as weighing 0.92 lb. He carried 4 of these lbows in his box of tools. How much did these 4 elbows weigh?

Rounded to the nearest whole number, 0.92 is early 1. Will the product of 4×0.92 be about 0.4 r about 4 or about 40? Explain. 0.92 rounded is 1, and $4 \times 1 = 4$

A
0.92
×4
3.68

In box A, when 2 hundredths are multiplied y 4, the product is 8 hundredths. In which column is the written? Explain all the multiplying.

When a decimal with hundredths as the fractional unit is ultiplied by a whole number, the fractional unit in the product hundredths. How do you show this? Write decimal point at left of second figure from the right.

2. Another piece of pipe was listed as weighing 0.798 lb. low much would 6 of these pieces weigh?

B
0.798
×6
4.788

When thousandths (box B) are multiplied by a hole number, the product shows thousandths

How many places to the right of one's place are sed to show thousandths? 3 Explain box B.

[W]

Copy and multiply the following:

	a	b	c	d	e	f
	0.13	0.47	0.254	0.73	0.462	4.9
	×4	×5	×3	×6	×7	×6
	0.52	2.35	0.762	4.38	3.234	29.4
	0.04	0.28	0.795	0.592	0.93	3.19
	×6	×3	×5	×7	×5	×4
	0.24	0.84	3.975	4.144	4.65	12.76
	0.37	0.69	0.686	0.284	0.086	5.621
	×2	×8	×4	×8	×6	×3
	0.74	5.52	2.744	2.272	0.516	16.863

When a decimal is multiplied by a whole number, the product must show the same fractional unit, and so the same decimal places, as the decimal factor.

[O]

6. Study the work in box C. hundredths

What fractional unit has the multiplicand? the product, 36.90? hundredths

Why can the decimal fraction .90 be reduced to .9? They are equal in value. Is the product in best form when we write it as 36.9? Yes, unless it is a measurement number.

C
6.15
×6
36.90, or 36.9

7. Study the work in box D. How is this multiplication different from that in box C? 2-place multiplier

Do we use decimal points in the partial products? No Explain the work.

What fractional unit do both the multiplicand and the product have? hundredths

D
1.24
×13
3 72
12 4
16.12

8. In box E, why do we have three decimal places in the product? There are three decimal places in multiplicand. To what can we reduce the decimal fraction in the product? .8

E
0.912
×25
4 560
18 24
22.800

[W]

Multiply. Be sure that each product has the correct fractional unit. Reduce decimal fractions in the products when you can.

	a	b	c	d	e	f
9.	0.8	0.3	4.23	0.4	27.3	5.36
	×5	×24	×8	×15	×20	×18
	4.0 or 4	7.2	33.84	6.0 or 6	546.0 or 546	96.48
10.	3.62	0.05	8.46	3.28	2.87	0.463
	×21	×26	×10	×21	×92	×7
	76.02	1.30 or 1.3	84.60 or 84.6	68.88	264.04	3.241
11.	8.3	27.6	96.7	3.62	7.286	8.039
	×26	×20	×54	×56	×5	×42
	215.8	552.0 or 552	5,221.8	202.72	36.430 or 36.43	337.638
12.	8.19	34.15	62.8	0.803	5.724	2.145
	×56	×18	×62	×14	×63	×24
	458.64	614.70 or 614.7	3,893.6	11.242	360.612	51.480 or 51.48

Extra Practice. For more practice, work Set 94.

ucts so they will realize it is often better that end zeros be retained. For example, in Ex. 1a below, the product could be given as \$81 or as \$81.00, but the latter form is more desirable, since it indicates that the original example contained hundredths of a dollar.

Set 94. Multiplying a decimal by a whole number; tenths, hundredths, and thousandths

	a	b	c	d	e	f
1.	\$6.75	3.468	202.9	2.5	\$20.50	18.7
	× 12	× 23	× 20	× 25	× 62	× 15
	\$81.00	79.764	4058.0	62.5	\$1,271.00	280.5
2.	1.602	\$0.85	0.083	1.025	0.09	8.650
	× 75	× 17	× 46	× 54	× 90	× 34
	120.150	\$14.45	3.818	55.350	8.10	294.100

Problems with Decimal Fractions

[W]

Write your work for these problems. Sometimes you will need to use numbers from other problems.

1. Mr. Griggs rides to work on the train a distance of 6.8 mi. In the 10 trips he makes during a week, how many miles does he ride? 68 mi.

2. A freight train averaged 11.5 mi. an hour for 12 hr. How many miles did it go in that length of time? 138 mi.

3. A passenger train averaged 45.875 mi. per hour during a 12-hour trip. How many miles did it travel in the 12 hr.? 550.5 mi.

4. Use your answers from Ex. 2 and 3 to find how much farther the passenger train went than the freight train in the same length of time. 412.5 mi.

5. One car in a 70-car train of coal cars held 56.25 T. of coal. How much coal could this whole train carry if each car were filled with the same amount? 3,937.5 T.

6. On the same railroad, the Marmion averaged 41.68 mi. per hour, and the Oglethorpe averaged 43.75 mi. per hour. How much farther can the Oglethorpe go in 12 hr. than the Marmion? 24.84 mi.

When the Multiplier is 10 or 100 or 1,000

Multiplying a decimal

1. As you study the work in box A, remember that any whole number can be written with a decimal point after it.

a. When 7. is multiplied by 10, you can think of the decimal point as moved 1 place to the right to make the product 70. Then 7 has been given 1 place-holder zero at its right.

b. When 7. is multiplied by 100, you can think of the decimal point as moved 2 places to the right to make the product 700. Then 7 has been given how many place-holder zeros at its right? 2

c. Tell what happens when 7. is multiplied by 1,000. The decimal point is moved 3 places to right

A	B
7.	3.82
a. $10 \times 7. = 70.$	a. $10 \times 3.82 = 38.2$
b. $100 \times 7. = 700.$	b. $100 \times 3.82 = 382.$
c. $1{,}000 \times 7. = 7000.$	c. $1{,}000 \times 3.82 = 3820.$

2. In box B explain what happens to the decimal point

a. in 10×3.82: moves 1 place to right b. in 100×3.82: moved 2 places to right c. in $1{,}000 \times 3.82$: moves 3 places to right

To find the product when the multiplier is 10 or 100 or 1,000, think of the decimal point as moved to the right as many places as there are 0's in the multiplier.

3. Multiply each of these numbers by 10:

5.8 58 0.725 7.25 2.409 24.09 0.77 0.009 0.09 452 4520 7.36 73.6

4. Multiply each of these numbers by 100:

7.16 716 3.2 320 0.95 95 0.044 0.880 736 73,600 5.04 504

5. Multiply each of these numbers by 1,000:

9.08 9,080 3.4 3,400 2.762 2,760 15.45 15,450 9.087 9,087 29.3 29,300 30.75 30,750

[W]

Extra Practice. For more practice, work Set 95.

Teaching Pages 236 and 237

Pupil's Objectives. (*a*) To work problems involving decimal numbers; (*b*) to learn how and why decimal numbers may be multiplied by 10, 100, or 1,000 by moving the decimal point.

Background. It has been said before that the decimal notational system is an extension of the whole-number notational system. Since this is so, any figure in either the decimal portion or the whole-number portion of a number can be thought of as one tenth of the figure immediately preceding it, or, in other terms, ten times as large as the number immediately following it. Because of this, we are offered a simple way of multiplying or dividing whole numbers or decimals by 10 or any power of 10. All that is necessary is to move the *figure* in question either to the right (to place-value columns of less value for division) or to the left (to place-value columns of greater value for multiplication), adding zeros and placing the decimal point as necessary.

In the text pupils think of multiplying whole numbers and decimals by 10 or a power of 10 in

terms of moving the *decimal point* to the right, adding place-holder zeros when necessary. Of course, this has the same result as moving the figure left to the appropriate place-value column, but it is probably easier for pupils to think in terms of moving the decimal point only.

Book Lesson (page 236). *Ex. 1–6:* Written work. Have pupils read through the problems so that they will have an opportunity to ask questions before starting written work.

Book Lesson (page 237). *Ex. 1–5:* Oral work. *Bottom of the page:* Written work. You may ask pupils to do the examples in box B on page 237 by the usual method of multiplying. They may then reduce answers and compare the multiplicands and products for each example. Let them state their own "discoveries" and compare their conclusions with the statement in dark print in the text.

Differentiations and Extensions

1. Have *all pupils* apply their understanding of multiplying decimal numbers by 10, 100, and 1,000 to problems involving money. After preliminary discussion, pupils may be encouraged to write original problems of this kind to present to the class.

2. Assign **Extra Practice Set 95** as needed.

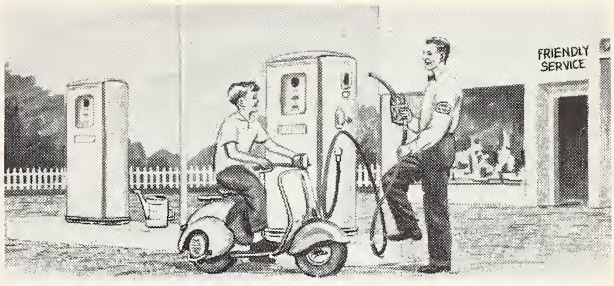
Set 95. Multiplying decimals by 10, 100, 1,000

	a	b	c	d	e
1.	6.3 × 10 = **63**	\$18.65 × 100 = **\$1,865**	83.052 × 1,000 = **83,052**	0.5 × 100 = **50**	\$7.38 × 10 = **\$73.80**
2.	\$98.75 × 1,000 = **\$98,750**	0.625 × 100 = **62.5**	2.6 × 1,000 = **2,600**	1.921 × 10 = **19.21**	\$0.75 × 1,000 = **\$750**
3.	0.07 × 10 = **0.7**	6.982 × 1,000 = **6,982**	9.5 × 100 = **950**	1.804 × 100 = **180.4**	0.8 × 1,000 = **800**

Teaching Pages 238 and 239

Pupil's Objective. To learn how to multiply whole numbers by decimals.

Background. Multiplying a whole number by a decimal may be thought of as multiplying a whole number by a common fraction. This procedure is used on page 232 in the text to help pupils realize that the product must show tenths. This generalization is similar to the one developed in multiplying a whole number times a decimal. On page 239, the idea is extended to include hundredths and thousandths, and the generalization thus becomes *When a whole number is multiplied by a decimal, the product has the same number of places at the right of one's place as the decimal factor has.*



Multiplying a Whole Number by a Decimal

[O]

1. At Lee's Filling Station, Jim got 0.7 gal. of gasoline for his motor scooter. The scooter goes 51 miles on 1 gallon of gasoline. How far will it go on 0.7 gal.?

If you use $\frac{7}{10}$ for 0.7, you know that on $\frac{7}{10}$ of a gallon, it will go $\frac{7}{10}$ of 51 mi., or $\frac{7}{10} \times 51$ mi. You think the same way to find 0.7 of 51 mi. In box A at the right, when 0.7×1 is found, why will the 7 in the product be 7 tenths? *0.7 × 1 = 1 × 0.7 = 0.7*

A	B
51 × 0.7 = 7	51 × 0.7 = 35.7

In box B, explain all the multiplying.

When a whole number is multiplied by tenths, the product shows *tenths* We put a decimal point in the product to show this.

[W]

Copy and multiply.

	a	b	c	d	e
2.	32 × 0.4 = *12.8*	85 × 0.2 = *17.0, or 17*	49 × 0.8 = *39.2*	67 × 0.5 = *33.5*	75 × 0.6 = *45.0, or 45*
3.	45 × 2.4 = *108.0, or 108*	73 × 4.5 = *328.5*	86 × 5.3 = *455.8*	39 × 6.2 = *241.8*	24 × 7.6 = *182.4*
4.	128 × 0.5 = *64.0, or 64*	364 × 0.9 = *327.6*	432 × 0.7 = *302.4*	875 × 0.4 = *350.0, or 350*	643 × 0.6 = *385.8*

[O]

5. At 41 mi. an hour, how far will a train go in 0.35 hr.? $0.35 \times 41 = ?$

In box C we have multiplied by the 5 hundredths in 0.35, so the 5 in the first partial product is in hundredth's place. We do not put points in partial products. Explain this multiplying.

In box D, for the second partial product, we have multiplied by the 3 tenths in 0.35, so the 3 in this partial product will be in *tenths* place. Do we use the decimal point in the partial product? *No*

Read the final product in box D. When any whole number is multiplied by hundredths, the product shows *hundredths*

C	D
41	41
×0.35	×0.35
2 05	2 05
	12 3
	14.35

E
286
×0.875
1 430
20 02
228 8
250.250

6. $0.875 \times 286 = ?$ Study the work in box E. Why is the first partial product thousandths? *Multiplier 5 means thousandths.* the second, hundredths? *Multiplier 7 means hundredths* the third, tenths? *Multiplier 8 means tenths* Why does the final product show 3 places at the right of one's place? In best form, the answer is *250.25* *To make product show thousandths*

[W]

Copy, and place decimal points correctly in the products.

	a	b	c
7.	$1.82 \times 237 = 43134$	$0.6 \times 138 = 828$	$0.407 \times 184 = 74888$
8.	$2.06 \times 247 = 50882$	$0.02 \times 54 = 108$	$0.843 \times 17 = 14331$

Copy and multiply.

9.	0.72×431 *310.32*	0.35×293 *102.55*	2.25×475 *1,068.75*
10.	0.875×424 *371*	0.375×856 *321*	0.625×784 *490*
11.	0.25×84 *21*	0.125×648 *81.0*	5.75×824 *4,738*

Extra Practice. For more practice, work Set 96.

When a whole number is multiplied by a decimal, the product has the same number of places at the right of one's place as the decimal factor has.

Book Lesson. *Ex. 1:* Oral work. *Rows 2–4:* Written work. *Ex. 5 and 6:* Oral work. *Rows 7–11:* Written work.

1. Help pupils discover that in Ex. 1 the multiplier is less than 1, so the product will be less than 1 times the multiplicand, or, in this problem, less than 51.

2. Have pupils determine which examples in rows 2 to 4 will produce answers larger than the multiplicand, and which will produce answers smaller than the multiplicand.

3. When Ex. 7 to 11 are completed, have pupils estimate the products to determine if their answers are sensible.

Differentiations and Extensions

1. After the written work has been corrected, have pupils try to explain why they made some of the errors they did (if any). Ask them then to summarize the particular types of errors so that they can avoid them in the future.

2. *More capable pupils* may work out multiplication examples of their own that are within the limits studied. Have partners check products for accuracy. Then have the examples put on the board with the decimal points omitted in the products. Let the other pupils in the class place the decimal points correctly, considering both the rule at the bottom of page 239 in the text and the reasonableness of the answers.

3. Assign **Extra Practice Set 96** as needed.

Set 96. Multiplying a whole number by a decimal; tenths, hundredths, and thousandths

	a	b	c	d	e	f
1.	36	42	15	16	80	24
	× 0.5	× 1.6	× 0.25	× 0.875	× 1.05	× 3.75
	18	67.2	3.75	14	84	90
2.	18	50	79	53	61	99
	× 1.625	× 0.50	× 21.5	× 14.50	× 100.8	× 0.005
	29.25	25	1,698.5	768.5	6,148.8	0.495

Teaching Pages 240 and 241

Pupil's Objectives. (*a*) To multiply mixed decimals by whole numbers by reversing factors when there is an advantage in doing so; (*b*) to review understandings and to practice several types of examples previously learned.

Background. Pupils have already learned in working with whole numbers and fractions that factors may be reversed in multiplication whenever it is convenient to do so. The same principle applies in multiplying decimals, and this idea is developed on page 240 in the text.

Book Lesson (page 240). *Ex. 1–3:* Oral work. *Rows 4–8:* Written work.

1. Remind pupils that when they reversed factors with whole numbers and with fractions it did not affect the products. You may want them to do some examples of both kinds to refresh their memories. Relate this understanding to multiplication with decimals as in Ex. 1 and 2 (and Boxes A and

Reversing Factors

Mixed decimal × whole number [O]

1. With the factors 18 and 2.314, would it be easier to multiply as in box A or as in box B? Why? 2 partial products easier to add than 4

2. In box B, when the factors are reversed we get different partial products. Does this change the final product? No

3. For rows 4–8 below, tell which examples will be easier to do if the factors are reversed. (R) Then tell how many decimal places each product should show.

A	B
18	2.314
×2.314	×18
72	18 512
18	23 14
5 4	41.652
36	
41.652	

When factors are reversed, the final product is not changed.

When one of two factors is a decimal, the product shows the same number of places at the right of one's place as this decimal factor does.

[W]

Copy and multiply. Reverse factors when you want to.

	a	b	c	d	e
	27	34	20	1.6	47
	×0.9	×8.3	×0.4	×3	×0.046
	24.3	282.2	8	4.8	2.162
R	0.02	89	0.8	3.9	R 5
	×36	×0.6	×7	×6	×2.59
	0.72	53.4	5.6	23.4	12.95
	42	8.3	74	R 37	214
	×2.6	×25	×6.2	×3.21	×0.076
	109.2	207.5	458.8	118.77	16.264
	863	R 12	R 65	43	R 75
	×0.52	×0.062	×0.146	×2.07	×2.456
	448.76	0.744	9.49	89.01	184.2
	53	400	R 42	R 0.005	R 124
	×0.03	×0.8	×1.426	×47	×3.582
	1.59	320	59.892	0.235	444.168

Just to Remind You!

[O]

1. Read these numbers: 4,166,285; 16.01; 43.865; 6.720; 1,500,000,000; 0.004; $\frac{5}{8}$ mi.; $9\frac{3}{4}$ T.; 6.4

2. Add mentally: **a.** 43 + 18 (*think*, "43, 53, 61"); **b.** 35 + 17; 52 **c.** 56 + 22; 78 **d.** 45 + 36; 81 **e.** 18 + 56; 74 **f.** 25 + 37. 62

3. Change to best form: $\frac{16}{5}$; $35\frac{14}{4}$; $3\frac{16}{8}$; 2 $\frac{20}{14}$. $1\frac{3}{7}$

4. Round to tenths: 7.62; 7.6 4.027; 4.0 61.66; 61.7 9.97 10.0

5. Tell the meaning or illustrate: ★**a.** picture graph; **b.** fractional unit; **c.** number line; **d.** factor; **e.** reference measure.

6. What reference measure would be useful in estimating the height of a tall plant? Height of a lamp or a table or a window sill, etc.

7. Tell the products of: **a.** 100 × 684; 68,400 **b.** 1,000 × 84.2; 84,200 **c.** 7.03 × 100; 703 **d.** 10 × 0.05; 0.5 **e.** 1.06 × 1,000; 1,060 **f.** 10 × 1.5 15

8. Tell how to find **n** in

a. 6.5 + n = 11.4; n = 11.4 − 6.5 **b.** 17.5 − n = 8.3; n = 17.5 − 8.3 **c.** n + 2.25 = 6.75 n = 6.75 − 2.25

9. How many zeros will there be in the denominator of the common fraction equivalent to 0.7? 1 to 0.39? 2 to 0.423? 3

Keeping Up Your Skill

[W]

Copy and work Ex. 1–19.

1. 489	2. 1.95	3. 80.6	4. 781	5. 8,000
340	8.26	2.9	−93	−5,928
658	1.08	31.4	688	2,072
35	3.67	7.2	6. 35	7. 49.3
+106	+0.35	+53.6	−2.86	−18.5
1,628	15.31	175.7	32.14	30.8

8. 34 × 87 2,958 9. 26 × 3.5 91 10. 0.04 × 96 3.84 11. 81)29,634 365.869

12. $2\frac{3}{4} \div 5\frac{1}{2}$ 13. $\frac{4}{5} \div 3\frac{1}{5}$ 14. $4\frac{2}{3} \times 1\frac{3}{8}$ 6 $\frac{5}{12}$ 15. $\frac{2}{5} \times 7\frac{1}{2}$ 3

16. $8\frac{1}{3} + 16\frac{3}{4}$ v $25\frac{1}{12}$ 17. $5\frac{3}{8} + \frac{7}{12}$ 5 $\frac{23}{24}$ 18. $6 - 3\frac{1}{8}$ 2 $\frac{7}{8}$ 19. $4\frac{1}{6} - 2\frac{3}{4}$ $1\frac{5}{12}$

B) on page 240 in the text. Point out that examples of this kind may be checked by reversing factors.

2. Ask pupils to find on pages 233, 234, and 239 statements that apply to fractional units and to the number of decimal places in a product. They may place on the board examples to illustrate each statement. Lead them to the realization that the second statement found in dark print on page 240 includes all three of the previous statements.

Book Lesson (top of page 241). *Ex. 1–9:* Oral work. Notice those exercises in the oral work for which pupils give the best responses and those for which they give the poorest responses. Make a note

(*Continued on page 252*)

★Answers Not on Reproduced Page 241

5. a. *Picture graph.* A symbol or a picture stands for a number or unit, and pictures are arranged in rows to show the numbers in the data.

b. *Fractional unit.* The size of one of the equal parts into which a whole (thing or group) is divided.

c. *Number line.* A line on which numbers representing distances are marked in order.

d. *Factor.* One of the two or more numbers which when multiplied together give a product, or total.

e. *Reference measure.* A familiar object easily compared to another object in estimating the latter's height, weight, etc.

of the weak areas which indicate the need for re-teaching or review.

Book Lesson (bottom of page 241). *Ex. 1–19:* Written work. Use your understanding of your pupils to anticipate their needs before they start the written work. Suggest certain measures to promote accuracy. For example, ask them what change is needed to work Ex. 6.

Differentiations and Extensions

1. Ask *more capable pupils* to estimate answers for some of the examples on these two pages. Encourage them to do this mentally when possible and to defend their estimates with an explanation of the procedure they used.

2. Have *slower learners* illustrate with devices some of Ex. 8–19 at the bottom of page 241.

Teaching Pages 242 and 243

Pupil's Objective. To discover the meaning of multiplying a decimal by a decimal (tenths by tenths).

Background. It is sometimes difficult for pupils to realize that multiplying tenths times tenths will give hundredths in the product. This is so whether the situation involves common fractions or decimal fractions. When an illustration is used, such as the one in the text which shows $\frac{3}{10}$ of $\frac{5}{10}$, the idea that a smaller fractional unit results is not difficult to understand. Similar work of this nature has already been covered in work with common fractions and, therefore, common fractions are used here to introduce the multiplication of tenths times tenths written as decimals.

Perhaps one of the most important things for pupils to realize in multiplying tenths times tenths is that the multiplier is less than 1 and, therefore, the product must be less than the multiplicand (i.e., only part of the multiplicand is taken). This whole idea is developed to the point where pupils realize that the product must have as many places at the right of one's place as there are decimal places in both factors together, as stated in the generalization in dark print at the bottom of page 243 in the text.

Teacher's Preparation. Provide means for clarifying the idea that multiplying tenths by tenths yields hundredths. The drawing at the top of the second column on page 253 in the Manual illustrates how 0.3×0.2 may be shown using graph paper. See pages 311 and 312 and 314 and 315 of this Manual for the Bead Board or Peg Board which may be used in a similar fashion.

Pre-book Lesson. With a device such as that illustrated on page 253, have pupils demonstrate the multiplication for an example like 0.3×0.2. Then point out that whereas $1 \times 0.2 = 0.2$, changing the multiplier to 0.3 (0.3×0.2) would yield a smaller product. Pupils should multiply using fractions ($\frac{3}{10} \times \frac{2}{10} = \frac{6}{100}$) to verify this.

Multiplying One Decimal Fraction by Another

Meaning: tenths × tenths

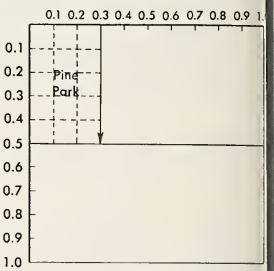
1. Bob's town is 4 mi. long and 3 mi. wide. In school one day he found the area by multiplying 3×4 sq. mi. = 12 sq. mi.

Explain from the drawing why he worked this way. 3 rows of 4 sq. mi. each equal 3×4, or 12 sq. mi.

2. Bob wanted to find the area of Pine Park. The speedometer on his father's car showed that the park was 0.3 mi wide and 0.5 mi. long. Bob knew that he probably should multiply 0.5 sq. mi. by 0.3. He *thought* "$3 \times 5 = 15$," but he didn't know what to do next.

Here are some ways Bob could have found the answer:

a. This square shows an enlarged diagram of 1 square mile in Bob's town. The heavy line from left to right marks off 0.5 of the square mile. A line drawn down from 0.3 (see arrow) marks off 0.3 of the 0.5 sq. mi.



The diagram showed Bob that the park was smaller than the 0.5 sq. mi., so he decided that the 15 he got by multiplying 3×5 must be 0.15 sq. mi. Explain why 0.15 is smaller than 0.5. 0.15 has only 1 tenth whi... 0.5 has 5 tenths.

b. To check his work, Bob used common fractions this way: $\frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$. Was Bob's 0.15 sq. mi. the correct answer? Ye...

c. In b above, multiplying tenths times tenths gives hundredths. So for 0.3×0.5, the answer is 0.15.

d. From the box tell an easy way to find the number of decimal places in the product. The number of decimal places in the product is the sum of the number of places in the multiplier and multiplicand.

1 place	1 place	2 places
0.5	× 0.3	= 0.15

3. Boxes A and B show two steps in multiplying $.3 \times 0.5$ when written in vertical form.

Box A shows that for the first step we *think* $3 \times 5 = ?$" instead of "$0.3 \times 0.5 = ?$"

In box B, we can *think* "tenths × tenths = undredths," so the product must show hundredths. ut we usually use the quicker way of noticing that ach of the factors (0.5 and 0.3) has 1 decimal place, o the product must show 2 decimal places.

A	B
0.5	0.5
×0.3	×0.3
15	0.15

The product 0.15 is less than the multiplicand 0.5. Why ould this be? The multiplier is less than 1. How does the 0 in 0.15 help to show that the roduct has no whole number? It means no ones.

4. $0.2 \times 0.4 = ?$ Study the work in box C.

How many decimal places must there be in the roduct? 2 In box C, how do we show the 2 places here the product of 2×4 is a 1-place number? By using zero as place-holder in tenths place.

C
0.4
×0.2
0.08

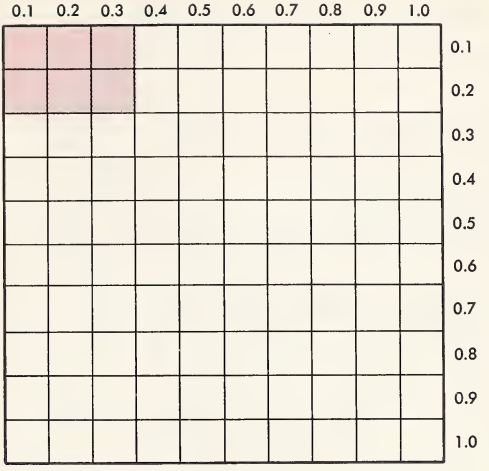
5. For each example in row 6 below, tell how many decimal laces the product should have.

[W]

Copy and multiply in rows 6–8. Check each product to be ure it has the correct number of decimal places.

	a	b	c	d	e	f
.	0.3	0.7	0.8	0.4	0.6	0.2
	×0.4	×0.2	×0.2	×0.5	×0.9	×0.5
	0.12	0.14	0.16	0.20	0.54	0.10
.	0.4	0.8	0.6	0.6	0.3	0.2
	×0.6	×0.5	×0.3	×0.2	×0.7	×0.9
	0.24	0.40	0.18	0.12	0.21	0.18
.	0.9	0.3	0.9	0.5	0.2	0.7
	×0.8	×0.3	×0.7	×0.6	×0.4	×0.8
	0.72	0.09	0.63	0.30	0.08	0.56

When two decimals are multiplied, the product has as many places at the right of one's place as there are decimal places in both factors together.



Differentiations and Extensions. Encourage your pupils, particularly the *more capable pupils*, to anticipate the next step in learning the process of multiplying decimals. Ask them how they would place the decimal point if they were multiplying hundredths by tenths, or hundredths by hundredths, basing their thinking on the rule at the bottom of page 243. If some of them wish to study the next page independently, let them do so. If they become perplexed, assure them that you will help them at the proper time.

NOTES

Book Lesson. *Ex. 1–5:* Oral work. *Rows 6–8:* Written work.

1. You may wish to have the pupils do by the fraction method, either on the board or on their papers, at least one row of the examples in the written work in order to compare products with those obtained using decimals.

2. Check the results of Ex. 8**b** and 8**e** carefully to make sure pupils are able to use a place-holder in decimal products when necessary (as presented in Ex. 4 and box C on page 243 in the text).

3. *More capable pupils* ought to be able to give answers for the examples in rows 6–8 orally, since only the locating of decimal points is new.

Multiplying Two Mixed Decimals

Estimating product [O]

1. Mr. Lee's car averages 14.3 mi. per gallon of gasoline. At that rate, how far will it go on 8.6 gal.?

Explain all the work in box A. Does the product have as many decimal places as there are in the two factors together? Yes

You can test the product to see if it is sensible. You round the factors to the nearest whole numbers and multiply, as in box B. From the product 126, how would you know where to put the decimal point in 12298 (box A)? The whole-number part must show hundreds.

A	B
14.3 ×8.6 858 1144 122.98	Rounding: 14 ×9 126

2. A kilometer is a unit of length used in many countries. A mile is 1.61 kilometers, so 3.8 miles are equal to how many kilometers? $3.8 \times 1.61 = ?$

$38 \times 161 = 6118$. Does $3.8 \times 1.61 =$ (6.118) or 61.18?

Show on the board that your answer is sensible. 4 × 2 = 8; whole-number part must be less than 8 [W]

In rows 3 to 6, first estimate the products by rounding the numbers and multiplying. Then copy each example, multiply, and check the number of decimal places in the product.

	a	b	c	d	e
3.	4.87 (10) ×2.4 11.688	3.45 (9) ×2.8 9.66	7.34 (14) ×1.6 11.744	9.2 (9) ×1.3 11.96	6.7 (18) ×2.9 19.43
4.	5.3 (35) ×7.1 37.63	2.9 (12) ×3.54 10.266	4.12 (8) ×1.6 6.592	4.914 (45) ×9 44.226	28.7 (232) ×7.6 218.12
5.	16.2 (96) ×5.5 89.1	3.7 (4) ×1.42 5.254	7.42 (63) ×9.3 69.006	4.2 (32) ×7.57 31.794	38.1 (76) ×2.2 83.82
6.	5.43 (20) ×3.7 20.091	3.61 (12) ×2.8 10.108	21.6 (88) ×3.9 84.24	8.275 (64) ×8 66.2	12.7 (65) ×4.6 58.42

Extra Practice. For more practice, work Sets 97–99.

Is Your Product Sensible?

Each of the five problems below calls for the product of tw decimals. The product figures are shown. Read each produ with its decimal point correctly placed. Then tell why yo think your answer is sensible.

1. The cruising speed of an airplane is 3.5 mi. a minut How many miles will it fly in 1.8 min.? 630 mi.

2. A family buys, on the average, 12.5 qt. of milk a wee At $0.275 a quart, how much was a week's milk bill? $3437

3. A ranch 2.6 mi. long and 1.7 mi. wide contains how man square miles? 442 sq. mi.

4. A bag that now costs $5.65 used to cost 1.8 of that amoun What was its former cost? $10170

5. Tom's dog weighs 24.75 lb. Tom himself weighs 3 times as much. How much does Tom weigh? 94050 lb.

For each example in rows 6–8, find an (estimated product) b rounding. Then copy the example and multiply. [V]

	a	b	c	d	e
6.	2.84 (3) ×4.7 13.348	3.67 (8) ×1.8 6.606	2.08 (14) ×7.4 15.392	2.844 (105) ×35 99.54	$12.5 ×4 $52.
7.	3.64 (4) ×1.2 4.368	7.9 (32) ×3.7 29.23	4.28 (32) ×8.4 35.952	3.22 (15) ×4.5 14.49	$8.1 ×5 $45.
8.	2.6 (18) ×5.7 14.82	3.81 (12) ×3.4 12.954	10.30 (30) ×2.6 26.78	5.026 (15) ×3 15.078	$8.1 ×7 $59.9

Teaching Pages 244 and 245

Pupil's Objectives. (*a*) To learn how to multiply mixed decimals when both tenths and hundredths occur; (*b*) to determine the correct position for decimal points in products by estimating and by rule; (*c*) to tell why answers in problems which call for the product of two mixed decimals are sensible.

Background. The generalization developed on page 243 for multiplying tenths by tenths can also be used in multiplying mixed decimals containing either tenths, hundredths, or thousandths, or any combination of these. The important point for pupils to realize is that the product will have as many decimal places as there are in both factors together. Of course, there are times when the final decimal place needs to be dropped or used in rounding to the next place on the left, as in Ex. 1**a**, Set 97, under Differentiations and Extensions on page 255.

Estimating is very useful in multiplying mixed numbers. In most cases the estimating involves rounding to the nearest whole number and multi-

plying whole numbers. This is shown in box B on page 244 and is used by the pupils on page 245 in deciding where it would be sensible to place decimal points in products.

Book Lesson (page 244). *Ex. 1 and 2:* Oral work. *Rows 3–6:* Written work.

1. Before pupils do the written work, give additional help in rounding numbers and estimating.

2. *Slower learners* may need to estimate at least two rows of examples orally in a small group before they attempt the written computation.

Book Lesson (page 245). *Ex. 1–5:* Oral work. *Rows 6–8:* Written work.

Differentiations and Extensions

1. Encourage *more capable pupils* to:

a. Obtain information about the prefix *kilo*, found in Ex. 2 on page 244. Ask them to find and explain terms of measures in which this prefix is used (such as kilocycle, kilowatt, kilogram).

b. Investigate some of the uses of decimals in everyday activities, especially uses that require multiplication. The problems on page 245 may help them think of typical situations.

2. Further work in multiplying decimals is provided in **Extra Practice Sets 97, 98, and 99.**

Set 97. Multiplying a decimal times a decimal; tenths and hundredths; some money numbers

	a	b	c	d
1.	\$0.75 × 0.4 = **\$0.30**	0.9 × 0.9 = **0.81**	0.80 × 0.6 = **0.48**	\$0.18 × 0.5 = **\$0.09**
	e. 0.5 × 0.8 = **0.4**	f. \$0.45 × 0.2 = **\$0.09**	g. 0.4 × 0.9 = **0.36**	h. 0.6 × 0.25 = **0.15**
2.	a. 0.6 × 0.80 = **0.48**	b. \$0.20 × 0.5 = **\$0.10**	c. 0.4 × 0.07 = **0.028**	d. \$0.90 × 0.5 = **\$0.45**
	e. 0.7 × 0.01 = **0.007**	f. 0.3 × 0.16 = **0.048**	g. 0.02 × 0.9 = **0.018**	h. 0.1 × 0.3 = **0.03**

Set 98. Multiplying a decimal times a decimal; tenths and hundredths; some money numbers

	a	b	c	d	e
1.	3.6 × 1.4 = **5.04**	\$1.50 × 2.2 = **\$3.30**	\$50.75 × 3.4 = **\$172.55**	4.5 × 10.6 = **47.7**	\$2.08 × 12.5 = **\$26.00**
2.	1.5 × 6.25 = **9.375**	26.4 × 2.38 = **62.832**	6.8 × 15.70 = **106.76**	\$18.25 × 4.8 = **\$87.60**	\$30.82 × 7.5 = **\$231.15**

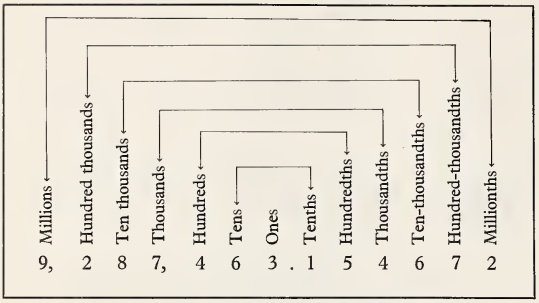
Set 99. Multiplying a decimal times a decimal; tenths and hundredths; some money numbers

1.	8.2 × 1.4 = **11.48**	\$17.25 × 8.6 = **\$148.35**	201.2 × 5.9 = **1,187.08**	\$50.20 × 2.5 = **\$125.50**	1.8 × 6.75 = **12.15**
2.	1.96 × 3.8 = **7.448**	41.5 × 4.42 = **183.43**	\$6.50 × 2.7 = **\$17.55**	4.06 × 1.25 = **5.075**	\$3.48 × 6.5 = **\$22.62**

NOTES

Smaller Decimals

Reading, through millionth [O]



This number chart extends farther to the right than those you have had before in this book.

1. In the chart, what is the name of the place that is farthest left? farthest right? in the middle? *millions* *millionths* *one's*

2. Tell the name of each place in the whole-number part of the chart and then the name of the place that balances it in the decimal-fraction part.

Begin this way: "Ten's place is balanced by tenth's place; hundred's place is balanced by *hundredths* place"; and so on.

3. We read the number in the chart this way:

9 million 287 thousand 463 and 154,672 *millionths*.

a. What place gave a name for the decimal fraction? In any decimal, does the name of the last used place farthest to the right always show what the fractional unit is? *Yes* Explain.★

b. What does the word "and" stand for as you read the number? *the decimal point*

4. Use the chart if you need to, and read the following:

a. 6.2573 **b.** 9.709 **c.** 72.00625 **d.** 824.00075

It will help you to read and understand decimals of many places if you first say the decimal places beginning with tenth's.

5. In the chart on page 246, say the decimal-place names at the right of one's place: "Tenth's, hundredth's," and so on.

6. Name each decimal place in Ex. a–d. Then read each of the numbers.

a. 7.0059 **b.** 0.006342 **c.** 0.01479 **d.** 16.3006

7. A very small unit of weight called a *gram* equals about 0.002205 lb. Read this number. *2,205 millionths of a pound*

8. A *micron* is a unit of length in microscope work. It equals 0.000039 inch. Can you read this number? *Thirty-nine millionths of an inch*

Products with More than Three Decimal Places

[O]

1. Find the product of 3.25 and 0.95.

a. Look at the box and *think*, "2 decimal places in one factor and 2 in the other; that's 4 places for the product. I count 4 places from right to left and then put in the decimal point." This is called "pointing off." The product is *3.0875*

	3.2 5
×	0.9 5
	1 6 2 5
	2 9 2 5
	3 0 8 7 5
	④ ③ ② ①

b. Now say the work as you estimate the product. Why is 3.0875 a sensible answer? *Using numbers rounded to 1, 3, 1, 3 — The product should be about 3.*

2. Tell how to point off each product. Read each product.

a. 0.46 × 1.05 = *0.4830* **c.** 0.3 × 0.1264 = *0.03792*

b. 0.42 × 3.2764 = *1.376088* **d.** 0.46 × 0.0285 = *0.013110*

[W]

Copy and multiply to find

	a	b	c	d
3.	0.075 × 4.25 *0.31875*	0.25 × 5.875 *1.46875*	9.5 × 0.0375 *0.35625*	0.375 × 4.8 *1.8000, or 1.8*
4.	0.875 × 8.96 *7.84000, or 7.84*	0.625 × 0.34 *0.21250, or 0.2125*	0.125 × 9.78 *1.22250, or 1.2225*	1.83 × 36.6 *66.978*

Extra Practice. For more practice, work Sets 100 and 101.

Teaching Pages 246 and 247

Pupil's Objectives. (*a*) To learn to read smaller decimal numbers (to millionths); (*b*) to learn how to point off products having more than three decimal places.

★Answers Not on Reproduced Page 246

3. a. Since we express decimal fractions in but one fractional unit, it must be the smallest, or the one shown by the last used place farthest right (i.e., instead of expressing 0.29 in two fractional units, as 2 tenths 9 hundredths, we express it in the smallest fractional unit, shown by the decimal place farthest right, as 29 hundredths).

Background. The chart at the top of page 246 in the text is simply an extension of the chart shown in box B on page 223. Pupils will find it useful in learning the values of the decimal places beyond thousandths. Notice that at this time pupils learn only to read these very small decimals since they are useful in situations such as those given in Ex. 7 and 8 at the top of page 247 and in similar situations that frequently arise in everyday affairs.

At the bottom of page 247, pupils learn that "pointing off" is counting from right to left in the product to make the product have as many decimal places as the two factors together. At the same time, they will discover that very small decimals can result in multiplication, but they should

be able to read these with ease because of the previous work with smaller decimals on page 246.

Book Lesson (page 246 and top of page 247). *Ex. 1–8:* Oral work.

Book Lesson (bottom of page 247). *Ex. 1 and 2:* Oral work. *Rows 3 and 4:* Written work.

Differentiations and Extensions

1. Ask *more capable pupils* to use decimal figures in writing statements like those at the right.

a. A meter equals thirty-nine and thirty-seven hundredths inches. (39.37)

b. A kilometer equals sixty-two thousand one hundred thirty-seven hundred-thousandths of a mile. (0.62137)

c. A liter equals one and five hundred sixty-seven ten-thousandths quarts. (1.0567)

d. A kilogram equals two and two hundred five thousandths of a pound. (2.205)

2. Assign **Extra Practice Sets 100 and 101** as needed.

Set 100. Multiplying smaller decimals; some money numbers

	a	b	c	d	e	f
1.	0.25 × 0.7 = **0.175**	1.875 × 0.8 = **1.5**	0.25 × 0.25 = **0.0625**	2.605 × 0.12 = **0.3126**	\$3.75 × 42 = **\$157.50**	4.12 × 0.75 = **3.09**
2.	2.125 × 0.015 = **0.031875**	20.98 × 0.6 = **12.588**	84 × 0.16 = **13.44**	0.8 × 0.20 = **0.16**	0.646 × 0.32 = **0.20672**	32.59 × 0.105 = **3.42195**

Set 101. Multiplying smaller decimals; some money numbers

1.	0.725 × 0.9 = **0.6525**	1.048 × 1.50 = **1.572**	\$0.60 × 1.5 = **\$0.90**	30.45 × 0.125 = **3.80625**	\$75.20 × 34 = **\$2,556.80**	\$72.00 × 0.9 = **\$64.80**
2.	12.875 × 0.102 = **1.31325**	10.375 × 0.20 = **2.075**	86 × 0.9 = **77.4**	5.208 × 0.375 = **1.953**	90 × 0.65 = **58.5**	0.255 × 0.16 = **0.0408**

NOTES

Problems with Decimals

[O]

Tell the example you will write for each problem.

1. In 1940 the population of Atkins was 5,725. In 1950 it was 1.2 times as great. What was the population in 1950? 1.2 X 5,725 = 6,870.0, or 6,870

2. At 2.8 mi. an hour, how far can Joe walk in 1.5 hr.? 1.5 X 2.8 = 4.20, or 4.2 mi.

3. Shoes which once cost $3.75 now cost 2.4 times as much. What is the present cost? 2.4 X $3.75 = $9.000, or $9

4. At the rate of 160 words a minute, how many words can Jack read in 3.75 min.? 3.75 X 160 = 600.00, or 600

5. Remembering that $\frac{1}{2} = 0.5$ and $\frac{1}{4} = 0.25$, multiply $\frac{1}{4}$ by $7\frac{1}{2}$ using decimals instead of common fractions. 7.5 X 0.25 = 1.875

6. If $\frac{1}{4}$ lb. of fish is allowed at a restaurant for a portion, how much would be allowed for $4\frac{1}{2}$ portions? Show how to solve by common fractions and by decimals. $4\frac{1}{2} \times \frac{1}{4} = \frac{9}{2} \times \frac{1}{4} = \frac{9}{8} = 1\frac{1}{8}$ lb.
4.5 X 0.25 = 1.125 lb.

[W]

Now write your work for problems 1–6.

So You Won't Forget!

[O]

In Ex. 1–3, tell what number you will get if you

1. round to tenths: 5.29 5.3 0.74 0.7 70.258 70.3 3.98 4.0 8.02 8.0
2. round to hundredths: 0.254 0.25 7.637 7.64 2.397 2.40 1.703 1.70 2.040
3. round to thousandths: 4.06257 4.063 0.00738 0.007 2.0396

4. In the example in the box, the mixed decimal 8.72 is between 8 and 9. Which is it nearer? 9 Since 3 × 8 = 24, and 3 × 9 = 27, the answer must be between 24 and 27. Why is it nearer to 27? 8.72 is nearer to 9 than to 8.

8.72
×3
26.16

As in Ex. 4, estimate the products for

5. 6 × 2.13 nearer 12 than 18
6. 7 × 4.82 nearer 35 than 28
7. 12 × 2.61 nearer 36 than 24
8. 3.842 × 5 nearer 20 than 15
9. 2.305 × 15 nearer 30 than 45
10. 0.624 × 3 nearer 3 than 0

Work for Young Experimenters

Supplementary activity [O]

4.12 × 28.9 = ?

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	
	2	8.	9			
×		4.	1	2		
			5	7	8	(hundredths × tenths = thousandths)
		2	8	9		(tenths × tenths = hundredths)
1	1	5	6			(whole number × tenths = tenths)
1	1	9.	0	6	8	

1. Janet wrote multiplication examples with all the points in a straight line, as in addition and subtraction with decimals. Why is it sensible to write the multiplier where she did? To line up ones under ones, tenths under tenths, etc. place-wo

Study the following to help you think the partial products as Janet did:

First partial product:
2 hundredths × 289 tenths = $\frac{2}{100} \times \frac{289}{10} = \frac{578}{1,000}$, or 578 thousandths.
Or, as a short cut, *think*, "hundreds × tenths = thousandths."
Is Janet's first partial product written to show it as thousandths? Yes
(We do not use decimal points in partial products.)

Second partial product:
1 tenth × 289 tenths = $\frac{1}{10} \times \frac{289}{10} = \frac{289}{100}$, or 289 hundredths.
Or, as a short cut, *think*, "tenths × tenths = hundredths."
Is Janet's second partial product written to show it as hundredths? Yes

Third partial product:
4 × 289 tenths = $4 \times \frac{289}{10} = \frac{1,156}{10}$, or 1,156 tenths.
Or, as a short cut, *think*, "a whole number × tenths = tenths."
Is Janet's third partial product written to show it as tenths? Yes

2. At the board, work Janet's example in the way you have been using to see if you get the same product as Janet did.

[W]

Multiply in Janet's way. Then check by your own way.

3. 2.4 × 0.85 (2.04)
4. 0.39 × 6.7 (2.613)
5. 2.7 × 8.64 (23.328)
6. 3.85 × 7.92 (30.492)

Teaching Pages 248 and 249

Pupil's Objectives. (*a*) To solve problems with decimals; (*b*) to review rounding decimal numbers; (*c*) to estimate decimal products by determining the limits within which the products must lie; (*d*) to experiment, as a supplementary activity, with another way of writing the work in multiplying decimals.

Background. Notice that the work on page 249 is headed by a clue caption indicating a supplementary activity. It is the kind of work that you may offer to *more capable children* only, or that you may omit entirely if you wish.

Keep in mind that *more capable pupils* frequently will devise unusual methods of solution. This type of creative thinking should be encouraged.

Book Lesson (top of page 248). *Ex. 1–6:* Oral work. *Bottom of lesson:* Written work.

Book Lesson (bottom of page 248). *Ex. 1–10:* Oral work. You may give more help at the board with this kind of estimating:

	Estimate		*Exact*
8.72	8 and	9	8.72
× 3	× 3	× 3	× 3
	24	27	26.16

Product is between 24 and 27

Pre-book Lesson (page 249). You may present the following explanation at the board to accompany work at the top of page 249.

28.9 × 4.12 — The multiplier, 4 and 12 hundredths, means 4 (whole number) and 1 tenth (0.1) and two hundredths (0.02). Multiply the multiplicand by each part of the multiplier, obtaining partial products.

28.9 × 0.02	28.9 × 0.1	28.9 × 4
0.578	2.89	115.6

(hundredths × tenths = thousandths)
(tenths × tenths = hundredths)
(whole number × tenths = tenths)

Add partial products

0.578
2.89
115.6
119.068

Leave this work on the board for reference.

Book Lesson (page 249). *Ex. 1 and 2:* Oral work. *Ex. 3–6:* Written work. Refer to the work that was left on the board from the Pre-book Lesson as pupils study Ex. 1 and 2 and the accompanying box in the text.

Differentiations and Extensions

1. Ask your *more capable pupils* to work out an example such as 42.5 × 3.75 by the following methods: estimating, fraction method, the usual decimal method, Janet's way, and Peter's way. Have them suggest other possible ways which they might use.

2. If you decide to limit the work on page 249 to your *more capable pupils*, you might consider using the time that they are working independently to provide supplementary material for the other pupils. Too often these pupils fail to have a chance to see that arithmetic can be fun, for they have to practice skills which others have mastered. This also might be a good time to introduce some of the games described in Part IV of the Manual.

Practice with Whole Numbers

M. and D. [W]

Copy and multiply.

	a	b	c	d	e
1.	380 × 17 = 6,460	475 × 86 = 40,850	2,809 × 48 = 134,832	7,510 × 70 = 525,700	162 × 259 = 41,958
2.	735 × 204 = 149,940	527 × 369 = 194,463	780 × 356 = 277,680	405 × 780 = 315,900	295 × 600 = 177,000

Divide. Write any remainder with R.

3.	24)6,115 = 254,R19	16)612 = 38,R4	14)7,910 = 565	27)1,302 = 48,R6	214)1,712 = 8
4.	28)5,614 = 200,R14	36)981 = 27,R9	15)6,105 = 407	46)1,106 = 24,R2	189)1,521 = 8,R9

Practice with Fractions and Decimals

A., S., M., D. [W]

Add or subtract as the sign tells you in Ex. 1–8.

1.	3.6 + 2.8 + 7.0 + 9.6 + 0.9 = 23.9	2. 0.75 + 0.58 + 6.02 + 3.58 + 0.07 = 11.00	3. 6.042 − 1.597 = 4.445	4. 3.000 + 6.524 = 9.524
			5. 0.402 − 0.315 = 0.087	6. 2.004 + 1.584 = 3.588
7.	$3\frac{1}{4} + 2\frac{1}{2} = 5\frac{3}{4}$	$\frac{7}{15} + 8\frac{2}{3} = 9\frac{7}{15}$	$2\frac{3}{4} + 6\frac{1}{3} = 9\frac{1}{12}$	$8\frac{2}{5} + 4\frac{1}{2} = 12\frac{9}{10}$	$5 + 3\frac{3}{8} = 8\frac{3}{8}$
8.	$2\frac{7}{10} - \frac{3}{10} = 2\frac{2}{5}$	$2\frac{1}{4} + 1\frac{3}{4} = 4$	$7 - 6\frac{7}{8} = \frac{1}{8}$	$8\frac{3}{8} - 2\frac{5}{6} = 5\frac{13}{24}$	$6\frac{2}{3} + 4\frac{3}{4} = 11\frac{5}{12}$

Multiply or divide as the sign tells you in rows 9–11.

9. $2 \times \frac{3}{8} = \frac{3}{4}$ $\frac{5}{8} \div 10 = \frac{1}{16}$ $5\frac{5}{6} \times \frac{3}{5} = 3\frac{1}{2}$ $\frac{3}{4} \div \frac{9}{10} = \frac{5}{6}$ $3\frac{1}{3} \times 1\frac{1}{2} \times \frac{3}{5} = 3$

10. $1\frac{1}{8} \times 4 = 4\frac{1}{2}$ $\frac{3}{8} \times \frac{4}{5} = \frac{3}{10}$ $2 \times 4\frac{2}{3} = 9\frac{1}{3}$ $4\frac{1}{3} \times 6 = 26$ $6\frac{1}{5} \div 2\frac{1}{8} = 3\frac{1}{5}$

11. $24 \div 64 = \frac{3}{8}$ $4\frac{1}{2} \times \frac{2}{3} = 3$ $10 \times 1\frac{2}{3} = 16\frac{2}{3}$ $2\frac{1}{4} \times 1\frac{1}{3} = 3$ $4\frac{2}{3} \div 3\frac{1}{9} = 1\frac{1}{2}$

250

Background. This page is part of the maintenance program for Grade 6. You may find, however, that you wish to use it to discover which areas of the work so far covered need reteaching before the end-of-chapter tests for Chapter 6 are taken by your pupils.

If you decide to use this lesson as a pre-test, be sure to allot sufficient time for pupils to ask any questions about the work.

Book Lesson (top of page). *Rows 1–4:* Written work.

Book Lesson (bottom of page). *Ex. 1–11:* Written work.

Teaching Page 250

Pupil's Objectives. (*a*) To obtain practice in multiplication and division of whole numbers; (*b*) to obtain mixed practice with fractions and decimals.

Do You Understand?

Test of Information and Meaning 6

1. Write the following decimal fractions as common fractions:

a. 0.5; $\frac{1}{2}$ b. 0.64; $\frac{16}{25}$ c. 0.125; $\frac{1}{8}$ d. 0.08; $\frac{2}{25}$ e. 0.005; $\frac{1}{200}$ f. 0.075 $\frac{3}{40}$

2. Write as decimal fractions:

a. $\frac{3}{100}$; .03 b. $\frac{75}{1000}$; .075 c. $\frac{7}{1,000}$; .007 d. $\frac{345}{1,000}$; .345 e. $\frac{25}{1,000}$; .025 f. $\frac{6}{100}$ .06

3. Which is larger:

a. 1.01 or (1.10)? b. 0.984 or (1.03)? c. (6) or 5.875?

4. Round to tenths: a. 16.06; 16.1 b. 31.97; 32.0 c. 20.197 20.2

5. Round to hundredths: a. 0.651; .65 b. 3.896; 3.90 c. 0.073 .07

6. Copy this number and write above each figure the name of its place value: 1,493.8526

7. Write the number in Ex. 6 by copying and completing the following: one thousand four hundred ninety-three and eight hundred thousand five ten-thousandths twenty-six

8. What does 0 mean in the number 0.875? no ones

9. Copy the number 0.250 and reduce it to lower terms. 0.25

10. Write the following in columns and find answers: 1.25 ft.

a. 0.75 + 0.8 + 0.125 + 0.6 2.275 b. 2.75 ft. − 1.5 ft.

11. Write in lowest terms the common fraction which each of these equals:

a. 0.5 $\frac{1}{2}$ b. 0.375 $\frac{3}{8}$ c. 0.875 $\frac{7}{8}$ d. 0.25 $\frac{1}{4}$ e. 0.8 $\frac{4}{5}$ f. 0.75 $\frac{3}{4}$

12. How many places at the right of one's place will there be in the product of

a. 8 × 4.6? 1 b. 9.7 × 5? 1 c. 0.5 × 0.9? 2 d. 5.14 × 7.25? 4

13. What is the product of

a. 10 × 7.5? 75 b. 1,000 × 0.82? 820 c. 100 × 2.975? 297.5

14. Write the example you would use for estimating the product of 2.8 × 5.25 3 × 5 = 15

Can You Solve Problems?

Problem Test 6

1. Mr. Shaw lives 3.9 miles from the place where he works. He makes 2 round trips each day. How far does he drive in a day going to and from work? 4 × 3.9 = 15.6 miles

2. When milk is 26.5¢ a quart or 2 quarts for 50¢, how much money can be saved by buying 2 quarts at a time? 3¢

3. Elsa's sister, June, works in a store. Each time June cuts off a piece of yard goods from a bolt, or roll, of material, she has to write on the bolt the number of yards that are left. She keeps the record in decimals to make the figuring easier. If the last record is 30.75 yards and June cuts off 3.25 yards, what does she write on the bolt? 27.5 yds.

4. Bill's brother ran three times around the half-mile track. How far did he run? (Use a decimal for "half mile.") 1.5 mi.

5. Taking the weight of a gallon of water as 8.34 pounds, find to the nearest whole pound the weight of 3 gallons of water. 25 lb.

6. Joe's brother Jack goes to a shop class where they measure some lengths to the nearest hundredth of an inch. A rectangular steel plate measured 4.25 inches long and 2.75 inches wide. How much greater was the length than the width? 1.5 in.

7. How much will 2.5 pounds of cheese cost at $0.82 a pound? $2.05

8. How much candy is there in two candy bars, one of which weighs 1.5 ounces and the other 2.75 ounces? (2.8) 4.3 oz.

9. Tom's father bought 30.5 feet of Manila rope at 7¢ a foot. How much did he pay for the rope? $2.14

10. For baked seafood, Mrs. Hancort used 1 lb. of cooked haddock, a 7.5-ounce can of lobster meat, a 6.5-ounce can of crabmeat, and a 5.5-ounce can of shrimps. How much seafood did she use? 35.5 oz., or 2 lb. 3.5 oz.

Teaching Pages 251, 252, and 253

Pupil's Objective. To take the four regular end-of-chapter tests.

Pre-book Lesson. Although your pupils have already taken five series of end-of-chapter tests, there may be some pupils who still approach tests with fear. Take some time to point out again the values they should derive from these tests.

Book Lesson (page 251). *Ex. 1–14:* Written work. Test of Information and Meaning 6.

Book Lesson (page 252). *Ex. 1–10:* Written work. Problem Test 6.

Book Lesson (top of page 253). *Rows 1–7:* Written work. Diagnostic Test 6. Practice sets with answers are reproduced on previous pages in the Manual. For their location, refer to the heading *Practice* in the index to this Manual.

Book Lesson (bottom of page 253). *Ex. 1–20:* Written work. Computation Test 6. If necessary, this test may be used for still further practice as there is an alternate computation test on page 303 of the Manual.

Differentiations and Extensions. Use the table on the next page as aid in entering the scores on the test record cards.

Do You Make Mistakes?

Diagnostic Test 6

	Copy in columns and work.	Study Pages	Practice Sets
	a b c		
1.	$0.9 + 0.4$ 1.3 $1.7 + 3.5$ 5.2 $75.4 + 69.8$ 145.2	220	92
2.	$2.5 - 0.9$ 1.6 $7.3 - 2.8$ 4.5 $85.2 - 78.9$ 6.3	221	92
3.	$4.29 + 3.81$ 8.1 $3.42 - 1.29$ 2.13 $0.725 - 0.198$ 0.527	225	93
4.	$0.4 + 7.25$ 7.65 $9.4 - 6.25$ 3.15 $5.75 - 0.125$ 5.625	226, 227	
5.	8×0.5 4.0 4×1.75 7 3×5.286 15.858	232–235	94
6.	0.125×50 6.25 5.75×4 23 63.9×75 4,792.5	238–240	96
7.	5.7×0.9 5.13 25.75×1.8 46.35 4.65×0.25 1.1625	242–247	97–101

How Well Can You Figure?

Computation Test 6

Copy, and do what the signs tell you to do.

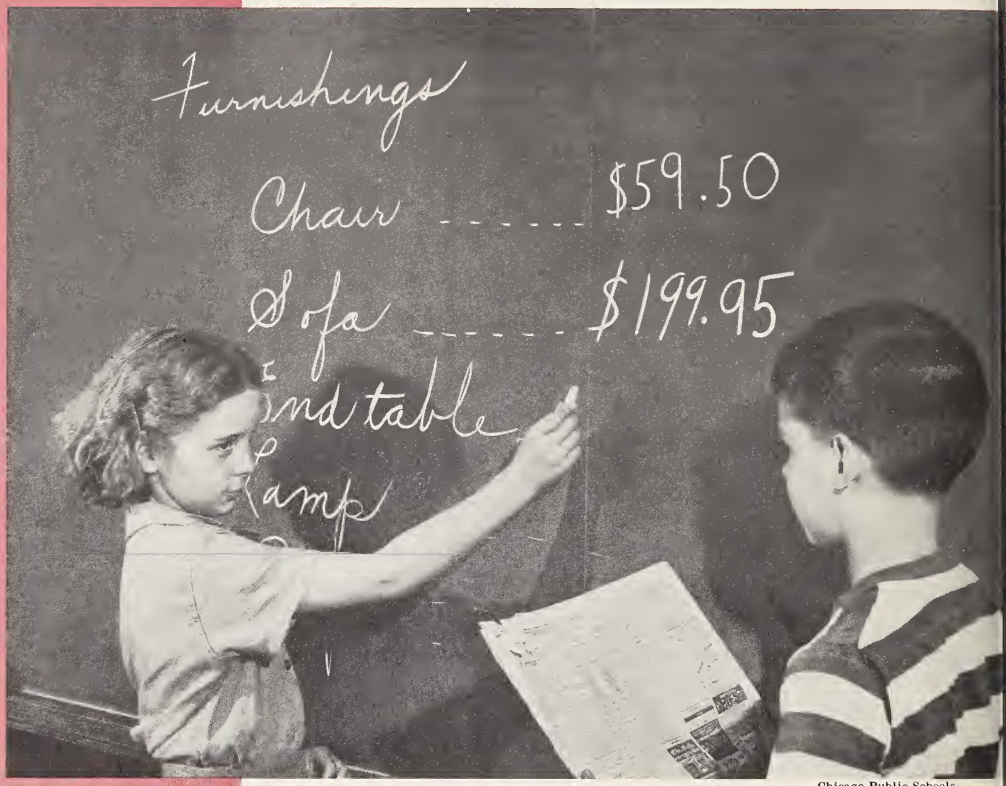
1. 21.7	2. 0.75	3. 0.9	4. 75	5. 8.005
$+5.2$	-0.29	$\times 4$	$\times 0.3$	-4.187
26.9	0.46	3.6	22.5	3.818
6. 95.5	7. 46.6	8. 0.28	9. 1.125	10. 72.25
$+8.75$	-5.875	$\times 5$	$\times 8$	$\times 1.5$
104.25	40.725	1.4	9.0	108.375
11. 9.6	12. 5.4	13. 69.8	14. 8.67	15. 24.7
$\times 3.8$	$\times 0.125$	$\times 4.29$	$\times 0.8$	$\times 0.003$
36.48	0.6750	299.442	6.936	0.0741
16. 10.06	17. 43.04	18. 0.03	19. 20.8	20. 4
-6.58	$+8.972$	$\times 0.7$	$+7.63$	-0.092
3.48	52.012	0.021	28.43	3.908

TABLE OF PER CENTS FOR CHAPTER 6 SCORES

Score	Per Cents for Problem Test 6
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100

Score	Per Cents for Computation Test 6	Score	Per Cents for Computation Test 6
1	5	11	55
2	10	12	60
3	15	13	65
4	20	14	70
5	25	15	75
6	30	16	80
7	35	17	85
8	40	18	90
9	45	19	95
10	50	20	100

NOTES



Chicago Public Schools

Newspapers, magazines, catalogues, and the like will often provide data useful in making up or solving arithmetic problems.

Teaching Chapter 7 of Grade 6

Introduction

The work in Chapter 7 is almost completely new. In the main, it covers division of decimals and short division. There are, of course, several lessons devoted specifically to problem-solving and to maintaining previously learned skills and abilities which cannot be considered new work.

I. Learning Outcomes in Chapter 7

1. Understanding of, and ability in dividing decimals
2. Ability to divide whole numbers and decimals a short way (short division)
3. Ability to solve problems
4. Understanding of various specific concepts, generalizations, relationships, and skills
5. Desirable emotionalized responses (attitudes, appreciations, values)

Understanding of, and Ability in Dividing Decimals (pages 255–259, 262–264, 266–269, 278)

Because decimal fractions are based on an extension of our whole-number notational system, they can be divided using the same principles as those used in dividing whole numbers. There are some new understandings and skills to be developed, but the learning load is not heavy. For the most part the new concepts are related to placing the decimal point in the quotient, to rewriting examples with decimal divisors so that they have whole-number divisors, and to rounding quotients to a specified place.

There is a summary of the different kinds of division examples with decimals on page 278 in the text. In the development for the division of fractions, each type of division example receives special attention. This is not necessary for the different types of examples in division of decimals, as can be seen in the outline of the learning sequence given below.

a. Whole number divided by a smaller whole number

b. Whole number divided by a larger whole number (changing a fraction to a decimal)

c. Decimal divided by a whole number (including rounding quotients to tenths, hundredths, or thousandths)

d. Whole number, decimal, or mixed decimal divided by a decimal (rewriting the example with a whole-number divisor)

e. Whole number, decimal, or mixed decimal divided by a mixed decimal

Pupils divide money numbers as preparatory work for dividing decimals. In the past, pupils have shown two places for cents in the quotient in dividing money numbers. Now they extend this idea by learning to show the same number of decimal places in the quotient as occur in the dividend. They also see how and why a whole number like \$78 may be written as \$78.00.

In dividing a whole number by a smaller whole number, pupils learn how zeros may be annexed so that the quotient will show a decimal rather than a common fraction. This approach has the advantage of demonstrating to pupils that annexing zeros and dividing decimals achieves the same result as writing remainders in a fraction, since both types of quotients are equivalent.

In dividing a whole number by a larger whole number, pupils must also annex zeros before dividing. When this kind of division has been mastered, pupils have a universal way of changing a fraction to a decimal. At this point they learn that a fraction may be used with a decimal (as $0.66\frac{2}{3}$) when the division does not come out even. They also learn to round to the nearest hundredth by first finding 3-place quotients.

Pupils next learn to divide a decimal by a whole number. In the previous work in dividing money numbers and in annexing zeros to whole numbers before dividing, they have really been dividing decimals, so this kind of example presents very few difficulties. Now, however, pupils learn a more general principle for rounding quotients—the dividend must be carried to one more place than will appear in the rounded quotient. The check for the division of decimals when there is a remainder is also given so pupils will see how similar it is to the procedure used for whole numbers.

The important idea in the next step, dividing by a decimal, is given in this generalization.

If the divisor is a decimal, you may multiply both divisor and dividend by 10, or 100, or 1,000, and so on, in order to make the divisor a whole number.

Pupils first see why this may be done, without changing the quotient, by referring back to the idea that both terms of the fraction $\frac{3}{4}$ may be multiplied by 2 in obtaining the equivalent fraction $\frac{6}{8}$. When the divisor is a decimal, pupils are to multiply by 10, 100, 1,000, and so on, by thinking of the decimal point as being moved an appropriate number of places to the right in both the divisor and dividend. No side work is therefore necessary, but most pupils will want to rewrite the example so that it has a whole-number divisor. Rewriting the example may be avoided by introducing the use of a caret ($\wedge$) to show the new position of the decimal points (as in $4.8\wedge\overline{)62.8\wedge32}$). Since there is a chance the caret will confuse pupils, it is not introduced in the text. However, if you think it worthwhile for your pupils, feel free to introduce its use.

The steps just described complete the sequence for dividing decimals. Once pupils have developed ability in dividing decimals, they will enjoy comparing the procedure with the division of fractions. They will discover that in most cases division involving decimals is much easier than division involving common fractions.

Ability to Divide Whole Numbers and Decimals a Short Way (pages 284–287)

In a sense the term *short division* is a misnomer. No steps are omitted in the thought process, but not as many are recorded as in the long-division form. This means that a great deal of information must be carried mentally in short division. This is why *slower learners* often find short division very difficult. For this reason, the section devoted to short division is placed near the end of the book so that you may leave it out or include it at your own discretion. Perhaps a good solution would be to present it to the *more capable children*, omitting it entirely for *slower learners*.

Short division at this level involves 1-place divisors only, but includes some work in dividing decimals. It is most useful in changing improper fractions to whole or mixed numbers and some work of this kind is therefore presented.

Ability to Solve Problems (pages 264, 271–274, 277, 281, 282)

According to the regular procedure, new skills and abilities are immediately applied to problem-solving situations as they are introduced. There are, therefore, many problems in this chapter involving the division of decimals.

Another important phase of the problem-solving program started in the previous chapter is continued in this chapter. The three types of problems that may occur in multiplication and division involving decimals are summarized in a chart much as was the case for the three kinds of problems involving whole numbers and fractions in Chapter 6. Pupils then differentiate the three types and go on to solve several problems of each type.

Pupils also receive practice in telling the example to use in division problems involving decimals, in telling what to do to solve problems without numbers, and in completing unfinished problems.

Understanding of Various Specific Concepts, Generalizations, Relationships, and Skills

1. *Concepts and essential technical terms and symbols:* divisible, short division

2. *Important relationships and generalizations.* The generalizations appearing in Chapter 7 will be found on the following pages in the text and on the corresponding pages in the Manual: pages 267, 271, 285.

3. *Skills beyond those already listed:*

a. Ability to write decimal equivalents for common fractions, using helpers if necessary (pages 260–261)

b. Ability to divide whole numbers by 10, 100, or 1,000 by thinking of the decimal point as being moved to the left (page 266)

c. Ability to recognize numbers divisible by 2, 3, 5, 6, and 9 (page 279)

d. Skill in finding n representing a missing factor in multiplication and division examples involving decimals (page 280)

Desirable Emotionalized Responses (Attitudes, Appreciations, Values)

You may remember that at the outset it was stated that emotionalized responses were not to be considered outcomes to be realized through the work of a few lessons, or even a whole chapter. In this chapter you will have your last opportunity this year to make sure your pupils have a healthy, objective viewpoint about arithmetic. To make sure that you make the most of it, reread some of the objectives listed on pages 13 and 14 in the Manual, and then make plans for continuing to meet these goals in whatever time remains.

Buying for the Baseball Team

Readiness for D. of decimals; dividing money numbers [W]

When the sixth-grade boys told Mr. King, the principal, that they would like to have a baseball team, he suggested that they find out what it would cost. The boys chose a committee to visit several stores on Saturday and report the costs to the class on Monday.

1. One store gave the committee a price of $59.22 for 9 uniforms. How much was this for a uniform? $6.58

2. At another store, the salesman thought the team would need to have extra uniforms and gave the committee a total price of $78 for 12 uniforms. How much was this for a uniform? (Use $78.00 for the dividend.) $6.50

3. Which of the two stores gave the committee the lower price per uniform? 2nd store

4. The boys wanted to buy 1 doz. baseballs and found that the cost of those would be $21.48. How much was this a ball? $1.79

Because the total cost seemed too great, the boys were about to give up planning when Mr. White, the father of one of the boys, said that he would provide $75 toward uniforms if the boys would earn money to buy the other equipment needed.

5. If the boys used the entire $75 to buy 9 uniforms, how much would they pay for each uniform? 8.33\frac{1}{3}$, or $8.34

6. Using just the $75, what could they pay for each uniform if they decided to buy 10? $7.50 12? $6.25

7. The committee said that one store had given them a price of $5.40 for 6 bats. How much is this for each bat? $0.90

8. If the team finally spent $21.48 for baseballs (Ex. 4), $5.40 for bats (Ex. 7), and $78 for uniforms (Ex. 2), what was the total cost? How much money did the boys have to raise? $104.88 $29.88

Teaching Pages 254 and 255

Pupil's Objective. To prepare for dividing decimals by solving problems involving division of money numbers.

Background. Pupils already know how to divide a money number by a whole number and so, in effect, they know how to divide a 2-place decimal by a whole number. The work on page 255, then, is review, but it is arranged in such a way that it will bring out some of the principles useful in learning how to divide decimals.

It should be noted that even though money numbers are decimals, it is best not to try to base

the program for learning how to divide decimals on the use of money numbers exclusively. The reason for this is that the meanings pupils attach to the various fractional parts of a dollar are not the meanings useful in working with decimals. For example, pupils are apt to think of a quarter as $\frac{1}{4}$ of a dollar rather than $\frac{25}{100}$ of a dollar. Moreover, even though children may think of a dime as one tenth of a dollar, it is written \$0.10 (ten hundredths of a dollar).

Teacher's Preparation

1. Put on the board statements such as the following:

a. Nylons are \$1.65 per pair, or \$4.50 for 3 pairs.
b. Potatoes are 50¢ a peck, or \$1.75 per bushel.
c. Handkerchiefs are 39¢ each, or 3 for \$1.00.

2. If possible, have on hand advertisements showing reduced prices for articles sold in quantity.

Pre-book Lesson. Read advertisements for articles being sold at reduced prices when purchased in larger quantities or read the statements placed on the board before the lesson. Have pupils figure the price per item when articles are bought in quantity so that this may be compared to the single-unit price.

Book Lesson. *Ex. 1–8:* Written work.

1. Have pupils study the illustration on page 254 to see if they can tell what the boys are doing.

2. Let pupils read through Ex. 1–8 on page 255 telling each time how the problem should be solved. Be sure to bring out the fact that a decimal point and two zeros may be annexed to the \$75 in Ex. 5 and 6 to make solving easier.

3. Assign Ex. 1–8 for individual written work. If it seems necessary, let *slower learners* work problems 1–3 on the board as a group project.

Differentiations and Extensions. *More capable pupils* might find the actual cost of each band uniform, gym suit, or baseball suit used by the school.

Teaching Pages 256 and 257

Pupil's Objectives. (*a*) To prepare for division involving decimals by learning how and why zeros may be annexed to money numbers in expressing fractional remainders as cents; (*b*) to learn how to divide a whole number by a smaller whole number so that the quotient appears as a mixed decimal rather than a mixed fraction.

Background. In dividing money numbers in earlier grades, pupils learned to place the cent point so as to show two places for cents. Now, on page 256, pupils will use what they have learned about decimals to analyze the placement of the quotient figures in dividing money numbers. The essential point for them to remember is that when they divide tenths (or hundredths, etc.) they must show a figure in the quotient in tenth's place (or hundredth's place, etc.). They also learn on page 256 (Ex. 3) that remainders in dividing money numbers do not need to be shown in a fraction, since zeros may be added and a decimal fraction obtained by continuing the division.

Pupils have many times divided a whole number by a smaller whole number and shown the remainder in a fraction as in boxes A and B on page 257. Now, as a continuation of the preparation received on page 256 in dividing money numbers, pupils will learn how to add a decimal point and a zero to the dividend so that the quotient may be written as a decimal. In so doing, of course, they will have worked into dividing tenths and hundredths in an easy, meaningful way.

Teacher's Preparation

1. Have on hand a flannel board and three circles of equal size, two of which have been divided into fifths and tenths respectively. Also prepare a ten-inch square of oak tag marked off in one-inch blocks as well as several other ten-inch squares cut into halves, fourths, and fifths.

2. Obtain play money amounting to \$78 in dollar bills and \$6 in fifty-cent pieces (see page 321 in this Manual for a possible source of supply).

Pre-book Lesson

1. By placing fractional parts on the flannel board, have different pupils show that $\frac{1}{5} = \frac{2}{10}$, $\frac{2}{5} = \frac{4}{10}$, $\frac{3}{5} = \frac{6}{10}$, $\frac{4}{5} = \frac{8}{10}$. When fractional parts such as $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, etc., are placed over the ten-inch square that has been divided into hundredths, the class can see easily that these fractions may be expressed as hundredths. Let pupils count squares to see just how many hundredths are equal to each fraction.

2. In a discussion bring out the fact that a half-dollar is also known as fifty cents. Then, using

When We Divide Money Numbers

Readiness [O]

1. On page 255, you were really dividing decimals. Box A shows the work for Ex. 1 on page 255.

In the last division (72 ÷ 9), were you dividing tenths, hundredths, or thousandths?

Does the last quotient figure, 8, show tenths, hundredths, or thousandths?

How does the decimal point in the dividend show where the decimal point in the quotient is to be placed? *Quotient point placed directly above dividend point.*

A

$$\begin{array}{r} \$6.58 \\ 9\overline{)\$59.22} \\ 54 \\ \hline 52 \\ 45 \\ \hline 72 \\ 72 \\ \hline \end{array}$$

2. In Ex. 2 on page 255, you had the whole number $78 to divide by the whole number 12. Why did we write $78 with two 0's, as in box B? *To change 78 to hundredths so as to have cents in quotient.*

What can you tell about the decimal points in the dividend and in the quotient? *They are in line, vertically.*

B

$$\begin{array}{r} \$6.50 \\ 12\overline{)\$78.00} \\ 72 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

$\frac{6}{12} = \frac{1}{2}$

C

$$\begin{array}{r} \$6\frac{1}{2} \\ 12\overline{)\$78} \\ 72 \\ \hline 6 \end{array}$$

D

$$\begin{array}{r} \$6.5 \\ 12\overline{)\$78.0} \\ 72 \\ \hline 60 \\ 60 \\ \hline \end{array}$$

E

$$\begin{array}{r} \$6.500 \\ 12\overline{)\$78.000} \\ 72 \\ \hline 60 \\ 60 \\ \hline 00 \end{array}$$

3. Study the work in box C.

a. Why does the quotient in box B mean the same as the quotient in box C? $\$\frac{1}{2} = 50¢ = \$0.50$

b. If you want to show cents instead of a fraction in a quotient, and the dividend is a whole number (as in box C), what can you do to the dividend? *Change dividend to hundredths by writing decimal point after ones place and adding 2 zeros at right of point.*

c. Why does $78.00 = $78? *Both mean 78 dollars and no cents.*

d. Could you have worked as in box D or in box E, instead of as in box B? *Yes* Why wouldn't you do so? *You would have to change quotients to show cents.*

Decimals in the Quotient

Whole number ÷ smaller whole number [O]

You know how to divide one whole number by another, as in boxes A and B.

A

$$\begin{array}{r} 8\frac{2}{5} \\ 5\overline{)42} \\ 40 \\ \hline 2 \end{array}$$

B

$$\begin{array}{r} 4\frac{1}{4} \\ 24\overline{)102} \\ 96 \\ \hline 6 \end{array}$$

$\frac{6}{24} = \frac{1}{4}$

1. Tell the remainder in box A; 2 in box B. 6

2. How do we show the division of the remainder in each box? *In a fraction (divide remainder by divisor).*

3. Now study the work in box C. Why can we say that the quotient, 8.4, equals the quotient, $8\frac{2}{5}$, in box A? $8.4 = 8\frac{4}{10} = 8\frac{2}{5}$

Why did we write only one 0 after the decimal point in the dividend? *Division comes out even to tenths.* Could we have written two 0's? *Yes*

C

$$\begin{array}{r} 8.4 \\ 5\overline{)42.0} \\ 40 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

The one 0 (box C) was brought down beside the remainder, 2 ones, making 20 tenths. 20 tenths divided by 5 is 4 tenths. The 4 was written in the quotient in *tenths* place.

4. Does the quotient in box D equal that in box B? *Yes* $(4.25 = \frac{425}{100} = 4\frac{1}{4})$

Why did we write two 0's after the decimal point in the dividend in box D? *So there would be no remainder.*

D

$$\begin{array}{r} 4.25 \\ 24\overline{)102.00} \\ 96 \\ \hline 60 \\ 48 \\ \hline 120 \\ 120 \\ \hline \end{array}$$

After you bring down the first 0, you divide 60 tenths by 24. Where in the quotient do you write 2? Why? *In tenths place* *tenths ÷ ones = tenths*

After you bring down the second 0, you divide 120 hundredths by 24. Where in the quotient do you write the quotient figure 5? Why? *In hundredths place* *hundredths ÷ ones = hundredths*

[W]

Divide as in boxes C and D.

5. $8\overline{)92}$ 11.5
6. $5\overline{)219}$ 43.8
7. $6\overline{)195}$ 32.5
8. $8\overline{)506}$ 63.25
9. $36\overline{)342}$ 9.5
10. $15\overline{)114}$ 7.6
11. $48\overline{)156}$ 3.25
12. $64\overline{)432}$ 6.75

play money, have pupils divide $78 one-dollar bills into twelve equal groups until each group has $6 with six bills left over. Let pupils decide how to allocate the $6 left over so that each of the twelve groups will receive an equal amount. They should see that a half of each dollar bill, or fifty cents, should be given to each group. Write the quotient on the board as $6\frac{1}{2}$ and as $6.50.

Book Lesson (page 256). *Ex. 1–3:* Oral work.

1. In discussing Ex. 2, refer pupils to the prebook activity involving the dividing of $78 into 12 equal groups. The quotients written on the board may be compared with those in boxes B and C.

2. Be sure pupils realize that the quotient in money numbers is not expressed as shown in boxes D and E, but rather as in box B. This can be made clear by having the answers in boxes D and E read aloud and by asking pupils if they have heard people say 0.5 or 0.500 of a dollar.

Book Lesson (page 257). *Ex. 1–4:* Oral work. *Ex. 5–12:* Written work.

1. Use each step on this page to develop the method of expressing a remainder as a decimal fraction. Be sure pupils realize that adding a decimal point and any number of zeros to the right of a whole number does not change the value of the number. Relate this to the two forms often used for writing one dollar: $1 and $1.00.

2. The division steps will be simple because they are the same as when dividing with dollars and cents. If the meaning of the decimal fraction is not completely clear, turn back to the flannel board and again show that a remainder of $\frac{2}{5}$ is equal to 0.4 and that a remainder of $\frac{1}{4}$ is equal to 0.25.

Differentiations and Extensions

1. *Slower learners* may discuss the following situation:

Tom looked up the precipitation for New York City in the month of January and found it to be 3.6 inches. Henry read a rainfall graph for the same month and found it to be $3\frac{3}{5}$ inches. Are the two readings the same? Explain.

2. *More capable pupils* may also write the quotients in Ex. 5–12 at the bottom of page 257 with the remainders in a fraction. These may be used to check the quotients written as decimals.

Teaching Pages 258 and 259

Pupil's Objectives. (*a*) To learn how to change a common fraction to a decimal fraction by dividing the numerator by the denominator; (*b*) to learn how to round decimal quotients, or write them with a fraction, when it is obvious that continued division would never give an even quotient.

Background. Dividing a whole number by a *larger* whole number is very similar to the work on page 257 in dividing a whole number by a *smaller* whole number. Of course, if pupils also realize that $\frac{3}{4}$ can be written as $3 \div 4$, they will discover that they have a way of changing a fraction to a decimal by simply adding zeros enough to make the division come out even and performing the division indicated. As explained at the top of page 259, the division may never come out even. In this case, it is necessary to add only as many zeros as is desirable and either round the quotient or show the remainder in a fraction. In the latter case, pupils should realize that a fraction does not hold a decimal place. That is, in box E, $0.66\frac{2}{3}$ does not mean $66\frac{2}{3}$ *thousandths*. Rather, the fraction $\frac{2}{3}$ refers to the fractional unit for the decimal part of the quotient and the whole quotient means $66\frac{2}{3}$ *hundredths*.

Notice that at the bottom of page 259, in rows 9 and 10, pupils obtain practice in rounding quotients to the nearest hundredth. In this case, they are told that they must carry the division to three places. Later on (page 263) they will learn that they must always carry the quotient one more place than the number of places to which they wish to round.

Pre-book Lesson. If you wish, present the common-denominator method of changing a fraction to a decimal. For example, write the fraction $\frac{1}{2}$ on the board, and then see if you can lead pupils to suggest that in order to write $\frac{1}{2}$ as a decimal it

Changing a Fraction to a Decimal

Whole number ÷ larger whole number [O]

1. Three of these 4 boys went skiing. What decimal part of the group was that?

First you can *think*, "$\frac{3}{4}$ of the boys went skiing." Then *think*, "I could write $\frac{3}{4}$ as $3 \div 4$, or $4\overline{)3.}$" See how the division is written in box A.

In box A, why have we written a decimal point and two 0's after the 3? *to make the quotient a decimal* Explain the division.

Explain the check. Why does 3.00 equal 3? *Both mean 3 ones and no hundredths.*

A	B
$\frac{3}{4}$ = what decimal?	0.625
0.75	$8\overline{)5.000}$
$4\overline{)3.00}$	48
28	20
20	16
20	40
	40
Check	Check
0.75	0.625
$\times 4$	$\times 8$
3.00	5.000

2. Explain how the fraction $\frac{5}{8}$ is changed to its equivalent decimal in box B. Explain the check.

3. On the board, change the following to their equivalent decimals:

a. $\frac{1}{4}$ *0.25* **b.** $\frac{1}{2}$ *0.5* **c.** $\frac{1}{8}$ *0.125* **d.** $\frac{7}{8}$ *0.875* **e.** $\frac{3}{8}$ *0.375*

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Sometimes, in changing a common fraction to a decimal, the division never ends. In box C, no matter how long we kept on writing 0's in the dividend and dividing, there would always be a remainder of 2.

C	D
0.666 3)2.000 18 20 18 20 18 2	0.333 3)1.000 9 10 9 10 9 1

4. What remainder in box D would always be the same no matter how many times you divide? /

5. Round to hundredths the quotient in box C; 0.67 in box D. 0.33

6. In Ex. 5, you rounded the three-place quotients in boxes C and D to the nearest hundredths. Boxes E and F show another way to express the quotients to hundredths. Explain. In each case, the fraction means part of a hundredth. [W]

E	F
$0.66\frac{2}{3}$ 3)2.00 18 20 18 2	$0.33\frac{1}{3}$ 3)1.00 9 10 9 1

Change the following to decimal fractions. After each dividend, write a decimal point and use as many 0's as are needed to make the division come out even.

	a	b	c	d	e	f	g	h
7.	$\frac{2}{5}$ 0.4	$\frac{3}{25}$ 0.12	$\frac{5}{16}$ 0.3125	$\frac{1}{5^2}$ 0.2	$\frac{3}{16}$ 0.1875	$\frac{11}{20}$ 0.55	$\frac{4}{5}$ 0.8	$\frac{7}{10}$ 0.7
8.	$\frac{3}{20}$ 0.15	$\frac{3}{5}$ 0.6	$\frac{7}{40}$ 0.175	$\frac{9}{100}$ 0.09	$\frac{5}{32}$ 0.15625	$\frac{9}{16}$ 0.5625	$\frac{8}{25}$ 0.32	$\frac{3}{10}$ 0.3

For each example in rows 9 and 10, find a three-place quotient and then round it to the nearest hundredth.

9.	$\frac{5}{6}$ 0.83	$\frac{7}{12}$ 0.58	$\frac{1}{9}$ 0.11	$\frac{5}{7}$ 0.71	$\frac{2}{15}$ 0.13	$\frac{5}{9}$ 0.56	$\frac{1}{7}$ 0.14	$\frac{1}{6}$ 0.17
10.	$\frac{3}{7}$ 0.43	$\frac{7}{9}$ 0.78	$\frac{4}{7}$ 0.57	$\frac{2}{9}$ 0.22	$\frac{8}{9}$ 0.89	$\frac{2}{7}$ 0.29	$\frac{4}{9}$ 0.44	$\frac{6}{7}$ 0.86

Now write each answer for rows 9 and 10 as a two-place decimal with the remainder in a fraction, as in boxes E and F. ★

Extra Practice. For more practice, work Sets 102–103.

must first be changed so that its denominator is 10 or some power of 10. They should suggest for $\frac{1}{2}$ that the denominator be multiplied by 5. Since this decreases the size of the fractional unit, the numerator must also be multiplied by the same number in order to make more of the smaller pieces. When this is shown as $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$ pupils will see that they are really multiplying $\frac{1}{2}$ by $\frac{5}{5}$, or by 1, and that this does not change the value of the fraction. Once the fraction $\frac{5}{10}$ has been obtained, it is a simple matter to write it as the decimal 0.5. Present other fractions such as $\frac{1}{5}$ (multiply by $\frac{2}{2}$), $\frac{3}{4}$ (multiply by $\frac{25}{25}$), $\frac{3}{25}$ (multiply by $\frac{4}{4}$), and $\frac{1}{3}$. For the latter case, help pupils realize that there is no whole-number multiplier that will give an exact power of 10.

Book Lesson. *Ex. 1–6:* Oral work. *Rows 7–10:* Written work.

1. Discuss each step in the first problem. Then ask "If 2 of the 4 boys went fishing, what fractional part of the group went fishing?" The answer will be quickly given, but have a pupil verify it by writing the solution on the board.

2. After Ex. 1 and 2, draw from the class the rule for changing a common fraction to its equivalent decimal fraction. Write the rule (*Divide the numerator by the denominator*) on the board.

3. Have examples **a**–**e** at the bottom of page 258 worked at the board one at a time. Have each pupil explain how he got his answer.

4. For Ex. 5, have pupils put the two examples on the board showing the quotients as 0.666 and 0.67, and as 0.333 and 0.33. Explain that the two decimals are not exactly equal.

5. Have a pupil read and explain Ex. 6. Ask the class which answers are exact, those shown rounded to hundredths on the board, or those in boxes E and F. Be sure pupils understand that the $\frac{2}{3}$ in $0.66\frac{2}{3}$ means $\frac{2}{3}$ of a hundredth, and that the $\frac{1}{3}$ in $0.33\frac{1}{3}$ means $\frac{1}{3}$ of a hundredth. If these two decimal fractions are read aloud, it may help to make clear the fact that the fractions are not in thousandth's place and that they show what part of a hundredth the remainder equals.

6. Before assigning rows 7–10 for written practice, review the rule which the class formulated for changing a common fraction to a decimal fraction. *Slower learners* will probably need extra help with a few of the examples in rows 9 and 10 before attempting to do the written work individually.

Differentiations and Extensions. Assign **Extra Practice Sets 102 and 103** as needed. (See the first column on the next Manual page for these sets.)

★Answers Not on Reproduced Page 259

9. a. $0.83\frac{1}{3}$	**b.** $0.58\frac{1}{3}$	**c.** $0.11\frac{1}{9}$	**d.** $0.71\frac{3}{7}$
e. $0.13\frac{1}{3}$	**f.** $0.55\frac{5}{9}$	**g.** $0.14\frac{2}{7}$	**h.** $0.16\frac{2}{3}$
10. a. $0.42\frac{9}{7}$	**b.** $0.77\frac{7}{9}$	**c.** $0.57\frac{1}{7}$	**d.** $0.22\frac{2}{9}$
e. $0.88\frac{8}{9}$	**f.** $0.28\frac{4}{7}$	**g.** $0.44\frac{4}{9}$	**h.** $0.85\frac{5}{7}$

Set 102. Dividing a whole number by a larger whole number

Round to hundredths unless the quotient comes out even.

1. a. 0.75 $8\overline{)6}$ b. 0.58 $12\overline{)7}$ c. 0.4 $5\overline{)2}$ d. 0.59 $27\overline{)16}$
   e. 0.89 $9\overline{)8}$ f. 0.25 $48\overline{)12}$ g. 0.44 $16\overline{)7}$

2. a. 0.29 $35\overline{)10}$ b. 0.25 $84\overline{)21}$ c. 0.75 $4\overline{)3}$ d. 0.68 $25\overline{)17}$
   e. 0.14 $7\overline{)1}$ f. 0.20 $41\overline{)8}$ g. 0.55 $60\overline{)33}$

3. a. 0.11 $9\overline{)1}$ b. 0.77 $31\overline{)24}$ c. 0.86 $7\overline{)6}$ d. 0.6 $75\overline{)45}$
   e. 0.76 $55\overline{)42}$ f. 0.78 $32\overline{)25}$ g. 0.8 $15\overline{)12}$

Set 103. Changing common fractions to decimal fractions

Change to 2-place decimals. Give any remainder in a fraction.

1. a. $\frac{3}{4}$ 0.75	b. $\frac{5}{16}$ $0.31\frac{1}{4}$	c. $\frac{2}{7}$ $0.28\frac{4}{7}$
d. $\frac{16}{24}$ $0.66\frac{2}{3}$	e. $\frac{7}{12}$ $0.58\frac{1}{3}$	f. $\frac{1}{6}$ $0.16\frac{2}{3}$
g. $\frac{5}{8}$ $0.62\frac{1}{2}$	h. $\frac{9}{25}$ 0.36	i. $\frac{5}{12}$ $0.41\frac{2}{3}$
2. a. $\frac{1}{9}$ $0.11\frac{1}{9}$	b. $\frac{3}{8}$ $0.37\frac{1}{2}$	c. $\frac{15}{16}$ $0.93\frac{3}{4}$
d. $\frac{5}{7}$ $0.71\frac{3}{7}$	e. $\frac{7}{24}$ $0.29\frac{1}{6}$	f. $\frac{7}{16}$ $0.43\frac{3}{4}$
g. $\frac{9}{21}$ $0.42\frac{6}{7}$	h. $\frac{11}{12}$ $0.91\frac{2}{3}$	i. $\frac{9}{16}$ $0.56\frac{1}{4}$
3. a. $\frac{7}{8}$ $0.87\frac{1}{2}$	b. $\frac{6}{25}$ 0.24	c. $\frac{1}{3}$ $0.33\frac{1}{3}$
d. $\frac{5}{6}$ $0.83\frac{1}{3}$	e. $\frac{8}{15}$ $0.53\frac{1}{3}$	f. $\frac{1}{7}$ $0.14\frac{2}{7}$
g. $\frac{2}{3}$ $0.66\frac{2}{3}$	h. $\frac{7}{22}$ $0.31\frac{9}{11}$	i. $\frac{13}{20}$ 0.65

NOTES

Teaching Pages 260 and 261

Pupil's Objectives. (*a*) To use pictures in explaining why certain common and decimal fractions are equivalent; (*b*) to discover ways to remember the decimal equivalents for some of the more frequently used common fractions.

Background. Now that pupils have learned to change a fraction to a decimal by dividing numerator by denominator, they are presented with pictures which will help them see that decimal equivalents for fractions have fractional units with denominators that are 10 or a power of 10. Pupils will see this more clearly if they are asked to tell the fractional unit for the decimal fraction in each of pictures A to G on page 260, and then to go back and tell the fractional unit for each common fraction.

Most adults find it very useful to know the decimal equivalents for some of the more frequently-used common fractions. These equivalents can be memorized, or a small decimal-equivalent chart may be kept on hand for reference; but it is much more effective for pupils to learn the simplest decimal equivalents and then use these in finding equivalents for other fractions. Various techniques for accomplishing this are presented on page 261 in the text.

Teacher's Preparation. Have on hand enough pieces of $\frac{1}{4}$-inch graph paper cut into 100-block squares so that each pupil may be provided with three squares.

Pre-book Lesson. Pass three 100-block squares of graph paper to each pupil to be used with the following questions and activities:

How many blocks are there in each square? (100)

Therefore, each block will equal what part of the whole square? ($\frac{1}{100}$)

What other way can you write $\frac{1}{100}$? (0.01)

Now fold your square of paper exactly in half. How many blocks are there in each half? (50)

What decimal part of the whole is each half? ($\frac{1}{2}$ of $\frac{100}{100}$, or $\frac{50}{100}$, or 0.50)

Take another 100-block square. Color 0.20 of this block. What fractional part has been colored? ($\frac{1}{5}$) Prove it by folding.

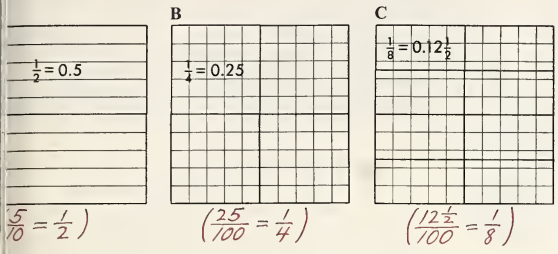
Take your last square and color 0.75 of it. What fractional part has been colored? ($\frac{3}{4}$) Show how you would fold your square to prove this.

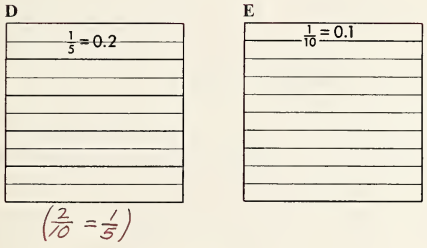
Book Lesson (page 260). *Boxes A–G:* Oral work.

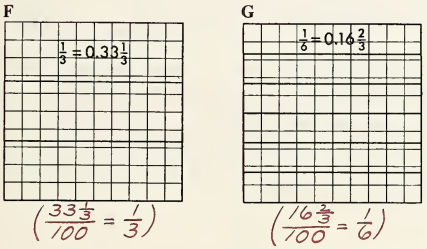
Equivalent Fractions and Decimals

[O]

Explain how each box shows that the common fraction equals the decimal fraction.







[O]

1. Charles couldn't remember the decimal equivalent of $\frac{1}{8}$, but he knew that $\frac{1}{4} = 0.25$. He *thought*, "$\frac{1}{8} = \frac{1}{2}$ of $\frac{1}{4}$. Then the decimal equivalent of $\frac{1}{8}$ must be $\frac{1}{2}$ of 0.25, or 0.125."

2. After Charles had found the decimal equivalent of $\frac{1}{8}$, how could he quickly find the equivalent of $\frac{3}{8}$? of $\frac{5}{8}$? of $\frac{7}{8}$? Multiply 0.125 by 3, by 5, by 7 respectively.

3. You know that the decimal equivalent of $\frac{1}{10}$ is 0.1 or 0.10. Tell how you would use this information to find a decimal equivalent for $\frac{7}{10}$; for $\frac{1}{5}$; for $\frac{4}{5}$. $\frac{4}{5} = \frac{8}{10} = 0.8$ $\frac{2}{10} = \frac{1}{5} = 0.2$ $7 \times 0.1 = 0.7$

4. Explain how you can use the fact that $\frac{1}{4} = 0.25$ to find the decimal equivalent of $\frac{1}{12}$; of $\frac{1}{16}$. $\frac{1}{16} \times 0.25 = 0.06\frac{1}{4}$ $\frac{1}{3} \times 0.25 = 0.08\frac{1}{3}$

5. How can you use the decimal equivalent of $\frac{1}{2}$ to find an equivalent for $\frac{1}{6}$? $\frac{1}{6} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \times 0.5 = 0.16\frac{2}{3}$

6. Explain how you can quickly find the decimal equivalent of $\frac{1}{20}$ when you know that $\frac{1}{5} = 0.2$ or 0.20. Then give the decimal equivalent of $\frac{7}{20}$; of $\frac{11}{20}$; of $\frac{19}{20}$. $\frac{1}{20} = \frac{1}{4} \times \frac{1}{5} = \frac{1}{4} \times 0.2 = 0.05$ 0.35 0.55 0.95

[W]

Copy and complete the following table. Use the boxes on page 260 if you need to. See if you know all the equivalents.

Halves Fourths Eighths	Halves Thirds Sixths	Halves Fifths Tenths
$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$
$\frac{1}{4} = 0.25$	$\frac{1}{3} = 0.33\frac{1}{3}$	$\frac{1}{5} = 0.2$
$\frac{3}{4} = 0.75$	$\frac{2}{3} = 0.66\frac{2}{3}$	$\frac{2}{5} = 0.4$
$\frac{1}{8} = 0.125$	$\frac{1}{6} = 0.16\frac{2}{3}$	$\frac{3}{5} = 0.6$
$\frac{3}{8} = 0.375$	$\frac{5}{6} = 0.83\frac{1}{3}$	$\frac{4}{5} = 0.8$
$\frac{5}{8} = 0.625$		$\frac{1}{10} = 0.1$
$\frac{7}{8} = 0.875$		$\frac{3}{10} = 0.3$
		$\frac{7}{10} = 0.7$
		$\frac{9}{10} = 0.9$

Book Lesson (page 261). *Ex. 1–6:* Oral work. *Bottom of the page:* Written work. Exercises 1–6 should be done orally, but wherever possible have work, such as the following for Ex. 2, placed on the board:

$\frac{1}{8} = 0.12\frac{1}{2}$ $\frac{3}{8} = 3 \times 0.12\frac{1}{2}$, or $0.37\frac{1}{2}$

Differentiations and Extensions

1. *More capable pupils* may use graph paper (in 100-block squares) and colored crayons to show $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{16}$. *Slower learners* may do the same for $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{4}{5}$. A few of the best of these may be mounted on oak tag or poster paper for display in the classroom.

2. *All pupils* should keep a completed copy of the chart at the bottom of page 261 in their arithmetic notebooks. *More capable pupils* may extend their charts to include twelfths and sixteenths. Commercial charts (see page 321 for a possible source of supply) showing decimal equivalents for common fractions may be displayed and discussed.

3. Divide the class into rows of approximately equal ability for a relay race. Prepare a sheet for each row with a list of decimal or common fractions. Taking the entries in order, each pupil should fill in one equivalent and then pass the paper to the child behind him. The first row to complete the list correctly is the winner.

Teaching Pages 262 and 263

Pupil's Objectives. To learn (*a*) to divide a decimal by a whole number; (*b*) that rounding decimal quotients requires carrying the work one place beyond that to which the quotient is to be rounded.

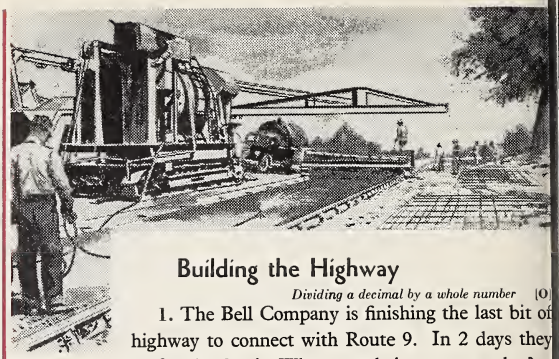
Background. In dividing a whole number by a smaller whole number on page 257 in the text, pupils added a decimal point and one or two zeros and divided as with whole numbers. Actually, this meant that pupils were dividing tenths and hundredths. Now, however, they will start with a decimal to be divided by a whole number. As long as the division comes out even to the number of places given by the original dividend, zeros do not need to be added, but if, as in Ex. 10 on page 263, the division does not come out evenly, zeros should be added so that the quotient may be rounded to a specified place. In Ex. 10**a**, pupils are led to realize that enough zeros must be added to give one more decimal place in the dividend than the number of places to which the quotient will be rounded. To avoid any confusion, use the example in box D on page 263 to show that once four decimal places have been obtained in the quotient, the remainder 12 may be discarded.

The reason decimals can be divided like whole numbers is that the decimal notational system is an extension of the whole-number notational system. This means that tenths may be divided like ones. You may enlarge upon this point by drawing 8 circles on the board and dividing them into two equal groups, getting the answer *4 circles*. Then show a square divided into tenths. Color 8 of the tenths and divide these into two equal groups, obtaining the quotient *4 tenths*.

Teacher's Preparation. Have on hand a supply of 10-inch squares of heavy paper.

Pre-book Lesson. Pass to the class the 10-inch squares of paper and ask pupils to pretend that they are cakes, pans of fudge, or squares of ice cream which are to be cut into 10 equal pieces. Ask them to mark off the ten pieces and write on each piece the part of the whole that it represents. Have them cut off two of the tenths and then attempt to solve the following problem:

If Mother says to share the rest of the cake equally with the family next door, what part of the whole cake would you take to them?



Building the Highway

Dividing a decimal by a whole number [O]

1. The Bell Company is finishing the last bit of highway to connect with Route 9. In 2 days they surfaced 0.8 mi. What was their average a day?

To find the average, why must we divide 0.8 mi. into 2 equal distances? *0.8 is total for 2 days.*

8 tenths ÷ 2 = *4* tenths, or 0.4. Explain the multiplying and subtracting in box A.

A
0.4
2)0.8
8

We divide tenths like ones. In box A, you divided tenths, so the quotient figure, 4, is written in *tenths* place.

Where do we put the decimal point in the quotient? *above point in dividend* Why do we write 0 before it? *To show that there are no ones in the quotient.*

2. When the Bell Company put down the road bed, it finished 4.2 mi. in 7 days. What was the average distance finished in a day? $\frac{1}{7}$ of 4.2 = 4.2 ÷ 7 (box B).

Think of 4.2 as 42 tenths. Explain the work in box B.

B
0.6
7)4.2
42

If you divide tenths by a whole number, why will the quotient show tenths? *$\frac{1}{7}$ of 42 (tenths) = 6 (tenths)*

How do you check the quotient? *Quotient times divisor equals dividend.* [W]

Write the work for these, as in boxes A and B, to find how many tenths in each quotient:

3. 0.9 ÷ 3 = *0.3*
4. 3.2 ÷ 8 = *0.4*
5. 2.4 ÷ 6 = *0.4*
6. 0.6 ÷ 2 = *0.3*
7. 0.7 ÷ 7 = *0.1*
8. 12.2 ÷ 2 = *6.*

Have the remaining 8 tenths cut into two equal sections. Then let a pupil show on the board the written work for solving the problem with common fractions, as below. The number of tenths in each half may be counted to verify the written work.

$$\frac{8}{10} \div 2 = \frac{8}{10} \times \frac{1}{2} = \frac{4}{10}, \text{ or } \frac{2}{5}$$

Now have pupils label each square of the paper with the decimal fraction 0.1. Then let them repeat the procedure described above, this time using decimal fractions so that they may derive the written record shown in box A in the text.

[O]

9. The Bell crew surveyed 1.26 mi. in 3 days. That was an average of _?_ mi. a day (box C).

Tenths: $12 \div 3 = 4$

Hundredths: $6 \div 3 = 2$

In boxes A to C, what do you notice about the number of decimal places in the dividend and in the quotient? *In each example, the same.*

10. It took 14 days to make a piece of road 0.06 mi. long through a swamp. Find the average distance made a day to the nearest thousandth of a mile (box D).

a. To find an answer to the nearest thousandth, how many decimal places must we first get in the quotient? *More; that is 4*

Why do we change 0.06 to 0.0600? *To get 4 decimal places in quotient*

b. Explain all the dividing in box D. To the nearest thousandth, is the answer 0.004 mi. or 0.005 mi.?

c. Study and explain the check.

C

$$3\overline{)1.26} \quad 0.42,\ 12,\ 6,\ 6$$

D

$$14\overline{)0.0600} \quad 0.0042,\ 56,\ 40,\ 28,\ 12$$

Check: $0.0042 \times 14 = 168$; 42; $0.0588 + 12 = 0.0600$

(Ex. 11e, 12e, and 13e) [W]

Divide and check. Round uneven quotients to hundredths.

	a	b	c	d	e
11.	6)28.8 (4.8)	9)47.7 (5.3)	21)7.287 (0.347)	52)87.36 (1.68)	33)0.712 (0.02)
12.	8)95.2 (11.9)	7)5.25 (0.75)	81)0.972 (0.012)	43)103.2 (2.4)	63)1.328 (0.02)
13.	5)0.005 (0.001)	6)4.02 (0.67)	42)1.554 (0.037)	62)192.82 (3.11)	93)2.519 (0.03)

Divide, rounding each answer to the nearest thousandth.

14.	82)1.984 (0.024)	53)7.82 (0.148)	16)28.41 (1.776)	28)0.9268 (0.033)	18)0.0628 (0.003)
15.	38)261.4 (6.879)	46)0.1429 (0.003)	72)1.1645 (0.016)	21)2.408 (0.115)	54)113.076 (2.094)

Extra Practice. For more practice, work Sets 104–105.

Book Lesson. *Ex. 1 and 2:* Oral work. *Ex. 3–8:* Written work. *Ex. 9 and 10:* Oral work. *Rows 11–15:* Written work.

1. After the pupils have read the problem in Ex. 1, point out that it is the same type of problem as they had in dividing 0.8 of a cake into 2 equal pieces.

2. In the oral work on page 262, use the idea that *we divide tenths like ones* to help pupils realize that all the procedures used in dividing whole numbers are used in dividing decimals by a whole number.

3. After Ex. 9, in which pupils review dividing hundredths (see page 256 in the text), write on the board the generalization that the quotient will have the same number of decimal places as the dividend. This idea will be called upon in Ex. 10 for an example having more than two decimal places.

4. Before Ex. 10, have pupils turn to page 248 in the text and use Ex. 1–3 at the bottom of the page in telling how many decimal places are needed in a number that is to be rounded to tenths, to hundredths, and to thousandths.

Differentiations and Extensions

1. *Slower learners* may need much individual help of the type suggested in the pre-book lesson.

2. See if you can explain to *more capable pupils* how it is possible to round the quotient to thousandth's place in box D without making the last division. They should see that after the 4 in the quotient has been obtained, the remainder of 4 (or $\frac{4}{14}$) indicates that the next division would give a quotient less than 5, which is all the information needed to round 0.004 to the nearest thousandth.

3. Assign **Extra Practice Sets 104 and 105** as needed.

Set 104. Dividing a decimal by a whole number; no remainder

	a	b	c	d	e
1.	4)8.4 (2.1)	5)6.25 (1.25)	3)10.125 (3.375)	8)8.64 (1.08)	96)10.272 (0.107)
2.	6)0.96 (0.16)	23)0.46 (0.02)	7)13.86 (1.98)	48)1.104 (0.023)	9)95.985 (10.665)
3.	12)3.6 (0.3)	15)3.45 (0.23)	2)0.8 (0.4)	5)72.875 (14.575)	25)3.575 (0.143)
4.	9)9.117 (1.013)	6)0.876 (0.146)	66)72.6 (1.1)	35)42.35 (1.21)	16)0.032 (0.002)

Set 105. Rounding quotients in dividing a decimal by a whole number

Give each quotient to the nearest thousandth.

1.	6)6.728 (1.121)	5)9.1625 (1.833)	16)12.525 (0.783)	25)100.37 (4.015)	12)1.435 (0.120)
2.	7)123.125 (17.589)	15)816.50 (54.433)	8)25.7275 (3.216)	32)18.72 (0.585)	4)56.109 (14.027)
3.	3)2.755 (0.918)	24)6.982 (0.291)	20)22.42 (1.121)	9)0.3702 (0.041)	36)1.828 (0.051)

Dividing Decimals

Divisor a whole number [W]

Write your work. If an answer does not come out even in thousandth's place, round it to the nearest hundredth.

1. Bob's father paid $2.88 for three water pails to use at camp. What was the price per pail? $0.96

2. A loaf of bread was marked "Net weight, 16.5 ounces." What was the weight of half a loaf? 8.25 oz.

3. On the speedometer of his father's car Billy measured the distance from his home to the postoffice. The round-trip distance going and coming the same way was 23.7 miles. How far was it from Billy's home to the postoffice? 11.85 mi.

4. Mr. Douglass divided his land along the edge of a lake into four house lots. The surveyor found the distance along the waterfront to be 526.35 feet. If Mr. Douglass made all the lots the same width, how much waterfront did each lot have? 131.59 ft.

5. Claire paid $2.94 for 6 yards of canvas to cover the porch chairs. What was the price per yard? $0.49

6. Spanish coins once were commonly used in some parts of the United States. The Spanish *real* (re-al) was one eighth of a dollar. When merchants were short of coins they sometimes cut a Spanish silver dollar into 8 parts, called "bits," and used the parts as *reals*. How much would one "bit" be worth in cents? $0.12\frac{1}{2}$, or 12$\frac{1}{2}$¢ (Do not round off.) How much is "two bits"? $0.25

7. In shop class, Joe had to divide a metal strip into four equal parts. He found that the strip was 8.376 inches long. How long should each part be? 2.094 in.

8. The three cottages which Ann's father rents in the summer are all connected on the same meter for electricity. Last year the season's bill for the cottages was $34.88. This was how much, on the average, for a cottage? $11.63

Have You Learned Well?

[W]

Copy and do what the signs say.

	a	b	c	d	e
1.	35.6	6.14	46.89	$91.76	$461.27
	91.7	3.76	7.46	0.89	24.42
	+1.3	+5.28	+39.72	+3.16	+38.97
	128.6	15.18	94.07	$95.81	$524.66
2.	6.807	2.096	4.024	$200.30	$741.50
	−5.962	−0.128	−0.176	−16.98	−264.87
	0.845	1.968	3.848	$183.32	$476.63
3.	2.09	741	5.309	$35.62	$62.18
	×0.76	×0.36	×1.04	×100	×4.91
	1.5884	266.76	5.52136	$3,562	$305.30

4. $1\frac{3}{4} + 2\frac{5}{8}$ $4\frac{3}{8}$ $7\frac{1}{3} + \frac{5}{6}$ $8\frac{1}{6}$ $3\frac{4}{5} + 1\frac{3}{10}$ $5\frac{1}{10}$ $5\frac{2}{3} + \frac{3}{4}$ $6\frac{5}{12}$ $\frac{4}{5} + 2\frac{1}{2}$ $3\frac{3}{10}$

5. $3\frac{1}{4} - 2\frac{1}{3}$ $\frac{11}{12}$ $6 - \frac{7}{10}$ $5\frac{3}{10}$ $4\frac{1}{5} - 1\frac{7}{10}$ $2\frac{1}{2}$ $2\frac{3}{16} - \frac{5}{8}$ $\frac{9}{16}$ $1\frac{5}{12} - \frac{2}{3}$ $\frac{3}{4}$

6. $1\frac{1}{2} \times 6$ 9 $2\frac{2}{3} \times \frac{5}{8}$ $\frac{2}{3}$ $\frac{5}{24} \times 1\frac{1}{2}$ $\frac{5}{16}$ $2\frac{2}{15} \times \frac{5}{8}$ $\frac{1}{3}$ $3\frac{25}{100} \times 2\frac{5}{10}$

7. $20 \div 6\frac{2}{3}$ 3 $\frac{5}{6} \div \frac{2}{3}$ $1\frac{1}{4}$ $\frac{7}{12} \div \frac{7}{8}$ $\frac{2}{3}$ $2\frac{4}{10} \div 6\frac{2}{5}$ $2\frac{3}{15} \div \frac{3}{5}$ $3\frac{2}{3}$

Find quotients to the nearest hundredth.

8. 7)61.8 → 8.83 | 21)96.26 → 4.58 | 13)42.18 → 3.24 | 7)$3.42 → $0.49 | 10)$5.68 → $0.57

Changing to Fractions or to Decimals

[W]

Give the equivalent common fraction in lowest terms.

	a	b	c	d	e	f
1.	0.16 $\frac{4}{25}$	0.64 $\frac{16}{25}$	0.35 $\frac{7}{20}$	0.375 $\frac{3}{8}$	0.075 $\frac{3}{40}$	0.80 $\frac{4}{5}$
2.	0.84 $\frac{21}{25}$	0.008 $\frac{1}{125}$	0.480 $\frac{12}{25}$	0.005 $\frac{1}{200}$	0.12 $\frac{3}{25}$	0.3125

Change to decimal form. Round any uneven quotient to the nearest hundredth. (those checked)

3. $2\frac{1}{4}$ 2.25 $9\frac{2}{5}$ 9.4 $\frac{3}{8}$ 0.375 $\frac{7}{8}$ 0.875 $3\frac{9}{20}$ 3.45 $\frac{3}{5}$ 0.6

4. $1\frac{1}{10}$ 1.1 $\frac{11}{20}$ 0.55 $\frac{9}{40}$ 0.225 $3\frac{7}{50}$ 3.14 $2\frac{3}{16}$ 2.19 $\frac{1}{8}$ 0.125

5. ✓$\frac{7}{12}$ 0.58 ✓$\frac{8}{9}$ 0.89 $1\frac{1}{4}$ 1.57 ✓$6\frac{1}{3}$ 6.33 ✓$3\frac{5}{11}$ 3.45 ✓$2\frac{1}{6}$ 2.17

6. ✓$\frac{4}{15}$ 0.27 $1\frac{9}{32}$ 1.29 $\frac{5}{18}$ 0.28 ✓$\frac{4}{21}$ 0.19 ✓$3\frac{11}{12}$ 3.92 ✓$\frac{3}{7}$ 0.43

Teaching Pages 264 and 265

Pupil's Objectives. (*a*) To solve problems involving division when the dividend is a decimal or dollars and cents and the divisor is a whole number; (*b*) to discover what progress has been made in learning to use fractions and decimals in the four basic processes.

Pre-book Lesson. Review at the board the steps involved in solving a division example such as 8)3.56. Using the same example, review rounding the quotient to the nearest hundredth. Be sure to stress the placement of the decimal point. The following examples may be used for further practice if it seems necessary: 4)2.66 22)25.3 46)103.5

Book Lesson (page 264). *Ex. 1–8:* Written work. Have each problem read aloud. Then have pupils tell what example to use in solving each.

Book Lesson (top of page 265). *Rows 1–8:* Written work.

Book Lesson (bottom of page 265). *Rows 1–6:* Written work.

Differentiations and Extensions

1. After making an analysis of the errors made in working page 265, divide the class into small groups and assign whatever work is needed.

2. *More capable pupils* may write and solve original problems similar to those on page 264.

Dividing by 10 or 100 or 1,000

Meaning [O]

1. To find quickly 10×3.82, look at the factor 3.82 and think the decimal point one place to the right, as in 38.2.

2. From Ex. 1, if the product, 38.2, is divided by the factor 10, we know that the quotient will be 3.82. Why? product ÷ one factor = other factor

In other words, to find $38.2 \div 10$, we can look at 38.2 and get the quotient, 3.82, by thinking of the decimal point as moved one place to the left.

3. In the box, if you think of the point as moved left,

a. 1 place (38.2), then 382 has been divided by 10;

b. 2 places (3.82), then 382 has been divided by 100;

c. 3 places (0.382), then 382 has been divided by 1000.

a. $382 \div 10$	=	38	2
b. $382 \div 100$	=	3	82
c. $382 \div 1{,}000$	=	0	382

4. Divide by 10: a. 6.2; 0.62 b. 0.98; 0.098 c. 17; 1.7 d. 3.108; 0.3108

5. Divide by 100: a. 364; 3.64 b. 2.86; 0.0286 c. 0.07; 0.0007 d. 102.3; 1.023

6. Divide the numbers in row 5 by 1,000. 0.364 0.00286 0.00007 0.1023

Extra Practice. For more practice, work Set 106.

Dividing by a Money Number

[O]

1. At \$0.27 apiece, how many melons can I get for \$2.00? 7 \$0.27's in \$2.00 = 7, R11

Why should 200 cents ÷ 27 cents give the same answer as \$2.00 ÷ \$0.27? \$2.00 = 200¢ \$0.27 = 27¢

Try to explain why it is easier to find $200 \div 27$ than $2.00 \div 0.27$ not easy to divide by a decimal

By what can we multiply both 0.27 and 2.00 to get the example 27)200? Work this example at the board. 100

0.27)2.00
equals
27)200

2. Work at the board: a. \$0.20)\$8.40; 42 b. \$0.64)\$1.92 3

Teaching Page 266

Pupil's Objectives. (*a*) To learn how to divide by 10, 100, or 1,000 by moving the decimal point to the left; (*b*) to divide by a money number as preparation for learning how to divide a whole number or a decimal by a decimal.

Background. In the work at the top of page 266, pupils review the idea that a number may be multiplied by 10 by thinking of the decimal point as moved one place to the *right*. On the basis of this understanding, pupils are then led to see that they can divide a number by thinking of the decimal point as moved to the *left*. As the decimal point is moved one place to the left the effect is to move all the figures one place to the right, or into place-value columns one tenth their previous value. This is the reason that numbers may be divided by ten by moving the decimal point to the left.

At the bottom of page 266, pupils again use money numbers in preparing for further work in dividing decimals. In this case, they learn that dividing 200 cents by 27 cents will give the same answer as dividing \$2.00 by \$0.27. The latter would be quite difficult unless common fractions were used. The point that pupils are being prepared for is that it is better to use a whole-number divisor and that this can be done by multiplying both the divisor and the dividend by some number that will make the divisor a whole number.

Teacher's Preparation. Have 2 one-dollar bills and 200 pennies in real or play money on hand.

Pre-book Lesson

1. Put the examples in box A below on the board.

A		
248	248	248
× 10	× 100	× 1,000

B		
2.48	2.48	2.48
× 10	× 100	× 1,000

Have pupils multiply the long way and then draw from them the rule for the quick way of multiplying (think of the decimal point as moved to the right as many places as there are 0's in the multiplier). Do the same for the examples in box B.

2. Ask the class what the relationship is between multiplication and division and try to lead them to suggest that they could divide a number by 10, 100, or 1,000 by thinking of the decimal point as moved to the *left* as many places as there are in the divisor.

Book Lesson (top of the page). *Ex. 1–6:* Oral work. Pupils might verify the work in the box at the top of the page by doing each example the long way.

Book Lesson (bottom of the page). *Ex. 1 and 2:* Oral work. For this lesson, it might be helpful to use 2 one-dollar bills and 200 pennies to illustrate the need for having the divisor and the dividend in like-form before trying to divide. Pupils can quickly show that, even though it is im-

possible to separate the bills into groups of 27¢ each, it is easy to divide if 200 pennies are substituted for the dollar bills.

Differentiations and Extensions

1. Present the following problem for *all pupils* to solve:

How do you find the weight of 10 oranges if on the average one orange weighs 2.6 ounces? How do you find the average weight of 100 oranges? of 1,000 oranges?

Emphasize moving the decimal point to the right and the fact that 10 oranges must weigh more than one, a hundred more than 10, etc.

Have pupils check the short method by multiplying the long way. They may also verify the answer by reasoning that the product must be larger than the multiplicand and that as the multiplier increases in size (the multiplicand remaining the same) the products must increase in size.

2. Discuss the relationship of multiplication to division. Then draw from the class the fact that the short method of dividing means moving the decimal point to the left. See if the pupils realize that this decreases the size of the number.

3. Ask pupils to solve the following problems:

a. If 10 oranges weigh 28 ounces, how much does one orange weigh on the average?

b. If 100 oranges weigh 250 ounces, how much does one weigh on the average?

c. If 1,000 oranges weigh 2,700 ounces, how much does one weigh on the average?

Stress the fact that the one orange must weigh much less than 10, 100, or 1,000 oranges weigh together and that in this case each quotient must be smaller than its dividend.

4. **Extra Practice Set 106** provides more work in dividing by 10, 100, and 1,000.

Set 106. Dividing a decimal by 10, 100, or 1,000

1. **a.** $10\overline{)0.30}$ = 0.03 **b.** $100\overline{)1.05}$ = 0.0105

c. $1{,}000\overline{)0.375}$ = 0.000375 **d.** $10\overline{)4.2}$ = 0.42

2. **a.** $100\overline{)93.6}$ = 0.936 **b.** $1{,}000\overline{)85.60}$ = 0.0856

c. $10\overline{)5.75}$ = 0.575 **d.** $100\overline{)0.50}$ = 0.005

When the Divisor Is a Decimal

Introduction [O]

1. You know how to divide by a whole number and by a money number. Now you will learn to divide by a decimal fraction or a mixed decimal, as in box A.

A
a. $0.4\overline{)42}$
b. $0.015\overline{)0.24}$
c. $10.6\overline{)38.69}$

Tell what kind of decimal each divisor is in Ex. a–c in the box. a, c: tenths; b: thousandths

With money numbers, you found that it is better to change the example so as to make the divisor a whole number. We can work the same way with any example that has a decimal divisor.

2. If you think of $4\overline{)3}$ as $\frac{3}{4}$, you can write, "$\frac{3}{4} = \frac{6}{8}$." This means that $4\overline{)3} = 8\overline{)6}$. What has been done to $4\overline{)3}$ to get the equivalent division $8\overline{)6}$? Both dividend and divisor multiplied by 2

3. By thinking the decimal point one place to the right, multiply these numbers by 10: 2.7; 6.3; 5.91; 0.35; 0.06; 7. 27 63 59.1 3.5 0.6 70

4. By thinking of the decimal point as moved two places to the right, multiply by 100: 2.47; 3.8; 0.5; 0.62; 0.004 247 380 50 62 0.4

5. By thinking of the point as moved three places to the right, multiply by 1,000: 0.85; 0.042; 7.318; 12.0042; 3.2 850 42 7,318 12,004.2 3,200

6. In box B, by what are divisor and dividend multiplied in changing $0.4\overline{)42}$ to $4\overline{)420}$? 10 Why is 0 put at the right of 42? ✓ $10 \times 42 = 420$

B
a. $0.4\overline{)42} = 4\overline{)420}$ 105
b. $0.015\overline{)0.24} = 15\overline{)240}$ 16
c. $10.6\overline{)38.69} = 106\overline{)386.90}$ 3.65

Now explain the changes in Ex. b and c. b. M. by 1,000 c. M. by 10

[W]

7. Divide in Ex. a–c. Use the form at the right in box B—that is, the one with divisors changed to whole numbers.

If the divisor is a decimal, you may multiply both divisor and dividend by 10 or 100 or 1,000, and so on, in order to make the divisor a whole number.

Teaching Page 267

Pupil's Objective. To learn how to change a divisor to a whole number as preparation for dividing by decimals.

Background. As far as the actual computation is concerned, we do not divide by a decimal. We first change the divisor so that it is a whole number. Since pupils are used to changing fractions to higher terms, and since they realize that a fraction may mean division, they can use these understandings, as in Ex. 2, to help them see why the dividend and divisor of a division example may be multiplied by the same number without effecting its value. With this meaning established, pupils

need only multiply the divisors in Ex. **a**, **b**, and **c** (box B in the text) by 10, 100, and 1,000 by moving the decimal point to the right. Pupils are capable of dividing the examples that result because of their previous work in learning how to divide a decimal or a whole number by a whole number.

Teacher's Preparation. Have on hand 10-inch strips of oak tag or heavy paper.

Pre-book Lesson. Pass out 10-inch strips of oak tag and instruct pupils to:

a. Cut as many $2\frac{1}{2}$-inch strips as possible from the 10-inch strip.

b. Change $2\frac{1}{2}$ to 2.5 and find how many 2.5's there are in 10 by subtracting.

c. Check this result by adding 2.5 four times and by multiplying (2.5 by 4).

d. Work this example: $10 \div 2\frac{1}{2} = ?$

e. Attempt to show all the written work for this example: $2.5\overline{)10}$

Step *e* above will indicate to you which pupils have carried over from the lesson at the bottom of page 266 understandings useful in dividing by any decimal.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7:* Written work.

1. Follow each step on page 267 to develop the idea that both the divisor and the dividend can be multiplied by the same number to make the divisor a whole number without changing the value of the example.

2. Be sure *slower learners* realize why the 0 must be inserted as a place holder in **a** and **b** in box B.

Teaching Page 268

Pupil's Objective. To develop ability in dividing whole numbers and decimals by tenths, hundredths, and thousandths written as decimals.

Background. Now that pupils have been introduced to the idea that we change decimal divisors to whole numbers by multiplying by some power of ten to make a whole number, they will learn how to use this procedure in dividing by a decimal. Note in box A that instead of using a caret (for more about use of the caret, see page 264 in this Manual), pupils rewrite the example with the divisor as a whole number and then divide. If you have decided to teach the use of the caret, you probably ought to do so in a pre-book lesson, or in discussing with your pupils the second paragraph in the text.

The Divisor a Decimal

Tenths; hundredths; thousandths [O]

1. An airplane flew 125 mi. in 0.4 hr. This was at what rate? That is, at how many miles per hour? $0.4\overline{)125} = ?$

What did we do to get $4\overline{)1,250}$ in box A? Explain the dividing. *1. divisor and dividend by 10*

If $1,250.0 \div 4 = 312.5$, then $125 \div 0.4 = 312.5$. Explain. *The two examples are equivalent.*

Use the statement just above to help you explain the check in box A. In checking, why is it better to use the original divisor instead of the changed divisor? *mistakes might have been made in changing example.*

A

$0.4\overline{)125} = 4\overline{)1,250}$

312.5
$4\overline{)1,250.0}$
12
5
4
10
8
20
20

Check
312.5
×0.4
125.00

B

$0.2\overline{)0.75} = 2\overline{)7.5}$

3.75
$2\overline{)7.50}$
6
15
14
10
10

Check
3.75
×0.2
0.750

2. In a hard storm, rain fell at the rate of 0.2 in. a minute. In all, 0.75 in. of rain fell. How many minutes did it rain? $0.2\overline{)0.75} = ?$

a. Explain the work in box B.

b. On the board, check by common fractions: $\frac{75}{100} \div \frac{2}{10} = \frac{75}{100} \times \frac{10}{2} = \frac{15}{4} = 3.75$

3. For rows 4 and 7, read each example with whole-number divisor. *(Carets show changed position of decimal points.)*

[W]

Work and check as in boxes A and B.

	a	b	c	d
4.	$0.6\overline{)3.6}$ (6)	$0.8\overline{)1.44}$ (1.8)	$0.9\overline{)0.369}$ (0.41)	$0.16\overline{)80}$ (500)
5.	$0.12\overline{)10.80}$ (90)	$0.17\overline{)0.952}$ (5.6)	$0.005\overline{)0.65}$ (130)	$0.15\overline{)4.05}$ (27)
6.	$0.76\overline{)1,824.00}$ (2,400)	$0.056\overline{)24.080}$ (430)	$0.65\overline{)3.510}$ (5.4)	$0.93\overline{)3.441}$ (3.7)
7.	$0.025\overline{)0.075}$ (3)	$0.06\overline{)8.70}$ (145)	$0.054\overline{)8.10}$ (150)	$0.015\overline{)8.10}$ (540)

Extra Practice. For more practice, work Sets 107–108.

In checking division involving decimal divisors, the product of the quotient and the *original* divisor should be checked with the *original* dividend. It will be seen that this will give pupils additional practice in multiplying decimals.

Book Lesson. *Ex. 1–3:* Oral work. *Rows 4–7:* Written work.

1. In the oral work, point out that once the divisor has been changed to a whole number, zeros may be annexed to the changed dividend to make the division come out even (as in boxes A and B in the text).

2. Before assigning rows 4–7 for written work, review the method of changing the divisor to a whole number by multiplying the divisor and the dividend by 10, 100, or 1,000.

Differentiations and Extensions

1. *Slower learners* may first work some of rows 4–7 at the board.

2. *More capable pupils* may solve the following problems:

a. The city of Niagara Falls, N.Y., has an area of 12.7 square miles and had a population of 90,872 in 1950. How many people on the average would that be to each square mile? (Carry your answer to tenths.)

b. Moline, Illinois, had a population of 37,397 in 1950, and an area of 6.9 square miles. What was its population per square mile? (Carry your work to tenths.)

c. Troy, New York, had a population of 72,311 in 1950, and an area of 9.3 square miles. What was its population per square mile? (Carry your work to tenths.)

d. Find the population of your town or city and the area. Then find the population per square mile.

3. Assign **Extra Practice Sets 107 and 108** as needed.

Set 107. Dividing a decimal or a whole number by tenths, hundredths, or thousandths

	a	b	c	d	e
1.	12 / 0.4)4.8	12.2 / 0.6)7.32	5 / 0.8)4	25.5 / 0.5)12.75	10.97 / 0.9)9.873
2.	1.25 / 0.7)0.875	162 / 0.2)32.4	26 / 0.25)6.5	3.3 / 0.12)0.396	112 / 0.125)14
3.	36.528 / 0.35)12.785	1,250 / 0.114)142.5	0.2 / 0.9)0.18	2,310 / 0.32)739.2	7.5 / 0.19)1.425

Set 108. Dividing a decimal or a whole number by tenths, hundredths, or thousandths; rounding quotients

If remainders continue, give quotients to the nearest hundredth.

1.	4.21 / 0.9)3.785	1.67 / 0.12)0.2	0.14 / 0.7)0.098	1.52 / 0.25)0.38	100 / 0.007)0.7
2.	43.08 / 0.06)2.585	2.56 / 0.5)1.28	1,123.33 / 0.15)168.5	0.625 / 0.9)0.5625	14,880 / 0.025)372

Set 109. Dividing a whole number or a decimal by a mixed decimal

	a	b	c	d	e
1.	5 / 1.5)7.5	4 / 3.25)13	0.13 / 6.2)0.806	59.39 / 1.65)98	7 / 12.5)8.75
2.	1.8 / 2.04)3.672	2.1 / 42.1)88.41	115 / 1.14)131.1	2.3 / 4.6)10.58	101.45 / 7.75)786.2
3.	21.7 / 18.5)401.45	12.32 / 6.7)82.524	14 / 5.5)77	54 / 12.8)691.2	25 / 1.38)34.5

Set 110. Dividing a whole number or a decimal by a mixed decimal; rounding quotients

If necessary, round quotients to the nearest thousandth.

1.	4 / 1.6)6.4	2.347 / 37.5)88	63 / 1.2)75.6	19.142 / 6.7)128.25	0.55 / 4.2)2.31
2.	0.063 / 10.4)0.65	1.24 / 1.5)1.86	5.3 / 4.25)22.525	1.189 / 8.2)9.75	0.1 / 5.8)0.58

The Divisor a Mixed Decimal

[O]

1. To work Ex. A–C below, we first changed each divisor to whole number. Explain how this was done. (See boxes.)

A	B	C
M. 1.2 and 46 by 10 1.2)46 38.333 12)460.000 36 100 96 40 36 40 36 4 38.33, Ans.	M. 1.26 and 1.29 by 100 1.26)1.129 0.896 126)112.900 100 8 12 10 11 34 760 756 4 0.90, Ans.	M. 2.08 and 0.087 by 100 2.08)0.087 0.041 208)8.700 8 32 380 208 172 0.04, Ans.

2. The answers in boxes A–C are rounded to the nearest hundredths. Why did we first carry each quotient to thousandths? To determine how to round in hundredths place.

3. In box B, why are there two 0's at the right of the dividend 12.9? Explain how the quotient 0.896 is rounded. 89 to 90 (hundredths) upward from decimal places needed in quotient

4. In C, to divide 8.70 we *think*, "870 hundredths." Why? 87 (tenths) not divisible by 208 in tenths place; 870 (ones) not divisible by 208 in ones place. [W]

Divide. Round uneven quotients to the nearest hundredth. (those checked)

a	b	c	d	e
0.21 ✓2.4)0.5	31 1.5)46.5	8.82 ✓1.02)9	22.45 ✓1.1)24.7	0.21 2.7)0.57
3.6 2.4)8.64	0.44 3.2)1.408	500 0.006)3	6.42 ✓2.18)14	965 0.19)183.35
0.6 37)22.2	0.36 3.8)1.352	17.81 0.32)5.7	13.14 0.07)0.92	0.06 1.6)0.096

Extra Practice. For more practice, work Sets 109–110.

Teaching Page 269

Pupil's Objectives. (*a*) To learn how to divide a whole number or a decimal by a mixed decimal; (*b*) to review rounding decimal quotients; (*c*) to learn that some quotients require a zero as a place-holder after the decimal point.

Background. When the divisor is a mixed decimal, we work just as when dividing by tenths, hundredths, or thousandths. The divisor is first changed to a whole number by moving the decimal point to the right (multiplying by 10 or a power of 10), and the decimal point in the dividend is also moved to the right the same number of places. This results in a whole number or a decimal that is to be divided by a whole number and involves no new steps. In some cases, quotients must be rounded or place-holder zeros must be inserted, as in box C on page 269 in the text, but these are not new ideas and should give very little trouble to your pupils.

Pre-book Lesson

This lesson may be introduced by means of the following problem:

John practiced for track every morning and afternoon. He ran around the practice field 4.5 miles each day. The distance around the track was 1.5 miles. How many times did John run around the field?

First see if the class can discover ways of finding the answer by using a diagram or a number line. When the class has worked out the answer in this way, ask that the example only be written in correct form on the board.

Book Lesson. *Ex. 1–4:* Oral work. *Rows 5–7:* Written work.

1. After discussing Ex. 1, have pupils explain the work in each box. Make sure everyone understands how to change a mixed decimal to a whole number. Have a pupil explain what a mixed decimal is.

2. Discuss Ex. 2–4, making sure the use of a zero as a place-holder in the quotient shown in box C is understood by all. Then go back to the problem used in the pre-book lesson and have pupils show all the work for dividing 4.5 by 1.5.

3. *Slower learners* ought to do several of the examples in rows 5–7 on the board.

Differentiations and Extensions. Assign **Extra Practice Sets 109 and 110** as needed. (These sets are reproduced with answers on the preceding Manual page.)

NOTES

Practice in Dividing with Decimals

[W]

Copy each example. Divide until there is no remainder.

	a	b	c	d
1.	0.02)0.064 (3.2)	0.8)0.48 (0.6)	4.8)6 (1.25)	0.016)4.816 (301)
2.	0.50)35 (70)	2.4)96 (40)	8.1)0.81 (0.1)	0.009)30.6 (3,400)
3.	0.004)10 (2,500)	0.6)300 (500)	2.14)53.5 (25)	0.048)7.2 (150)
4.	0.0022)55 (25,000)	1.08)2.7 (2.5)	0.015)90 (6,000)	3.2)13.44 (4.2)

Find answers for these to the nearest hundredth:

5.	7)0.815 (0.12)	3)1.48 (0.49)	9)8.208 (0.91)	6)32.62 (5.44)
6.	11)0.32 (0.03)	36)285 (7.92)	24)166 (6.92)	1.4)3.82 (2.73)
7.	0.72)0.4 (0.56)	0.21)3.8 (18.10)	5.7)3.52 (0.62)	8.6)2 (0.23)

Give answers for these as hundredths and show any remainder in a common fraction in lowest terms:

8.	6)7 ($1.16\frac{2}{3}$)	12)5 ($0.41\frac{2}{3}$)	16)9 ($0.56\frac{1}{4}$)	1.3)0.62 ($0.47\frac{9}{13}$)
9.	2.1)0.96 ($0.45\frac{5}{7}$)	0.15)0.37 ($2.46\frac{2}{3}$)	2.2)1.5 ($0.68\frac{2}{11}$)	3.2)1 ($0.31\frac{1}{4}$)

In rows 10 and 11, change the common fractions to 2-place decimals (hundredths) and show any remainder in a fraction.

	a	b	c	d	e	f	g	h	i
10.	$\frac{1}{2}$ 0.50	$\frac{1}{3}$ $0.33\frac{1}{3}$	$\frac{2}{5}$ 0.40	$\frac{1}{6}$ $0.16\frac{2}{3}$	$\frac{24}{25}$ 0.80	$\frac{3}{10}$ 0.30	$\frac{1}{4}$ 0.25	$\frac{3}{8}$ $0.37\frac{1}{2}$	$\frac{2}{3}$ $0.66\frac{2}{3}$
11.	$\frac{5}{6}$ $0.83\frac{1}{3}$	$\frac{9}{10}$ 0.90	$\frac{7}{8}$ $0.87\frac{1}{2}$	$\frac{1}{5}$ 0.20	$\frac{3}{4}$ 0.75	$\frac{5}{8}$ $0.62\frac{1}{2}$	$\frac{3}{5}$ 0.60	$\frac{1}{8}$ $0.12\frac{1}{2}$	$\frac{7}{10}$ 0.70

12. 0.7 is how many times as large as 0.07? 10

13. 0.002 is what part of 0.08? $\frac{1}{40}$

14. How many places to the right do you *think* a decimal point to multiply a number by 100? 2 by 1,000? 3 by 10? 1

15. Does $250 \div 5$ have the same quotient as $2.5 \div 0.05$? Yes

16. What part of 3 is 0.3? $\frac{1}{10}$ 17. What part of 2.5 is 0.025? $\frac{1}{100}$

Three Kinds of Problems in M. and D.

With decimals [O]

You have worked the three kinds, or types, of problems in multiplication and division with whole numbers and common fractions. You work exactly the same way with decimal fractions. Study the three types below.

Finding the product. Mr. Saks planted 0.75 of his 12-acre field to corn. How many acres did he plant to corn? $0.75 \times 12 = n$ **You know the two factors.** $n = 9$ **You multiply to find the product.**	Type I 12 $\times 0.75$ Multiply
Finding the factor which shows a relationship, or ratio. Mr. Saks planted 9 acres of his 12-acre field to corn. What decimal part of his whole field did he plant to corn? $n \times 12 = 9$ **You know the product and one factor.** $n = 9 \div 12 = 0.75$ **You divide to find the missing factor.** In this division, the quotient shows relationship by showing *what part* 9 is *of* 12. Sometimes a quotient shows *how many times.*	Type II $12\overline{)9}$ Divide
Finding the other factor when the ratio factor is given. Mr. Saks planted 9 acres of his field to corn. He said that this was 0.75 of the whole field. How many acres are in the whole field? $0.75 \times n = 9$ **You know the product and one factor.** $n = 9 \div 0.75 = 12$ **You divide to find the missing factor.**	Type III $0.75\overline{)9} =$ $75\overline{)900}$ Divide

Tell which type of decimal problem each of these is:

1. Sue's story is 4 pages long. She has copied 2 pages of it. What decimal part of it has she copied? II

2. Joe was to have 0.4 of all the cherries he picked. If his share was 18 qt., how many quarts did he pick in all? III

3. Bob sold 0.5 of his 32 marbles. How many did he sell? I

Teaching Pages 270 and 271

Pupil's Objectives. (*a*) To review skills involved in dividing decimals; (*b*) to discover that Type I, Type II, and Type III problems also occur in decimals and are solved in the same way as with whole numbers or common fractions.

Background. On page 205 in the text, pupils saw summarized in a chart the three types of problems in multiplication and division involving fractions. Since decimals are also fractions, the same three types of problems occur in multiplication and division involving decimals. If pupils have grasped these problems earlier, they should have no trouble.

Book Lesson (page 270). *Ex. 1–17:* Written work.

1. Assign the page for individual written work; then observe pupils as they work to assure yourself that everyone understands the directions for each type of example.

2. The results of each pupil's work for this page may be used in determining grouping for remedial instruction.

Book Lesson (page 271). *Top of page and Ex. 1–3:* Oral work.

1. Refer pupils to the chart on page 205 so they will see that nothing but the use of decimals is new in the chart on page 271.

2. After pupils have identified the problems at the bottom of the page in the text, have each problem solved and a written record placed on the board, as below. Selected pupils may explain each solution to the class.

Problem 1. *Type II.* Finding the factor which shows a ratio (a how-many-times or what-part-of number)

$n \times 4 = 2$
$n = 2 \div 4$
$n = 0.5$

$$4\overline{)2.0} \quad 0.5$$
$$\underline{2\,0}$$

Problem 2. *Type III.* Finding the other factor when the ratio factor is given

$0.4 \times n = 18$
$n = 18 \div 0.4$
$n = 45$

$$0.4\overline{)18} = 4\overline{)180} \quad 45$$
$$\underline{16}$$
$$\underline{20}$$
$$\underline{20}$$

Problem 3. *Type I.* Finding the product

$0.5 \times 32 = n$
$n = 16$

$$\begin{array}{r} 32 \\ \times\ 0.5 \\ \hline 16.0 \end{array}$$

Differentiations and Extensions

1. *More capable pupils* who, on the basis of the work on page 270 in the text, do not need remedial instruction may solve the following problems:

a. If one candy bar weighs 1.3 ounces and a large box of them weighs 31.2 ounces, how many bars are there in the large box?

b. Mary found that her tomatoes averaged 3.3 ounces each. She filled several bags to sell and weighed one of them. She found that it weighed 26.4 ounces. About how many tomatoes were in the bag?

c. John's mother bought a large basket of apples that weighed 16.1 pounds. John counted the apples and found that there were 70 in the basket. On the average, what was the weight of one apple? (Round your answer to the nearest tenth.)

These pupils may then explain the problems and solutions to the class as a whole.

2. After the lesson on page 271, *all pupils* may use the information below in making up problems to illustrate the three types of problems in multiplication and division involving decimals.

Type I. 0.4 of the pupils in a school having an enrollment of 350 were absent.

Type II. Of the 400 tickets issued for a concert, 250 were sold.

Type III. 350 bottles of milk were sold on Monday. This was 0.35 of the total for the week.

Problems with Decimals

Type I: Finding the product [O]

For each problem tell the example to use.

1. The sales tax on a second-hand car is 0.02 of its price. How much is the tax on the car marked $400? 0.02 X $400 = $8.00

2. A second-hand car that was marked $640 is now reduced 0.125 of that price. How much has it been reduced? 0.125 X $640 = $80

3. On the car marked $750, Mr. Snead made a down payment which was 0.34 of the marked price. What was the down payment on this second-hand car? 0.34 X $750 = $255

4. The price of a Puritan car one year was $1,580. The next year, a new Puritan cost 0.15 more. How much was the increase in price? 0.15 X $1,580 = $237

5. Sam's brother Bob has saved 0.8 of the $575 price asked for the car he wants to buy. How much has Bob saved? 0.8 X $575 = $460

[W]

Now go back and solve these type I problems.

Teaching Pages 272 and 273

Pupil's Objective. To practice solving problems involving the multiplication and division of decimals including all of these three types of problems: (*a*) finding the product; (*b*) finding the ratio factor; (*c*) finding the other factor when the ratio factor and the product are given.

Book Lesson. *Ex. 1–5:* Oral work. *Bottom of page 272:* Written work. *Ex. 6–12:* Oral work. *Middle of page 273:* Written work. *Ex. 13–18:* Oral work. *Bottom of page 273:* Written work.

1. Have pupils read the problems in Ex. 1–5. Review Type I problems by writing the example

Type II: Finding the ratio factor [O]

Tell the example for finding the missing ratio factor.

6. If 46 words out of 50 are spelled right, what part of the words are right? $46 \div 50 = 0.92$

7. Lois blew out 7 of the 12 candles on her birthday cake. What decimal part of the candles did she blow out? $7 \div 12 = 0.58\frac{1}{3}$

8. Arthur lost 4 of his 12 marbles. How many hundredths of his marbles did he lose? $4 \div 12 = 0.33\frac{1}{3}$

9. Hal needs 300 ft. of kite string. He has 250 ft. What decimal part has he of the length he needs? $250 \div 300 = 0.83\frac{1}{3}$

10. The Royals have won 11 baseball games of the 16 they have played. What decimal part of the 16 games have they won? $11 \div 16 = 0.68\frac{3}{4}$

11. 18 is what part of 21? $18 \div 21 = 0.85\frac{5}{7}$

12. What part of 75 is 10? $10 \div 75 = 0.13\frac{1}{3}$ [W]

Now go back and solve these Type II problems. Give each answer as a 2-place decimal with any remainder in a common fraction.

Type III: Finding the other factor when the ratio factor is given [O]

Tell the example you will use to find the missing factor.

13. The hatchery man says that 0.80 of the eggs he sets will hatch. How many eggs must he set to get 4,000 chicks? $4000 \div 0.8 = 40{,}000 \div 8 = 5{,}000$

14. Jack found a tree that was 2 ft. thick. His father said this would be 0.32 of the distance around it. How many feet was it around the tree? (Give the answer to hundredths.) $2 \div 0.32 = 200 \div 32 = 6.25$ ft.

15. Mr. Rose figures that rent should cost no more than 0.15 of his salary. At that ratio, if he pays $720 a year for rent, his salary should be at least how much? $\$720 \div 0.15 = \$72{,}000 \div 15 = \$4{,}800$

16. 0.22 of the weight of corn is water. A sample of corn lost 1.76 lb. when dried. Before drying, it weighed 8 lb. $1.76 \div 0.22$

17. 36 is 0.03 of 1200. $36 \div 0.03$

18. 0.8 of 30 = 24. $24 \div 0.8$ [W]

Now go back and solve these Type III problems.

Differentiations and Extensions. *More capable pupils* may work these problems:

a. If at one time 66,000 of Nevada's population of 109,802 lived in rural areas, what decimal fraction of the population lived in the rural areas?

(Carry to the nearest tenth.)

$$109{,}802\overline{)66{,}000.00} = 0.60, \text{ or } 0.6$$

65 881 2

118 80

b. Out of 165,000 people gainfully employed in Arizona, 14,950 were engaged in the extraction of minerals. What decimal fraction of the population were occupied in mining?

$$165{,}000\overline{)14{,}950.00} = 0.09$$

14 950 00

c. At one time, 713,564 people of Rhode Island lived in cities. This was 0.916 of the total population. What was the total population?

$$0.916\overline{)713{,}564} \qquad 916\overline{)713{,}564{,}000} = 779{,}000$$

641 2

72 36

64 12

8 244

8 244

Teaching Pages 274 and 275

Pupil's Objectives. (*a*) To obtain practice in recognizing the three kinds of multiplication and division problems involving decimals; (*b*) to discover what progress has been made in mastering the understandings and techniques related to using and working with decimals.

Book Lesson (top of page 274). *Ex. 1–6:* Oral work. *Bottom of lesson:* Written work. Have each problem read aloud. Then ask the class to identify each type of problem. Make a list on the board similar to the one below.

Type I. Finding the product. Ex. 3, 4
Type II. Finding the ratio factor. Ex. 2
Type III. Finding the other factor when ratio factor is given. Ex. 1, 5, 6

Book Lesson (bottom of page 274). *Ex. 1–8:* Oral work. If there is any need of clarification, this work may be done at the board and explained by a pupil.

Book Lesson (top of page 275). *Ex. 1–7:* Written work.

for the first problem on the board in the two ways shown below.

$0.02 \times \$400 = ?$

$$\begin{array}{r} \$400 \\ \times\ 0.02 \end{array}$$

Have examples for the rest of the problems given orally. (*Slower learners* may need to work out each example as a group project. These examples may be written on the board and erased.)

2. Have pupils discuss the second type of problem (Ex. 6–12). Let them go through each example orally, expressing each in the various ways shown below for Ex. 6.

$n \times 50 = 46$ $\quad 46 \div 50$ $\quad \frac{46}{50}$ $\quad 50\overline{)46}$

Which Kind of Decimal Problem?

Differentiating Types I, II, and III [O]

Tell which kind of problem each is, as "Type II, finding the ratio factor." Then tell the example to use.

1. One day a merchant's sales-tax collections were $19.29. If the tax was 0.03 of his sales, how much were his sales? III; 19.29 ÷ 0.03 = $643

2. One rainy day 5 pupils out of 40 in Miss Hill's class were late. What decimal part of the class was late? II; 5 ÷ 40 = 0.125

3. Yellow brass is 0.3 zinc. How many pounds of zinc are used in making 68 lb. of yellow brass? I; 0.3 × 68 = 20.4 lb.

4. Powdered egg weighs 0.14 as much as eggs in shells. How much powdered egg will 45 lb. of eggs in shells make? I; 0.14 × 45 = 6.3 lb.

5. How much must Dick earn in order to save $60 if he saves 0.30 of his earnings? III; $60 ÷ 0.30 = $200

6. An oven-dressed turkey weighs 0.7 as much as it weighed when alive. What was the live weight of Mrs. Jones's turkey if it weighed 21 lb. after dressing? III; 21 ÷ 0.7 = 30 lb.

[W]

Now go back and solve decimal problems 1–6.

Oral Practice

1. For 4.1 × 0.016, how many decimal places will there be in the product? 4

2. What 2-place decimal equals $\frac{4}{5}$? 0.80 $\frac{3}{4}$? 0.75 $\frac{2}{3}$? 0.66 $\frac{2}{3}$

3. In 314)11,034, what is the first partial dividend? 110 tens

4. Is 0.8 × 14 more or less than 14? Why? 0.8 is less than 1

5. Add mentally: a. 42 + 16 + 20; 78 b. 57 + 31 + 35 123

6. Which is larger, 0.1 or 0.01?

7. Say the products: 10 × 4.1; 44 100 × 0.016; 1.6 42.5 × 1,000; 42,500

8. Say the quotients: 7.2 ÷ 10; 0.72 0.8 ÷ 100; 0.008 0.05 ÷ 1,000, 0.00005

Practice with Decimals

[W]

Find answers for these:

	a	b	c	d	e
1.	10.3	8.26	72.5	0.125	0.019
	45.8	0.98	36.8	0.376	0.036
	+9.2	+1.50	+7.1	+0.058	+0.081
	65.3	10.74	116.4	0.559	0.136
2.	19.2	1.32	0.418	3	17.105
	−1.8	−0.85	−0.165	−2.78	−3.627
	17.4	0.47	0.253	0.22	13.478
3.	13.2	0.023	2.65	4.10	0.0021
	×4.8	×15	×1.2	×2.53	×6
	63.36	0.345	3.18	10.373	0.0126
4.	438	8.15	0.4	0.03	7.21
	×0.02	×0.038	×0.5	×0.04	×0.07
	8.76	0.3097	0.2	0.0012	0.5047

5. Divide: a. 36.94 ÷ 4; 9.235 b. 9.632 ÷ 0.32; 30.1 c. 0.8424 ÷ 0.018; 46.8 d. 1.65 ÷ 0.5; 3.3 e. 936 ÷ 1.28. 731.25

6. Change to decimals (nearest thousandth): $\frac{3}{8}$; 0.375 $\frac{1}{16}$; 0.063 $\frac{5}{24}$; 0.208

7. Write answers only: a. 10 × 3.8; 38 b. 0.016 × 100; 1.6 c. 1,000 × 3.12; 3,120 d. 7.6 ÷ 10; 0.76 e. 97 ÷ 100. 0.97

Just to Remind You

[O]

1. The last decimal place used gives a decimal its name. Explain this for 1.46; ★ for 3.245; ★ and for 0.0028. ★ for decimal point

2. In reading mixed decimals, how do you use "and"? To show what you mean, read: a. 1.25; b. 284.3; c. 7,050.02

3. What decimal part of a dollar is: 9¢? 0.09 $0.37? 0.37

4. Read: 1,280,537,000. 1 billion 281 million 537 thousand

5. In 206 × 431, how many partial products will there be? 2 for multiplier 206, 3 for multiplier 431.

6. In which of these will the first trial quotient be too large? a; c;

(a) 43)1,627 (b) 24)784 (c) 36)1,418

Book Lesson (bottom of page 275). *Ex. 1–6:* Oral work. If necessary, have Ex. 5 and 6 worked out on the board.

Differentiations and Extensions

More capable pupils may solve the problems below. After each problem they should state its type and then work the problem.

a. The population of Boston was 896,000. This was 0.191 of the population of Massachusetts. What was the population of Massachusetts? (Carry to one's place and then round to the nearest thousand.)

b. The population of the mountain states was 5,074,998. Of that number, 0.549 lived in cities. How many people lived in the cities? (Round to the nearest thousand.)

c. In 1947 there were 35,404 million board feet of lumber cut in the United States. Indiana cut 180 million board feet. What decimal part of the total cut in Indiana? (Carry your work to thousandth's place only.)

★Answers Not on Reproduced Page 275

Bottom of page

1. In 1.46, 6 is in hundredth's place, so 1.46 means 1 and 46 hundredths.

In 3.245, 5 is in thousandth's place, so 3.245 means 3 and 245 thousandths.

In 0.0028, 8 is in ten-thousandth's place, so 0.0028 means 28 ten-thousandths.

Using What You Have Learned

[W]

1. Divide 7 by 8. Give the quotient as a 3-place decimal. 0.875

2. Change these common fractions to decimal form:

a. $\frac{5}{10}$; 0.5 b. $2\frac{20}{100}$; 2.2 c. $\frac{625}{1,000}$ 0.625

3. Write "sixty-four hundredths" as a common fraction in lowest terms and as a decimal. $\frac{16}{25}$ 0.64

4. Write with dollar sign and decimal point:
a. five dollars and six cents; $5.06 b. five cents; $0.05 c. ten cents $0.10

5. When hundredths are multiplied by tenths, the product has 3 decimal places. $0.2 \times 0.12 = 0.024$

6. Write estimates only for the products:

a. 0.9×3; 3 b. 6×4.1; 24 c. 9×9.8 90

7. Round these mixed decimals to the nearest whole number:

a. 6.96; 7 b. 8.21; 8 c. 12.52 13

8. Write 0.3 and 0.54 with a common denominator. 0.30 0.54

9. Is 0.7×18 more or (less) than 18? Why? 0.7 is less than 1

10. Write only the answers: a. 10×2.8; 28 b. $0.6 \div 100$; 0.006 c. $1 \div 100$; 0.01 d. 6.28×100; 628 e. $0.08 \times 1{,}000$; 80 f. $0.46 \div 1{,}000$ 0.00046

11. Change $\frac{3}{16}$ to a four-place decimal. 0.1875

12. To multiply a number by 100, *think* its decimal point 2 places to the right

13. Write each of these as a common fraction:

a. 0.8; $\frac{8}{10}, \frac{4}{5}$ b. 0.94; $\frac{94}{100}, \frac{47}{50}$ c. 0.059; $\frac{59}{1000}$ d. 0.05 $\frac{5}{100}, \frac{1}{20}$

14. In each pair of numbers which is larger: a. 0.84 or (1)? b. (3) or 2.98? c. 0.08 or (0.18)? d. (0.38) or 0.0048?

15. Copy the following and put in decimal points to make products correct: a. $0.5 \times 15 = 75$; b. $12 \times 0.26 = 312$; c. $0.68 \times 0.07 = 476$; d. $3.103 \times 2.8 = 86884$

16. Divide each number by 100. Write only the quotient.
a. 8; 0.08 b. 20; 0.20 c. 2.7; 0.027 d. 0.59; 0.0059 e. 12.5; 0.125 f. 3.087, 0.03087

17. Write a. 26 as tenths; 260 tenths b. 41 as hundredths. 4,100 hundredths

18. Write a. 34 as thousandths; 34,000 thousandths b. 2 as ten-thousandths. 20,000 ten-thousandths

Division Problems

[O]

Tell the example to use in solving each problem.

1. $3.80, the cost of food for a hike, was shared equally by 10 boys. How much did each boy pay? $3.80 ÷ 10 = $0.38

2. A gallon of sea water makes 0.25 lb. of salt. How many gallons will it take to make 100 lb. of salt? 100 ÷ 0.25 = 400 gal.

3. Sue bought 10 oz. of pepper in 2.5-ounce boxes. How many boxes did she buy? 10 ÷ 2.5 = 4

4. Our car averages 16.7 mi. per gallon of gasoline. How many gallons do we use in going 100 mi.? 100 ÷ 16.7 = 5.988, or about 6 gal.

5. Jack, a 4-H Club boy, raised 356.4 bu. of corn on 4.7 A., an average of 75.8 bu. an acre, to the nearest tenth. 356.4 ÷ 4.7 = 75.829

6. Sam, also a 4-H boy, has 7 bu. of seed wheat. If 1.25 bu. will seed an acre, how many acres can he plant? 7 ÷ 1.25 = 5.6A

7. A millimeter, a measure of length used by scientists, is about 0.04 in. How many millimeters are there in 1 ft.? 1 ft = 12 in. 12 ÷ 0.04 = 1,200 ÷ 4 = 300

[W]

Now go back and solve problems 1–7.

Teaching Pages 276 and 277

Pupil's Objective. To discover what progress has been made in working with decimals and in solving problems involving decimals.

Book Lesson (page 276 and top of page 277). *Ex. 1–18:* Written work.

1. In this lesson pupils should be reminded that it is very important for them to read the directions carefully.

2. After assigning Ex. 1–18 for written work, circulate among pupils to see that *slower learners* are following directions. Sometimes reading difficulties may be helped by reading an exercise aloud.

Book Lesson (bottom of page 277). *Ex. 1–7:* Oral work. *Bottom of lesson:* Written work. Each problem should be read aloud and the proper example set up by the pupils. A discussion of the reasons for using each example will be helpful.

Dividing with Decimals

A summary [O]

This chart shows all the kinds of examples you have been working in division with decimals. As with common fractions, some quotients tell *how many times* and some tell *what part*. Where two examples appear in a box, there is one of each kind.

For each example in the chart tell just by looking whether the quotient will be a how-many-times or a what-part-of number.

(T) (P)

Dividend	Divisor		
	Whole number	Decimal fraction	Mixed decimal
Whole Number	T 1. 15 ÷ 6 2.5 P 2. 7 ÷ 8 0.875	6. T 56 ÷ 0.4 140	10. T 8 ÷ 1.15 6.96 11. P 5 ÷ 25.5 0.196 or 0.2
Decimal Fraction	P 3. 0.75 ÷ 5 0.15	7. T 0.85 ÷ 0.5 1.7 8. P 0.5 ÷ 0.95 0.526, or 0.53	12. P 0.2 ÷ 4.28 0.046…, or 0.05
Mixed Decimal	T 4. 24.8 ÷ 8 3.1 P 5. 3.06 ÷ 6 0.51	9. T 2.25 ÷ 0.75 3	13. T 4.6 ÷ 1.15 4 14. P 2.4 ÷ 19.2 0.125

[W]

1–14. Work each example in the chart. When a quotient does not end by thousandth's place, round the quotient to the nearest hundredth.

Copy and divide in rows 15–18. Round any uneven quotient to the nearest hundredth.

	a	b	c	d
15.	0.09)395.1 (4,390)	0.05)2.689 (53.780)	0.07)0.0028 (0.04)	1.3)2.6 (2)
16.	7)0.5572 (0.0796)	7)0.4543 (0.0649)	13)0.26 (0.02)	3.2)96 (30)
17.	0.15)1.632 (10.88)	8)676.8 (84.6)	0.4)0.284 (0.71)	0.24)4.8 (20)
18.	4.5)153 (34)	0.6)0.7926 (1.321)	12)9.6 (0.8)	0.04)47.6 (1,190)

Extra Practice. For more practice, work Sets 111–112.

Teaching Page 278

Pupil's Objectives. (*a*) To see summarized all the different kinds of division examples involving decimals; (*b*) to obtain practice in working the different kinds of examples and identifying quotients as how-many-times or what-part-of numbers.

Background. The chart on this page is similar to the one which deals with fractions on page 199 in the text. One point that can be made with this chart is that since in working with decimals we always change the divisor to a whole number, all of the types of examples shown in 6–14 in the chart reduce to dividing a whole number or decimal by a whole number.

A summary of this kind can also be used as a diagnostic test, since it will give you an opportunity to discover what types of examples are giving pupils difficulty.

Pre-book Lesson. Have pupils turn to pages 184 and 185 in the text. Review the generalization there regarding how-many-times and what-part-of numbers in comparison division.

Then ask pupils the name of the term that is written first when a division example is expressed horizontally. After the correct response has been obtained write on the board "dividend ÷ divisor = quotient." Ask pupils how they can tell whether the quotient will be more or less than 1. Also ask which situation results in a how-many-times number and which in a what-part-of number.

Book Lesson. *Top of page:* Oral work. *Ex. 1–18:* Written work.

1. As pupils discuss the chart, have them compare it with the similar chart for fractions on page 199. Try to lead them to the realization that the same kinds of problems exist for both fractions and decimals.

2. As pupils tell whether quotients for the examples in the chart will be how-many-times or what-part-of-numbers, have the information written on the board.

3. Before assigning the written work, check to make sure all pupils understand rounding decimal quotients to the nearest hundredth. Remind pupils to compare their answers for Ex. 1–14 with the information on the board to see whether the quotients are the kinds predicted.

Differentiations and Extensions. Assign **Extra Practice Sets 111 and 112** as needed.

Set 111. Division, including all the different kinds of decimal examples; rounding decimal quotients

Round any uneven quotient to the nearest hundredth.

	a	b	c	d
1.	16)20 (1.25)	5)8.695 (1.74)	0.9)1,458 (1.620)	1.5)69 (46)
2.	0.35)8.645 (24.7)	18)92.52 (5.14)	24)564 (23.5)	2.25)38.25 (17)
3.	13.2)32.076 (2.43)	0.264)46.2 (175)	75)8,499 (113.32)	29)75.806 (2.61)

Set 112. Division, including all the different kinds of decimal examples; rounding decimal quotients

Round any uneven quotient to the nearest hundredth.

	a	b	c	d
1.	4.8 15)72	0.71 7)5	0.10 9)0.875	171.11 0.45)77
2.	4.17 2.4)10	1.29 7)9	0.79 14)11	0.11 16)1.7326
3.	5,844 0.125)730.5	7.10 1.32)9.375	127.14 65)8,264	0.32 25)8
4.	107.04 6)642.25	13.21 0.70)9.25	20.26 10.5)212.75	1.44 9)13
5.	0.13 32)4	0.16 83)12.98	511.76 0.017)8.7	0.11 3.40)0.375

Teaching Page 279

Pupil's Objectives. (*a*) To discover ways of telling whether a number, especially a larger number, may be divided evenly by any of the numbers 2, 3, 5, 6, and 9; (*b*) to work several Extra Practice Sets as a review of a portion of the year's work.

Background. As for other lessons with the title "Work for Young Experimenters," decide which pupils should be assigned this lesson and which should not. Since the rules for divisibility are very useful and not very difficult, they should probably be presented to *all pupils* in most cases.

With the arithmetic program for Grade 6 very near its completion, you will want material to be used for review work. This is provided in the eleven Extra Practice Sets mentioned at the bottom of the text page. As usual, these sets are reproduced in the Manual.

Pre-book Lesson. Write on the board a large fraction which can be reduced, such as $\frac{35}{486}$, and see which pupils are able to reduce it and how they attack the problem. In this way you can cite the need for rules of divisibility while discovering what skill your pupils have already developed in this area. Do not insist that the fraction you have written on the board be reduced. Leave it there though, to be used after the work in the text in deciding the effectiveness of the lesson in the book.

Work for Young Experimenters

Supplementary activity: rules for divisibility [O]

Charlie and Ted had trouble knowing what numbers they could divide by in reducing fractions to lowest terms. Miss Pike gave them these rules to work by:

1. An even number is divisible by 2. [For example, 714].

2. If the sum of the figures in a number is divisible by 3, the number is divisible by 3. [For example, 285 is divisible by 3 because 2 + 8 + 5 = 15, which is divisible by 3.]

3. Any number ending in 0 or 5 is divisible by 5. [For example, 930 or 935]

4. If the sum of the figures in an even number is divisible by 3, the number is divisible by 6. [For example, 534 is divisible by 6 because it is an even number and 5 + 3 + 4 = 12, which is divisible by 3.]

5. If the sum of the figures in a number is divisible by 9, the number is divisible by 9. [For example, 189 is divisible by 9 because 1 + 8 + 9 = 18, which is divisible by 9.]

[W]

Copy each of the following numbers. Beside each, write first the numbers by which it can be divided and then, in parentheses, the numbers of the rules that give the reasons.

1. 82 2 (1)	8. 198 2,3,6,9 (1,2,4,5)	15. 936 2,3,6,9 (1,2,4,5)	22. 9,870 (1,2,3,4) 2,3,5 (1)
2. 69 3 (2)	9. 275 5 (3)	16. 852 2,3,6 (1,2,4)	23. 1,628 (1,2,4,5) 2,3,6
3. 25 5 (3)	10. 834 2,3,6 (1,2,4)	17. 642 2,3,6 (1,2,4)	24. 3,456 (1,2,4) 2,3
4. 48 2,3,6 (1,2,4)	11. 384 2,3,6 (1,2,4)	18. 950 2,5 (1,3)	25. 6,294 (2) (1-5) 2,3,5
5. 72 2,3,6,9 (1,2,4,5)	12. 693 3,9 (2,5)	19. 753 3 (2)	26. 9,270 (1)
6. 39 3 (2)	13. 914 2 (1)	20. 486 2,3,6,9 (1,2,4,5)	27. 5,846 (1) (2,3,4) 2,3
7. 45 3,5,9 (2,3,5)	14. 540 2,3,5,6,9 (1-5)	21. 185 5 (3)	28. 4,020

Extra Practice. As a review of some of the work you have had during the year, work Sets 117–127.

279

Book Lesson. *Top of the page:* Oral work. *Ex. 1–28:* Written work.

1. A number is given as an example for each of the rules in the box. Have pupils do the dividing on the board for each of these numbers.

2. After the five rules in the box have been discussed, have pupils tell how they would decide whether a number could be divided by 1, by 2, by 3, and so on, to dividing by 10. From this activity pupils will have an opportunity to enlarge upon the rules in the book to this extent:

a. Any number may be divided by 1 or by itself (point out that this information is not needed in reducing fractions).

b. Any number ending in zero is divisible by 10.

They will also discover that no rule for deciding if a number is divisible by 4, 7, and 8 is given in the text.

Differentiations and Extensions

1. Let pupils use the principles given in the box in the text as the basis for writing several reducible fractions. These may be exchanged with partners and reduced.

2. From time to time throughout this Manual, we have called your attention to the fact that *more capable pupils* often develop ingenious rules and techniques for solving problems and for speeding up computation. It might be profitable to ask these pupils to discuss with the rest of the class some of the methods they have devised and to illustrate each with several examples. In this way, *all pupils* will have an opportunity to benefit from the work done by *more capable pupils*.

3. You may want to suggest for use now some of the games described in Part IV of this Manual.

4. Assign **Extra Practice Sets 117–127** for review. Use your judgment in making this assignment, limiting the number of examples for *slower learners* and the scope of the work for *more capable pupils*.

Set 117. Multiplying a whole number times a mixed number; vertical form

	a	b	c	d	e	f	g	h
1.	$6\frac{1}{2}$	$2\frac{2}{3}$	$4\frac{3}{5}$	$7\frac{5}{8}$	$5\frac{1}{3}$	$9\frac{8}{9}$	$6\frac{4}{5}$	$2\frac{7}{8}$
	$\times 8$	$\times 6$	$\times 7$	$\times 5$	$\times 4$	$\times 1$	$\times 7$	$\times 9$
	52	**16**	**$32\frac{1}{5}$**	**$38\frac{1}{8}$**	**$21\frac{1}{3}$**	**$9\frac{8}{9}$**	**$47\frac{3}{5}$**	**$25\frac{7}{8}$**
2.	$3\frac{4}{7}$	$7\frac{2}{9}$	$3\frac{1}{7}$	$8\frac{3}{4}$	$2\frac{5}{6}$	$8\frac{1}{4}$	$5\frac{3}{8}$	$1\frac{1}{9}$
	$\times 2$	$\times 4$	$\times 5$	$\times 2$	$\times 4$	$\times 9$	$\times 3$	$\times 3$
	$7\frac{1}{7}$	**$28\frac{8}{9}$**	**$15\frac{5}{7}$**	**$17\frac{1}{2}$**	**$11\frac{1}{3}$**	**$74\frac{1}{4}$**	**$16\frac{1}{8}$**	**$3\frac{1}{3}$**

Set 118. Multiplying a whole number times a mixed number; vertical form

	a	b	c	d	e	f
1.	$10\frac{1}{2}$	$4\frac{5}{6}$	$7\frac{5}{8}$	$8\frac{5}{14}$	$14\frac{7}{8}$	$2\frac{3}{11}$
	$\times 2$	$\times 8$	$\times 7$	$\times 8$	$\times 4$	$\times 6$
	21	**$38\frac{2}{3}$**	**$53\frac{3}{8}$**	**$66\frac{6}{7}$**	**$59\frac{1}{2}$**	**$13\frac{7}{11}$**
2.	$6\frac{3}{10}$	$11\frac{4}{5}$	$13\frac{3}{5}$	$9\frac{11}{16}$	$18\frac{2}{5}$	$7\frac{5}{12}$
	$\times 7$	$\times 9$	$\times 5$	$\times 2$	$\times 3$	$\times 5$
	$44\frac{1}{10}$	**$106\frac{1}{5}$**	**68**	**$19\frac{3}{8}$**	**$55\frac{1}{5}$**	**$37\frac{1}{12}$**
3.	$12\frac{2}{3}$	$3\frac{1}{7}$	$9\frac{3}{16}$	$12\frac{1}{9}$	$15\frac{3}{8}$	$5\frac{3}{4}$
	$\times 4$	$\times 3$	$\times 6$	$\times 3$	$\times 2$	$\times 6$
	$50\frac{2}{3}$	**$9\frac{3}{7}$**	**$55\frac{1}{8}$**	**$36\frac{1}{3}$**	**$30\frac{3}{4}$**	**$34\frac{1}{2}$**

Set 119. Multiplying a whole number times a mixed number; horizontal form

	a	b	c	d	e
1.	$2 \times 9\frac{1}{2}$ **19**	$5 \times 5\frac{2}{3}$ **$28\frac{1}{3}$**	$7 \times 2\frac{4}{5}$ **$19\frac{3}{5}$**	$9 \times 4\frac{3}{4}$ **$42\frac{3}{4}$**	$2 \times 3\frac{7}{8}$ **$7\frac{3}{4}$**
2.	$3 \times 8\frac{5}{6}$ **$26\frac{1}{2}$**	$4 \times 6\frac{5}{8}$ **$26\frac{1}{2}$**	$6 \times 1\frac{1}{8}$ **$6\frac{3}{4}$**	$7 \times 3\frac{8}{9}$ **$27\frac{2}{9}$**	$5 \times 7\frac{1}{5}$ **36**
3.	$6 \times 7\frac{3}{8}$ **$44\frac{1}{4}$**	$2 \times 8\frac{2}{5}$ **$16\frac{4}{5}$**	$5 \times 2\frac{5}{7}$ **$13\frac{4}{7}$**	$3 \times 4\frac{3}{5}$ **$13\frac{4}{5}$**	$1 \times 9\frac{5}{9}$ **$9\frac{5}{9}$**
4.	$8 \times 3\frac{1}{7}$ **$25\frac{1}{7}$**	$9 \times 1\frac{1}{3}$ **12**	$4 \times 5\frac{4}{9}$ **$21\frac{7}{9}$**	$8 \times 6\frac{1}{4}$ **50**	$6 \times 4\frac{7}{8}$ **$29\frac{1}{4}$**

Set 120. Dividing a mixed number by a fraction; inversion method

Divide, using the inversion method.

1.	$8\frac{1}{2} \div \frac{3}{4}$ **$11\frac{1}{3}$**	$2\frac{7}{10} \div \frac{3}{5}$ **$4\frac{1}{2}$**	$12\frac{2}{3} \div \frac{2}{9}$ **57**	$7\frac{1}{5} \div \frac{3}{5}$ **12**	$3\frac{3}{4} \div \frac{5}{8}$ **6**
2.	$10\frac{1}{4} \div \frac{3}{8}$ **$27\frac{1}{3}$**	$4\frac{1}{3} \div \frac{2}{3}$ **$6\frac{1}{2}$**	$9\frac{3}{4} \div \frac{3}{4}$ **13**	$3\frac{5}{8} \div \frac{1}{4}$ **$14\frac{1}{2}$**	$9\frac{1}{6} \div \frac{2}{3}$ **$13\frac{3}{4}$**
3.	$6\frac{4}{9} \div \frac{5}{6}$ **$7\frac{11}{15}$**	$6\frac{1}{10} \div \frac{2}{5}$ **$15\frac{1}{4}$**	$5\frac{5}{12} \div \frac{5}{6}$ **$6\frac{1}{2}$**	$11\frac{1}{5} \div \frac{2}{5}$ **28**	$5\frac{3}{5} \div \frac{4}{5}$ **7**

Set 121. Dividing a mixed number by a fraction; inversion method

Divide, using the inversion method.

1.	$7\frac{1}{4} \div \frac{3}{4}$ **$9\frac{2}{3}$**	$8\frac{3}{4} \div \frac{1}{8}$ **70**	$5\frac{1}{4} \div \frac{7}{8}$ **6**	$3\frac{3}{8} \div \frac{3}{5}$ **$5\frac{5}{8}$**	$6\frac{1}{8} \div \frac{7}{12}$ **$10\frac{1}{2}$**
2.	$2\frac{5}{8} \div \frac{6}{7}$ **$3\frac{1}{16}$**	$6\frac{2}{3} \div \frac{5}{12}$ **16**	$8\frac{1}{2} \div \frac{3}{8}$ **$22\frac{2}{3}$**	$1\frac{3}{7} \div \frac{5}{7}$ **2**	$2\frac{4}{5} \div \frac{7}{15}$ **6**
3.	$7\frac{1}{5} \div \frac{9}{10}$ **8**	$9\frac{3}{4} \div \frac{3}{10}$ **$32\frac{1}{2}$**	$4\frac{2}{7} \div \frac{5}{6}$ **$5\frac{1}{7}$**	$5\frac{2}{5} \div \frac{2}{15}$ **$40\frac{1}{2}$**	$1\frac{7}{8} \div \frac{5}{6}$ **$2\frac{1}{4}$**

Set 122. Decimal quotients in dividing whole numbers; rounding to the nearest tenth

Round uneven quotients to the nearest tenth.

	a	b	c	d	e	f
1.	6.3 12)75	141.4 65)9,191	4.8 8)38	1,415.8 6)8,495	32.4 15)486	4.5 16)72
2.	112.1 25)2,803	43.5 4)174	4.6 32)148	7.1 8)57	48.3 4)193	13.5 6)81
3.	3.4 16)54	2.2 25)54	225 15)3,375	13.3 28)371	216.9 24)5,205	268.6 8)2,149

Set 123. Decimal quotients in dividing whole numbers; rounding to the nearest thousandth

Round quotients to the nearest thousandth unless the division ends sooner.

1.	27.75 8)222	19.6 45)882	112.75 12)1,353	2.333 18)42	50.286 7)352	70.375 8)563
2.	8.136 22)179	766.25 4)3,065	112.412 34)3,822	344.125 8)2,753	1.556 18)28	1.857 7)13
3.	279.778 9)2,518	1.625 8)13	580.692 13)7,549	12.75 32)408	106.4 35)3,724	4.75 24)114

Set 124. Dividing money numbers; rounding to the nearest hundredth

Round uneven quotients to the nearest hundredth.

	a	b	c	d
1.	$2.25 4)$9.00	1,000 $0.05)$50.00	$0.23 25)$5.75	$3.33 15)$50
2.	6.50 $0.10)$0.65	3 $1.50)$4.50	412.50 $0.04)$16.50	49.01 $1.01)$49.50
3.	$0.01 8)$0.06	2.46 $3.25)$8.00	1.94 $12.48)$24.25	$4.25 12)$50.96
4.	$50 0.60)$30.00	0.10 $0.50)$0.05	0.03 $36.00)$0.90	0.83 $6.00)$5.00

Set 125. Writing equivalents for measures

Copy and write the equivalents.

	a	b	c
1.	16 in. = $1\frac{1}{3}$ ft.	6 tsp. = 2 tbsp.	$31\frac{1}{2}$ in. = $\frac{7}{8}$ yd.
2.	800 lb. = $\frac{2}{5}$ T.	3 mo. = $\frac{1}{4}$ yr.	20 oz. = $1\frac{1}{4}$ lb.
3.	3 pt. = $\frac{3}{8}$ gal.	75 sec. = $1\frac{1}{4}$ min.	3,000 lb. = $1\frac{1}{2}$ T.

Change to

	a	b	c	d
1. ounces:	$\frac{1}{2}$ lb. **8**	3 lb. **48**	$2\frac{1}{2}$ lb. **40**	7 lb. 5 oz. **117**
2. feet:	24 in. **2**	$1\frac{1}{2}$ yd. **$4\frac{1}{2}$**	$\frac{1}{2}$ mi. **2,640**	2 rd. **33**
3. minutes:	$1\frac{1}{2}$ hr. **90**	30 sec. **$\frac{1}{2}$**	1 da. **1,440**	75 sec. **$1\frac{1}{4}$**
4. bushels:	16 qt. **$\frac{1}{2}$**	3 pk. **$\frac{3}{4}$**	10 qt. **$\frac{5}{16}$**	6 pk. **$1\frac{1}{2}$**
5. quarts:	$\frac{1}{2}$ pk. **4**	$\frac{1}{4}$ bu. **8**	3 pt. **$1\frac{1}{2}$**	$2\frac{1}{2}$ bu. **80**

Set 127. Finding areas of rectangles; some fraction measures

Find the areas of rectangles with the following dimensions:

1. 3 ft. long; 2 ft. wide **6 sq. ft.**

2. Width, $2\frac{1}{2}$ in.; length, 4 in. **10 sq. in.**

3. Length, $1\frac{1}{4}$ yd.; width, $\frac{3}{4}$ yd. **$\frac{15}{16}$ sq. yd., or $8\frac{7}{16}$ sq. ft.**

4. Base, 2 mi.; height, 1 mi. **2 sq. mi.**

5. Each side, 8 in. **64 sq. in.**

6. 8 ft. long; $3\frac{1}{4}$ ft. wide **26 sq. ft.**

7. 500 ft. by 150 ft. **75,000 sq. ft.**

8. 3 yd. long; 27 in. wide **$2\frac{1}{4}$ sq. yd.**

9. 36 in. by 36 in. **1,296 sq. in.**

10. $\frac{1}{2}$ mi. long; 1,320 ft. wide **$\frac{1}{8}$ sq. mi.**

11. 6″ by $3\frac{1}{2}$″ **21 sq. in.**

12. 4′ by 4′ **16 sq. ft.**

13. 9 ft. by 12 ft. **108 sq. ft.**

14. 3″ × 5″ **15 sq. in.**

15. $8\frac{1}{2}$″ × 11″ **$93\frac{1}{2}$ sq. in.**

16. 9 in. by 13″ **117 sq. in.**

17. 7 yd. by $\frac{1}{2}$ yd. **$3\frac{1}{2}$ sq. yd.**

18. 20 mi. by $3\frac{1}{2}$ mi. **70 sq. mi.**

19. 15″ by 15″ **225 sq. in.**

20. $4\frac{1}{2}$ ft. by 6 in. **$2\frac{1}{4}$ sq. ft.**

NOTES

Finding n, the Missing Factor

M. and D. with decimals [O]

Multiplication	Division
A $0.3 \times 0.9 = 0.27$ multiplier · multiplicand · product factors	B $0.27 \div 0.3 = 0.9$ $0.27 \div 0.9 = 0.3$ dividend · divisor · quotient

0.3, 0.9

1. In box A, name the factors, the product. 0.27

2. In box B, what are the two decimals which multiplied together give 0.27? 0.3 and 0.9

3. Then which two numbers in a division exercise may be thought of as factors? Which number, then, is a product? divisor and quotient dividend

4. In the multiplication exercises (box C), when **n** is either the multiplicand or the multiplier, how do we find **n**, the missing factor?

C	
$0.3 \times n = 0.27$	$n \times 0.9 = 0.27$
$n = 0.27 \div 0.3$	$n = 0.27 \div 0.9$
$n = 0.9$	$n = 0.3$

Divide product by known factor.

D	
$0.27 \div 0.3 = n$	$0.27 \div n = 0.9$
$n = 0.27 \div 0.3$	$n = 0.27 \div 0.9$
$n = 0.9$	$n = 0.3$

Divide dividend by other number.

5. In the division exercises (box D), when **n** is either the divisor or the quotient, how do we find **n**?

6. Then in either a multiplication or a division example with decimals, what do we do to find **n**, the missing factor? Divide

[W]

Write your work to find **n**, the missing factor, in Ex. 7–15.

7. $n \times 0.2 = 1.0$ (5)
8. $0.21 \div n = 0.7$ (0.3)
9. $0.24 \div 0.8 = n$ (0.3)
10. $0.36 \div n = 0.4$ (0.9)
11. $0.6 \times n = 0.24$ (0.4)
12. $n \times 0.5 = 0.45$ (0.9)
13. $0.3 \times n = 0.051$ (0.17)
14. $0.328 \div n = 0.8$ (0.41)
15. $n \times 0.5 = 0.125$ (0.25)

Teaching Page 280

Pupil's Objective. To find *n*, a missing factor, in multiplication and division with decimals.

Background. The work on this page reviews the product-factor relationship which was presented on pages 202 and 203 in the text in conjunction with work with fractions. The material on page 280 in the text, then, constitutes a review of this basic relationships and an extension to include decimals. You may find it helpful at this time to turn to pages 172–174 in this Manual for the lesson covering the presentation of the product-factor relationship with fractions.

Pre-book Lesson. Test your pupils' understanding of the product-factor relationship in multiplication and division by presenting examples similar to those in boxes C and D on page 280 in the text. If the class, without referring to the text, can derive the generalization given as the answer for Ex. 6, probably all but the *slower learners* may turn to the written work.

Book Lesson. *Ex. 1–6:* Oral work. *Ex. 7–15:* Written work.

Differentiations and Extensions. Let *more capable pupils* make up, for partners to solve, examples like 7–15 in the text.

Teaching Page 281

Pupil's Objective. To practice choosing the process to use in solving problems without numbers.

Pre-book Lesson. Review with the class whatever ways they know for making problem-solving easier. Include such things as drawing a diagram, using a number line, using actual objects or pieces of oak tag to represent objects, using fractional parts on the flannel board, and the like.

Book Lesson. *Ex. 1–10:* Oral work. Have each problem on the text page read aloud. Then allow a pupil to explain exactly what he needs to do in order to solve the problem. Whenever it is possible to use objects, a diagram, a number line, or the like, do so. If pupils still have difficulty in visualizing the problems, help them provide figures and solve the problems at the board.

Differentiations and Extensions. *Slower learners* may provide appropriate numbers for each problem in a class discussion and then solve independently. *More capable pupils* may complete the problems themselves, exchange papers with a partner, and solve.

Teaching Pages 282 and 283

Pupil's Objectives. (*a*) To write questions that will complete unfinished problems and to solve the problems thus made; (*b*) to choose the correct answer in a multiple-choice test involving fractions and decimals; (*c*) to obtain practice in computation with denominate numbers in the four basic processes.

(*Continued on page 292*)

Problems without Numbers

[O]

Tell what you would do to solve these problems:

1. You know the total number of cars that passed a given spot today and the number of cars a week ago. How would you find what decimal part one total number of cars is of the other? Divide smaller number by larger number.

2. If you know the number of cars which passed the given spot each day for a week, how would you find the average number of cars a day? Divide total by number of days.

3. If you know the cost of making one mile of road, how will you find the cost of making a road several miles long? Multiply cost of 1 mile by number of miles.

4. A ranch is in the form of a rectangle. You know its area and its length. How do you find the width of the ranch? Divide area by length.

5. You know the fraction of a pound of fertilizer which Ann uses for her plants every two weeks. How can you tell how long a large bag of the fertilizer will last? 2× pounds in bag pounds used every 2 weeks

6. You know the distance some boys are going to cover on a hike and the number of miles an hour they average. After they have gone a certain number of miles, how can you tell about how much longer the hike will take at the same rate? Divide miles yet to go by miles walked per hour.

7. You know the perimeter of a square garden. How can you tell the length of one side of the garden? Divide perimeter by 4.

HELPER. Are all sides of a square the same length? Yes

8. You know the cost of a yard of material. How can you find the cost of a fractional part of a yard? Multiply cost of 1 yard by fractional part.

9. You know the cost of a yard of material. You have less than that amount of money. How would you find what fractional part of a yard you could buy? Divide money you have by cost of 1 yard.

10. You know that the number of pupils in your room is a certain decimal fraction of the number of pupils in the school. How can you find the number of pupils in the school? Divide number in your room by fraction.

Completing Unfinished Problems

[W]

Copy each exercise and write a question to make a problem. ★

1. Woodstock had a population of 18,064 in 1940. In 1950 the population was 1.25 times as much.

2. Dot is saving money in order to buy a bag. She saves 75¢ a week. The bag costs $8.25.

3. On a trip last summer, Mr. Ames paid $5.39 for 1 quart of oil costing 35¢ and 16.8 gallons of gasoline.

4. A pint of milk weighs about 1.07 lb. A can of the same kind of milk weighs 21.4 lb.

5. Last year Tom grew 2.4 in. and Jim grew 1.8 in.

6. Milk costs 24¢ a quart. We bought 3 quarts of milk for the picnic supper.

7. A large ship had 2,000 passengers and a crew of 1,200.

8. Some lobster fishermen started the season with 140 lobster traps. At the end of the season they had only 112.

9. A dozen eggs weigh 1 lb. 10 oz. A case which will hold 30 dozen of these eggs weighs 5 lb.

10. A truckman charged $50 for hauling a load 100 miles.

11. In a dictionary, the picture of a robin is $1\frac{1}{4}$ inches long. The scale is $\frac{1}{8}$ *in.* = *1 in.*

12. The rainfall during 4 months was as follows: May, $2\frac{1}{2}$ in.; June, 4 in.; July, $5\frac{1}{2}$ in.; August, 3 in.

13. Mr. Hanson burned 3 T. of coal during January.

14. Mrs. Avery wants to buy a piece of rug 3 yd. by $6\frac{1}{2}$ yd. The rug material sells at $14 a square yard and any fraction of a square yard is counted as 1 sq. yd.

Now solve the problems you have made. ★

★Answers Not on Reproduced Page 282

Only one illustrative question is given for each of Ex. 1–14.

1. How large was the population in 1950? (22,580)

2. In how many weeks will she save enough to buy the bag? (11 wk.)

3. What was the price of the gasoline per gallon? (30¢)

4. How many pints of milk does the can hold? (20 pt.)

5. Tom grew how much more than Jim? (0.6 in.)

6. How much did we pay for milk for the picnic? ($0.72)

7. How many people were there on the ship? (3,200)

8. How many lobster traps were lost? (28)

9. How much will the case full of eggs weigh? ($53\frac{3}{4}$ lb.)

10. What was the average charge per mile? ($0.50)

11. How long was the real robin? (10 in.)

12. What was the total rainfall for the 4 months? (15 in.)

13. This was an average of about how many pounds per day? (194 lb.)

14. How much will the piece of rug cost? ($280)

Short Answer Test

[W]

Copy the right answer. If it is not given, write "Not given."

1. $0.3 \times 69 = ?$ 2.3; 20.7; 19.7
2. $0.4 \times 0.2 = ?$ 0.02; 0.8; 0.008 *not given*
3. $7.2 \times 0.5 = ?$ 35; 3.6; 0.36
4. $6\frac{1}{2} \div 2\frac{3}{4} = ?$ $2\frac{9}{22}$; $2\frac{4}{11}$; $4\frac{8}{11}$
5. $\frac{1}{2} \div 8 = ?$ $\frac{1}{16}$; 4; $8\frac{1}{2}$
6. $\frac{3}{4} \times 3 = ?$ $1\frac{1}{2}$; $\frac{6}{23}$; $7\frac{3}{4}$ *not given*
7. $8\frac{1}{4} - 2\frac{1}{3} = ?$ $5\frac{11}{12}$; $6\frac{2}{3}$; $6\frac{5}{6}$
8. $\frac{7}{8} = ?$ 0.7; 0.83 *not given*
9. $\frac{3}{16} = ?$ 0.16; $0.18\frac{3}{4}$
10. $0.02 \times 42 = ?$ 2.1; 0.84
11. $\frac{1}{6} = ?$ $0.16\frac{2}{3}$; 0.60
12. $\frac{4}{25} = ?$ 0.16; 0.18
13. $0.3 \times 1.6 = ?$ 0.48; 4.8
14. If 0.6 of a number is 18, the number is _?_ 10.8; 30
15. If 0.25 of a number is 50, the number is _?_ 12.5; 200
16. $\frac{5}{6}$ of 18 = _?_ 15; 90
17. $2\frac{1}{8} + 5\frac{3}{4} =$ _?_ $7\frac{7}{8}$; $8\frac{1}{8}$
18. $\frac{3}{4}$ of 24 = _?_ 12; 18
19. $5\frac{1}{2} - 2\frac{4}{5} =$ _?_ $2\frac{7}{10}$; $2\frac{3}{5}$

Practice with Measures

[W]

	a	b	c	d
1.	8 ft. 7 in. +2 ft. 9 in. *11 ft. 4 in.*	2 hr. 45 min. +5 hr. 37 min. *8 hr. 22 min.*	5 gal. 1 qt. −3 gal. 2 qt. *1 gal. 3 qt.*	8 yd. −2 yd. 2 ft. *5 yd. 1 ft.*
2.	5 T. 1600 lb. ×3 *17 T. 800 lb.*	2 lb. 5 oz. ×7 *16 lb. 3 oz.*	1 hr. 22 min. ×5 *6 hr. 50 min.*	8 ft. 3 in. ×12 *99 ft.*

3. 5)2 hr. 50 min. *34 min.*
4. 6)8 T. 200 lb. *1 T. 700 lb.*
5. 3)2 gal. 1 qt. *3 qt.*
6. In what century is the year 1805? *19th* 1999? *20th* 1901? *20th* 2001? *21st*

Book Lesson (page 282). *Ex. 1–14:* Written work. If you wish, each problem may be read aloud so that pupils may discuss the information given as a basis for formulating suitable questions. It may be necessary to write the questions on the board before *slower learners* attempt to solve.

Book Lesson (top of page 283). *Ex. 1–19:* Written work. Pupils should understand that selecting the correct answer is not a simple matter of "guessing." They should estimate or perform operations mentally before deciding which answer is correct. They should realize, too, that it is better to work difficult problems with pencil than to guess which answer is correct.

Book Lesson (bottom of page 283). *Ex. 1–6:* Written work. Before assigning this work, give a quick review of the following equivalents:

12 in. = 1 ft.	60 min. = 1 hr.	2,000 lb. = 1 ton
3 ft. = 1 yd.	4 qt. = 1 gal.	16 oz. = 1 lb.

Teaching Pages 284 and 285

Pupil's Objective. To discover a short way to divide 2- and 3-place numbers by 1-place divisors.

Background. Short division is presented on the next four pages in the text. It is placed here near the end of the book so that you may use your judgment in deciding whether or not you want to present it to your pupils.

Short division is not a trick process. It involves all the procedures that occur in so-called long division. The only aspect that has been shortened is the amount of material that is written in the actual computation. For this reason a great deal of information must be carried mentally. You may find, therefore, that some *slower learners* are simply incapable of using the short-division form.

Notice that the short division presented in the text deals with 1-place divisors only. It is possible to use the short division form with 2-place divisors, or even more, but if *more capable* children are able to do this it is better to let them discover the procedure themselves.

Pre-book Lesson. A quick review of the table numbers for dividing by 3, 4, 5, 6, 7, 8, and 9 will be most helpful. (For the reteaching of uneven division in Grade 6, see page 75 in this Manual.)

Book Lesson. *Ex. 1–19:* Oral work. *Rows 20–23:* Written work.

1. Follow each step on the text page. Ex. 2–9 are easy, but it might be a good idea to have pupils check by long division so they will be sure they are handling the short method correctly.

2. If you wish, assign rows 11–16 as written work for *all pupils.* Then work with *slower learners* independently so you may ask them to do their thinking aloud. In this way you may be able to determine whether they are having difficulty with uneven division facts, in expressing remainders, or in retaining mentally a greater amount of information than usual. If the latter is the case,

A Short Way to Divide

Even division [O]

1. Jack has found that in even division, like $3\overline{)693}$, he doesn't have to write all the steps, as in box A. He just thinks the steps, and his work looks like that in box B. This way of dividing is called **short division.** Explain the division in box B. *Steps same as in box A, but only quotient written.*

A

$$3\overline{)693}$$
6
9
9
3
3

B

$$3\overline{)693}$$

At the board, work Ex. 2–9 by short division.

2. $2\overline{)84}$ 42
3. $3\overline{)36}$ 12
4. $4\overline{)48}$ 12
5. $2\overline{)246}$ 123
6. $4\overline{)408}$ 102
7. $7\overline{)217}$ 31
8. $\frac{1}{6}$ of 240 40
9. $\frac{1}{3}$ of 279 93

Uneven D.; one-place quotient

Short division is no different from long division, except that more work is done mentally; that is, without writing everything down.

10. We can divide any whole number or decimal by a one-place divisor without writing all the steps.

C

$$7\overline{)25}\quad 3\tfrac{4}{7}$$

For Ex. C, in our minds we separate 25 into 21 and the remainder, 4. The 21 is the product of three 7's. Why is 21 the **table number** for $7\overline{)25}$? *21 is largest product of 7 contained in 25.*

The other part of 25 is $25 - 21$, or 4. So we see that $7\overline{)25} = 7\overline{)21} + 7\overline{)4}$, or $3\frac{4}{7}$. The answer may also be 3, R4.

Without pencil and paper tell the quotients, as in box C.

	a	b	c	d	e	f	g
11.	$5\overline{)24}$ $4\frac{4}{5}$	$3\overline{)17}$ $5\frac{2}{3}$	$9\overline{)14}$ $1\frac{5}{9}$	$6\overline{)33}$ $5\frac{1}{2}$	$7\overline{)48}$ $6\frac{6}{7}$	$6\overline{)58}$ $9\frac{4}{6}$	$7\overline{)59}$ $8\frac{3}{7}$
12.	$4\overline{)33}$ $8\frac{1}{4}$	$7\overline{)46}$ $6\frac{4}{7}$	$5\overline{)32}$ $6\frac{2}{5}$	$9\overline{)47}$ $5\frac{2}{9}$	$7\overline{)36}$ $5\frac{1}{7}$	$6\overline{)49}$ $8\frac{1}{6}$	$7\overline{)47}$ $6\frac{5}{7}$
13.	$9\overline{)76}$ $8\frac{4}{9}$	$9\overline{)38}$ $4\frac{2}{9}$	$8\overline{)57}$ $7\frac{1}{8}$	$7\overline{)26}$ $3\frac{5}{7}$	$9\overline{)79}$ $8\frac{7}{9}$	$8\overline{)59}$ $7\frac{3}{8}$	$7\overline{)65}$ $9\frac{2}{7}$
14.	$9\overline{)49}$ $5\frac{4}{9}$	$9\overline{)58}$ $6\frac{4}{9}$	$8\overline{)58}$ $7\frac{1}{4}$	$9\overline{)64}$ $7\frac{1}{9}$	$4\overline{)39}$ $9\frac{3}{4}$	$8\overline{)73}$ $9\frac{1}{8}$	$8\overline{)49}$ $6\frac{1}{8}$
15.	$6\overline{)45}$ $7\frac{1}{2}$	$7\overline{)58}$ $8\frac{2}{7}$	$8\overline{)67}$ $8\frac{3}{8}$	$6\overline{)44}$ $7\frac{1}{3}$	$7\overline{)39}$ $5\frac{4}{7}$	$8\overline{)27}$ $3\frac{3}{8}$	$6\overline{)56}$ $9\frac{1}{3}$
16.	$8\overline{)76}$ $9\frac{1}{2}$	$8\overline{)42}$ $5\frac{1}{4}$	$9\overline{)28}$ $3\frac{1}{9}$	$7\overline{)66}$ $9\frac{3}{7}$	$9\overline{)29}$ $3\frac{2}{9}$	$6\overline{)57}$ $9\frac{1}{2}$	$6\overline{)29}$ $4\frac{5}{6}$

Uneven D.

17. How many pecks are in 307 quarts? How many quarts are left over?

Why is 30 (tens) the first partial dividend in this example? *3 not divisible by 8; in hundreds place.*

Why is 24 the table number for $8\overline{)30}$? *Next smaller number evenly divisible by 8.*

Why is 6, the first remainder, 6 tens? *30 tens − 24 tens = 6 tens*

In long division the 6 tens and the 7 ones (box D), are added by bringing down the 7. In short division (box E), the 6 tens are added to the 7 ones by carrying the 6, that is, holding it in mind and mentally putting it with the 7.

What is the table number for $8\overline{)67}$? 64

307 quarts = 38 pecks 3 quarts.

D

$$8\overline{)307}\quad 38, \text{R}3$$
24
67
64
3

E

$$8\overline{)307}\quad 38, \text{R}3$$

F

$$5\overline{)423}\quad 84\tfrac{3}{5}$$

Check
84
×5
420
+3
423

G

$$6\overline{)1855}\quad 309\tfrac{5}{6}$$

18. Explain Ex. F and the check.

19. In Ex. G, 18 tens ÷ 6 = 3 tens.

Since there are no 6's in 5, we write 0 in the quotient in one's place and show the remainder in a fraction, as 5 divided by 6.

After the first quotient figure, there must be a figure in the quotient for every remaining figure in the dividend.

[W]

Copy these examples and work them by short division. Show any remainder in a fraction. Check your answers for row 23.

	a	b	c	d	e	f
20.	$3\overline{)72}$ 24	$5\overline{)270}$ 54	$2\overline{)67}$ $33\frac{1}{2}$	$4\overline{)96}$ 24	$6\overline{)156}$ 26	$8\overline{)368}$ 46
21.	$7\overline{)142}$ $20\frac{2}{7}$	$6\overline{)285}$ $47\frac{1}{2}$	$4\overline{)123}$ $30\frac{3}{4}$	$9\overline{)460}$ $51\frac{1}{9}$	$7\overline{)374}$ $53\frac{3}{7}$	$5\overline{)371}$ $74\frac{1}{5}$
22.	$3\overline{)127}$ $42\frac{1}{3}$	$8\overline{)187}$ $23\frac{3}{8}$	$6\overline{)384}$ 64	$7\overline{)235}$ $33\frac{4}{7}$	$4\overline{)162}$ $40\frac{1}{2}$	$9\overline{)947}$ $105\frac{2}{9}$
23.	$3\overline{)90}$ 30	$8\overline{)258}$ $32\frac{1}{4}$	$7\overline{)295}$ $42\frac{1}{7}$	$5\overline{)408}$ $81\frac{3}{5}$	$5\overline{)162}$ $32\frac{2}{5}$	$9\overline{)736}$ $81\frac{7}{9}$

Extra Practice. For more practice, work Sets 113, 114, 116.

consider eliminating the short method for the pupils in question.

3. The only really new concept on page 285 involves carrying the remainder to the next place. If each step on the text page is carefully developed and time is allowed for plenty of discussion, there should be no difficulty. Be sure *all pupils* understand the generalization in dark print. This is not a new idea as it applied to long division, but it may prove helpful, especially for *slower learners*, in making the new method clear.

4. Note that Ex. 22f has a 3-place quotient with a middle zero. If pupils have difficulty with this example, discuss it orally, then assign similar examples from Extra Practice Set 116.

Differentiations and Extensions

1. It may help some pupils to realize that short division often requires thinking of the dividend as being changed so that the first partial dividend is expressed in a number evenly divisible by the divisor. For the example in box E in the text, the dividend should be thought of as 24 tens and 6 tens + 7 ones, or 24 tens and 67 ones. If pupils attempt to separate 30 ten-bundles and 7 single tickets into 8 groups, they will see that there are only enough tens to make 8 equal groups of 3 tens each and that this uses just 24 of the tens, the 6 tens remaining being combined with the ones and thought of as 67 ones. This reasoning, of

course, also applies to the long-division method, but it may be helpful for pupils to repeat the procedure as the new short form for writing the division is shown on the board. If this is done, show the written record as below, for *slower learners* may find the crutch a useful device for lessening the amount of information that must be retained mentally.

$$8\overline{)3\ 0^{6}7}\qquad 3\ 8,\ \text{R}3$$

2. Assign **Extra Practice Sets 113, 114, and 116** as needed. *Slower learners* especially will need this practice in short division in order to eliminate the crutch. Be sure, however, that these pupils understand the process before assigning abstract practice.

Set 113. 1-place quotients in short division; remainders in a fraction

Work by short division.

	a	b	c	d	e	f	g
1.	$8\frac{1}{2}$ 2)17	$8\frac{7}{8}$ 8)71	$2\frac{8}{9}$ 9)26	$8\frac{1}{4}$ 8)66	$5\frac{7}{9}$ 9)52	$7\frac{4}{5}$ 5)39	$6\frac{3}{4}$ 4)27
2.	$4\frac{6}{7}$ 7)34	$7\frac{1}{2}$ 6)45	$4\frac{4}{5}$ 5)24	$8\frac{8}{9}$ 9)80	$5\frac{6}{7}$ 7)41	$8\frac{1}{2}$ 4)34	$7\frac{2}{3}$ 3)23
3.	$3\frac{5}{6}$ 6)23	$8\frac{6}{7}$ 7)62	$8\frac{3}{4}$ 4)35	$5\frac{2}{3}$ 3)17	$8\frac{2}{3}$ 6)52	$9\frac{2}{3}$ 3)29	$6\frac{1}{2}$ 6)39
4.	$8\frac{2}{5}$ 5)42	$9\frac{1}{2}$ 2)19	$5\frac{3}{4}$ 8)46	$6\frac{1}{2}$ 2)13	$6\frac{2}{3}$ 9)60	$7\frac{3}{8}$ 8)59	$8\frac{1}{7}$ 7)57

Set 114. 2-place quotients in short division; remainders in a fraction

Work by short division.

	a	b	c	d	e	f
1.	$28\frac{2}{3}$ 3)86	$95\frac{4}{5}$ 5)479	$63\frac{2}{7}$ 7)464	$23\frac{3}{4}$ 4)95	$32\frac{1}{3}$ 6)194	$54\frac{2}{9}$ 9)488
2.	$57\frac{1}{4}$ 8)458	$21\frac{1}{2}$ 2)43	$94\frac{1}{3}$ 6)566	$24\frac{1}{5}$ 5)121	$77\frac{1}{3}$ 3)232	$49\frac{2}{3}$ 6)298
3.	$98\frac{1}{7}$ 7)687	$94\frac{3}{4}$ 8)758	$33\frac{1}{9}$ 9)298	$96\frac{1}{2}$ 4)386	$87\frac{2}{9}$ 9)785	$34\frac{3}{4}$ 8)278
4.	$48\frac{1}{2}$ 2)97	$84\frac{2}{3}$ 3)254	$12\frac{5}{7}$ 7)89	$86\frac{1}{2}$ 4)346	$86\frac{1}{2}$ 2)173	$87\frac{4}{5}$ 5)439

Set 116. 2- and 3-place quotients in short division; remainders with R

Work by short division and show any remainder with R.

1.	166, R2 3)500	127, R2 4)510	78, R6 8)630	38, R8 9)350	57, R6 7)405	38, R7 8)311
2.	36, R5 6)221	27, R6 8)222	130, R2 7)912	133, R4 6)802	27, R2 4)110	68, R8 9)620
3.	75, R6 7)531	77, R2 4)310	66, R2 3)200	46, R8 9)422	83, R3 6)501	136, R2 3)410
4.	26, R8 9)242	45, R6 7)321	88, R4 6)532	177, R1 4)709	246, R2 3)740	63, R7 8)511

Short Division

Harder subtraction {O}

In division, whether long or short, there is a partial dividend for each quotient figure. In short division, at each partial dividend, you do three important things in your mind:

A

$$8\overline{)517} \quad 64\tfrac{5}{8}$$

a. You *think* the table number for the partial dividend.
b. You *think* the remainder (partial dividend minus table number).
c. You carry the remainder to the next figure in the dividend, getting the next partial dividend.

Say the table number and the remainder for each of these. For the first example say, "28, remainder 3."

	a	b	c	d	e	f
1.	4)31 *28,R3*	3)20 *18,R2*	6)22 *18,R4*	4)11 *8,R3*	9)23 *18,R5*	7)31 *28,R3* *45,R6*
2.	9)33 *27,R6*	7)13 *7,R6*	8)22 *16,R6*	6)40 *36,R4*	8)31 *24,R7*	9)51 *63,R8*
3.	8)52 *48,R4*	9)42 *36,R6*	7)20 *14,R6*	9)61 *54,R7*	6)51 *48,R3*	9)71

The following subtractions will help you in working short division. Say the remainders.

	a	b	c	d	e
4.	21 − 18 *3*	41 − 36 *5*	52 − 48 *4*	53 − 49 *4*	22 − 16 *6*
5.	33 − 28 *5*	55 − 49 *6*	61 − 56 *5*	30 − 24 *6*	32 − 28 *4*
6.	31 − 27 *4*	53 − 48 *5*	73 − 64 *9*	80 − 72 *8*	90 − 81 *9*

2- and 3-place quotients

For these short-division examples with two partial dividends, tell quotients and give remainders with R.

	a	b	c	d	e	f
7.	*36* 5)180	*36,R2* 3)110	*35,R3* 6)213	*37,R3* 4)151	*32,R7* 9)295	*40,R3* 5)203
8.	*73,R3* 7)514	*72* 6)432	*88,R1* 8)705	*17,R3* 4)71	*82,R7* 8)663	*87,R3* 6)525
9.	*80,R3* 9)723	*52,R6* 7)370	*36,R5* 6)221	*65,R4* 8)524	*59,R4* 9)535	*78,R6* 7)552
10.	*29,R3* 9)264	*85,R2* 6)512	*47,R4* 7)333	*38* 8)304	*34,R7* 9)313	*46,R3* 7)325

Short division can be used with larger dividends and with decimals.

B

$$8\overline{)2,316} \quad 289\tfrac{1}{2}$$

C

$$4\overline{)6.260} \quad 6.26 \div 4 = ? \quad 1.565$$

D

$$7\overline{)4,210.000} \quad 421 \div 0.7 = ? \text{ (to nearest hundredth)} \quad 601.428$$

Ans., 601.43, to nearest hundredth

11. For Ex. B, what are the 3 partial dividends? Explain the division. *23(hundreds); 71(tens); 76(ones)*

12. In Ex. C, why is a 0 written after 6.26? Explain the division. *So the division will come out even.*

13. In Ex. D, before dividing, why do we multiply 421 and 0.7 by 10? *To make the divisor a whole number.* Why do we write 0's in tenth's, hundredth's, and thousandth's places in the dividend? Explain the work. *Decimal placement in rounding to hundredths place.*

{W}

Copy and divide, using short division. If uneven, round the quotient to the nearest tenth.

	a	b	c	d
14.	*447* 7)3,129	*484* 9)4,356	*1.31* 4)5.24	*$12.56* 5)$62.80
15.	*13.1* 0.6)7.86	*117.5* 8)940	*1.30* 7)9.15	*2,405* 0.04)96.2
16.	*2,487* 3)7,461	*0.4* 9)3.84	*0.9* 6)5.407	*4,050* 0.2)810
17.	*116* 8)928	*524.4* 7)3,671	*140.5* 6)843	*16.5* 0.5)8.25

18. Change to whole or to mixed numbers. Use short division.

a. $\frac{244}{7}$ *$34\frac{6}{7}$* b. $\frac{125}{5}$ *25* c. $\frac{208}{4}$ *52* d. $\frac{121}{8}$ *$15\frac{1}{8}$* e. $\frac{95}{6}$ *$15\frac{5}{6}$*

Use short division and change to decimals, rounding to the nearest hundredth.

	a	b	c	d	e	f
19.	$\frac{7}{8}$ *0.88*	$\frac{5}{9}$ *0.56*	$\frac{3}{4}$ *0.75*	$\frac{2}{3}$ *0.67*	$\frac{5}{6}$ *0.83*	$\frac{8}{9}$ *0.89*
20.	$\frac{4}{5}$ *0.8*	$\frac{3}{7}$ *0.43*	$\frac{1}{6}$ *0.17*	$\frac{2}{9}$ *0.22*	$\frac{10}{7}$ *1.43*	$\frac{13}{5}$ *2.6*
21.	$2\frac{1}{4}$ *2.25*	$3\frac{1}{9}$ *3.11*	$1\frac{5}{7}$ *1.71*	$6\frac{1}{2}$ *6.5*	$8\frac{2}{5}$ *8.4*	$10\frac{1}{3}$ *10.33*

Extra Practice. For more practice, work Set 115.

Teaching Pages 286 and 287

Pupil's Objectives. (*a*) To learn how to divide the short way when the subtraction involved is harder because of borrowing; (*b*) to learn how to divide the short way with decimals and in changing improper fractions.

Background. The important steps in the short-division procedure are shown in **a**, **b**, and **c** at the top of page 286. You will notice that step **b** involves mental subtraction. Since this step can be difficult, help is given in rows 4–6 where subtraction involving borrowing is given for review.

Short division can also be used in dividing decimals, in changing mixed numbers, and in changing improper fractions to whole or to mixed numbers. This material is presented on page 287, but you will very likely want to omit it for *slower learners*.

Book Lesson. *Top of page 286 and Ex. 1–13:* Oral work. *Ex. 14–21:* Written work.

1. As the class discusses the preliminary material on page 286, make sure that everyone understands that after *thinking* the table number in the partial dividend they must *think* and record the quotient figure before going on to steps **b** and **c**.

2. Use rows 1–3 for oral review of the work on pages 284 and 285.

3. Be sure pupils understand that the subtractions in rows 4–6 are similar to those they will encounter in short division. Show this for Ex. 4**a** by having pupils isolate the subtraction part of the division in the example $210 \div 6$. Then help pupils master subtraction involving borrowing if they have not already done so. If a pupil has difficulty with 53–49, he may make a small flash card of $13 - 9$ and drill on that fact plus all the other combinations which involve 3 and 9. (See pages 15 and 17 in the text for a review of subtraction.)

4. If pupils are slow in responding with quotients and remainders for rows 7–10, use this material as written work.

5. Omit the work on page 287 for *slower learners* if you feel it will be too difficult for them.

Differentiations and Extensions. Assign **Extra Practice Set 115** as needed.

Set 115. Decimals and money numbers in short division; rounding to hundredths

Work by short division. Round quotients to hundredths.

	a	b	c	d
1.	$6.67 \quad 3\overline{)\$20.00}$	$1.37 \quad 6\overline{)\$8.21}$	$16.75 \quad 0.4\overline{)6.7}$	$5.89 \quad 0.07\overline{)0.412}$
2.	$6.29 \quad 8\overline{)50.3}$	$7.64 \quad 0.8\overline{)6.11}$	$683.33 \quad 0.6\overline{)410}$	$16.67 \quad 0.03\overline{)0.5}$
3.	$2.37 \quad 0.3\overline{)0.710}$	$4.74 \quad 7\overline{)33.21}$	$27.79 \quad 0.09\overline{)2.501}$	$86.67 \quad 0.6\overline{)52}$
4.	$578.67 \quad 0.9\overline{)\$521}$	$2.59 \quad 9\overline{)23.3}$	$175.75 \quad 0.04\overline{)\$7.03}$	$574.57 \quad 0.7\overline{)402.2}$

Teaching Pages 288, 289, and 290

Pupil's Objective. To take the four regular end-of-chapter tests.

Book Lesson (page 288). *Ex. 1–13:* Written work. Test of Information and Meaning 7.

Book Lesson (page 289). *Ex. 1–9:* Written work. Problem Test 7.

Do You Understand?

Test of Information and Meaning 7

1. How do you think of moving the decimal point to get the product of

a. 100×4.1? *2 places to right* b. 10×0.6? *1 place to right* c. $1{,}000 \times 6.1$? *3 places to right*

2. How can you change any common fraction to a decimal fraction if you do not know the equivalent decimal? *divide numerator by denominator.*

3. For a decimal fraction with four figures at the right of the decimal point, what is the fractional unit? *ten-thousandths*

4. Work the following division by using common fractions:

$0.5\overline{)1.25}$ $\quad \frac{25}{100} \div \frac{5}{10} = \frac{1}{4} \div \frac{1}{2} = 2\frac{1}{2}$

5. If the remainder in Ex. A is to be shown in a decimal fraction in the quotient, what must you do next? *Add a decimal point and a zero after 33.*

A	B
4	0.3
$8\overline{)33}$	$2\overline{)0.6}$
32	6
1	

6. When the divisor is a mixed decimal, as in $1.4\overline{)42}$, how must you change the example before you divide? *Make divisor a whole number by multiplying divisor and dividend by same number.*

7. How would you check the quotient in Ex. B? *$0.3 \times 2 = 0.6$*

8. Does the fraction $\frac{7}{8}$ equal the decimal 0.75? *No*

9. Does $5\frac{1}{4} = 5.4$? *No*

10. If you multiply both the divisor and the dividend of a division example by 100, have you changed the quotient? *No*

11. When you want to express a quotient to the nearest hundredth, how many decimal places must you first get in the quotient? *3 (to thousandths)*

12. Give the decimal equivalent for each of the following:

a. $\frac{1}{3}$ *$0.33\frac{1}{3}$* b. $\frac{7}{8}$ *0.875* c. $\frac{2}{5}$ *0.4* d. $\frac{1}{2}$ *0.5* e. $\frac{3}{4}$ *0.75* f. $\frac{1}{6}$ *$0.16\frac{2}{3}$* g. $\frac{1}{4}$ *0.2*

13. How do you think of moving the decimal point to get the quotient for

a. $70.6 \div 10$? *1 place to left* b. $75 \div 1{,}000$? *3 places to left* c. $0.2 \div 100$? *2 places to left*

Book Lesson (top of page 290). *Rows 1–6:* Written work. Diagnostic Test 7. Practice sets with answers are reproduced on previous pages in the Manual. For their location, refer to the heading **Practice** in the index to this Manual.

Book Lesson (bottom of page 290). *Ex. 1–12:* Written work. Computation Test 7. If necessary, this test may be used for still further practice as there is an alternate computation test on page 303 of the Manual.

Differentiations and Extensions. Use the table on page 297 in the Manual as an aid in entering the scores on the test record cards.

Can You Solve Problems?

Problem Test 7

1. The distance around the park is 1.2 miles. In 8 trips around the park, how far has Frank ridden his bicycle? 9.6 mi.

2. The four men on a 1-mile relay track team ran in the following times: 48.6 sec., 47.3 sec., 47.2 sec., and 46.9 sec. What was their average time? 47.5 sec.

3. The cost of gasoline for a motor scooter was 25.6 cents for 42 miles. What was the cost per mile to the nearest tenth of a cent? 0.6 cents

4. A road-building crew averages 0.6 of a mile per day. At that rate, how long will it take the crew to build 12 mi. of road? 20 da.

5. At 22.8 miles per hour how many miles can a tractor travel in one day of 7 hours? 159.6 mi.

6. The cost of passenger air transportation for one aviation company was $0.0506 per passenger mile. On an average day, planes of that company carried passengers 60,000 passenger miles. What was the average daily cost of carrying passengers? $3,036

7. The wool in the average suit of clothes weighs $2\frac{1}{2}$ pounds. If the washed wool in cloth is only 0.4 of the weight of wool as it comes from the sheep, what was the original weight of the wool used in a suit of clothes? 6.25 lb.

8. When wool is washed, it loses about 0.6 of its original weight. How many pounds of washed wool will be obtained from 220 pounds of greasy wool? 88 lb.

9. Make up one of each of the following kinds of problems with decimals:

I. You must find a product;

II. You must find the ratio factor;

III. You must find the other factor when the ratio factor is given.

Do You Make Mistakes?

Diagnostic Test 7

		Study Pages	Practice Sets
1.	Write the equivalent decimal for a. $\frac{3}{4}$; b. $\frac{7}{8}$; c. $\frac{1}{4}$; d. $\frac{4}{5}$; e. $\frac{7}{10}$; f. $\frac{1}{2}$ 0.75 0.875 0.25 0.8 0.7 0.5	258–261	91
2.	Round any uneven decimal to the nearest hundredth. a 1.875 8)15; b 2.83 6)17; c 0.32 25)8; d 0.16 32)5; e 0.13 45)6	257–259	102
3.	Change to an equivalent decimal rounded to hundredths. $\frac{2}{9}$ 0.22; $\frac{3}{7}$ 0.43; $\frac{3}{14}$ 0.21; $\frac{2}{11}$ 0.18; $\frac{5}{17}$ 0.29	258–259	103
4.	a. 5)0.75 0.15; b. 9)1.899 0.211; c. 4)24.8 6.2 d. 15)0.45 0.03; e. 25)2.75 0.11	262–263	104, 105
5.	Round uneven quotients to the nearest hundredth. a. 0.7)0.98 1.4; b. 0.6)3.36 5.6; c. 2.4)0.768 0.32 d. 1.6)498 311.25; e. 1.4)379 270.07	267–269	107–112
6.	Use short division for these: a. 7)185 26.43; b. $\frac{325}{8}$ 40.625; c. 9.2 ÷ 4 2.3	284–287	113–116

How Well Can You Figure?

Computation Test 7

Divide. Round uneven quotients to nearest hundredth.

1. 17)38 2.24
2. 9)7 0.78
3. 4)1.29 0.32
4. 0.8)425 531.25
5. 25)86 3.44
6. 11)4.62 0.42
7. 45)92.87 2.06
8. 5.2)1.097 0.21
9. 0.5)0.045 0.09
10. 3.4)695 204.41
11. 0.3)197.82 659.4
12. 6.7)14.632 2.18

TABLE OF PER CENTS FOR CHAPTER 7 SCORES

Score	Per Cents for Problem Test 7
1	11
2	22
3	33
4	44
5	56
6	67
7	78
8	89
9	100

Score	Per Cents for Computation Test 7	Score	Per Cents for Computation Test 7
1	8	7	58
2	17	8	67
3	25	9	75
4	33	10	83
5	42	11	92
6	50	12	100

Term Test

An optional term test for the second portion of Grade 6 is provided on Manual pages 306–308. It consists of three parts to test separately learning outcomes of different kinds: information and meanings taught during the second portion of the year; problem-solving ability; and computational skills. The test should probably be mimeographed so that each child may have his own set of questions. As they are set up, the three parts of the term test are intended to be given at three sittings.

The 81 Addition Facts

Sums through 18

1	1	1	1	1	1	1	1	1
+1	+2	+3	+4	+5	+6	+7	+8	+9
2	3	4	5	6	7	8	9	10
2	2	2	2	2	2	2	2	2
+1	+2	+3	+4	+5	+6	+7	+8	+9
3	4	5	6	7	8	9	10	11
3	3	3	3	3	3	3	3	3
+1	+2	+3	+4	+5	+6	+7	+8	+9
4	5	6	7	8	9	10	11	12
4	4	4	4	4	4	4	4	4
+1	+2	+3	+4	+5	+6	+7	+8	+9
5	6	7	8	9	10	11	12	13
5	5	5	5	5	5	5	5	5
+1	+2	+3	+4	+5	+6	+7	+8	+9
6	7	8	9	10	11	12	13	14
6	6	6	6	6	6	6	6	6
+1	+2	+3	+4	+5	+6	+7	+8	+9
7	8	9	10	11	12	13	14	15
7	7	7	7	7	7	7	7	7
+1	+2	+3	+4	+5	+6	+7	+8	+9
8	9	10	11	12	13	14	15	16
8	8	8	8	8	8	8	8	8
+1	+2	+3	+4	+5	+6	+7	+8	+9
9	10	11	12	13	14	15	16	17
9	9	9	9	9	9	9	9	9
+1	+2	+3	+4	+5	+6	+7	+8	+9
10	11	12	13	14	15	16	17	18

The 81 Subtraction Facts

Minuends through 18

2	3	4	5	6	7	8	9	10
−1	−1	−1	−1	−1	−1	−1	−1	−1
1	2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10	11
−2	−2	−2	−2	−2	−2	−2	−2	−2
1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	10	11	12
−3	−3	−3	−3	−3	−3	−3	−3	−3
1	2	3	4	5	6	7	8	9
5	6	7	8	9	10	11	12	13
−4	−4	−4	−4	−4	−4	−4	−4	−4
1	2	3	4	5	6	7	8	9
6	7	8	9	10	11	12	13	14
−5	−5	−5	−5	−5	−5	−5	−5	−5
1	2	3	4	5	6	7	8	9
7	8	9	10	11	12	13	14	15
−6	−6	−6	−6	−6	−6	−6	−6	−6
1	2	3	4	5	6	7	8	9
8	9	10	11	12	13	14	15	16
−7	−7	−7	−7	−7	−7	−7	−7	−7
1	2	3	4	5	6	7	8	9
9	10	11	12	13	14	15	16	17
−8	−8	−8	−8	−8	−8	−8	−8	−8
1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
−9	−9	−9	−9	−9	−9	−9	−9	−9
1	2	3	4	5	6	7	8	9

Table of Facts

The facts for the four basic processes are included in the text for reference and practice. Most pupils in Grade 6 are familiar with the facts, but some pupils can profit from using these tables in a variety of ways.

For example, pupils who have difficulty remembering a set of facts may cover the answers and say the whole set. Pupils may also use these pages as they prepare study cards for hard facts. Moreover, if the tables are used in an oral lesson, most pupils will discover many interesting relationships that will help them remember hard facts.

For easy reference and use, you may wish pupils to keep a copy of each table in their notebooks.

NOTES

The 81 Multiplication Facts

Through 9×9

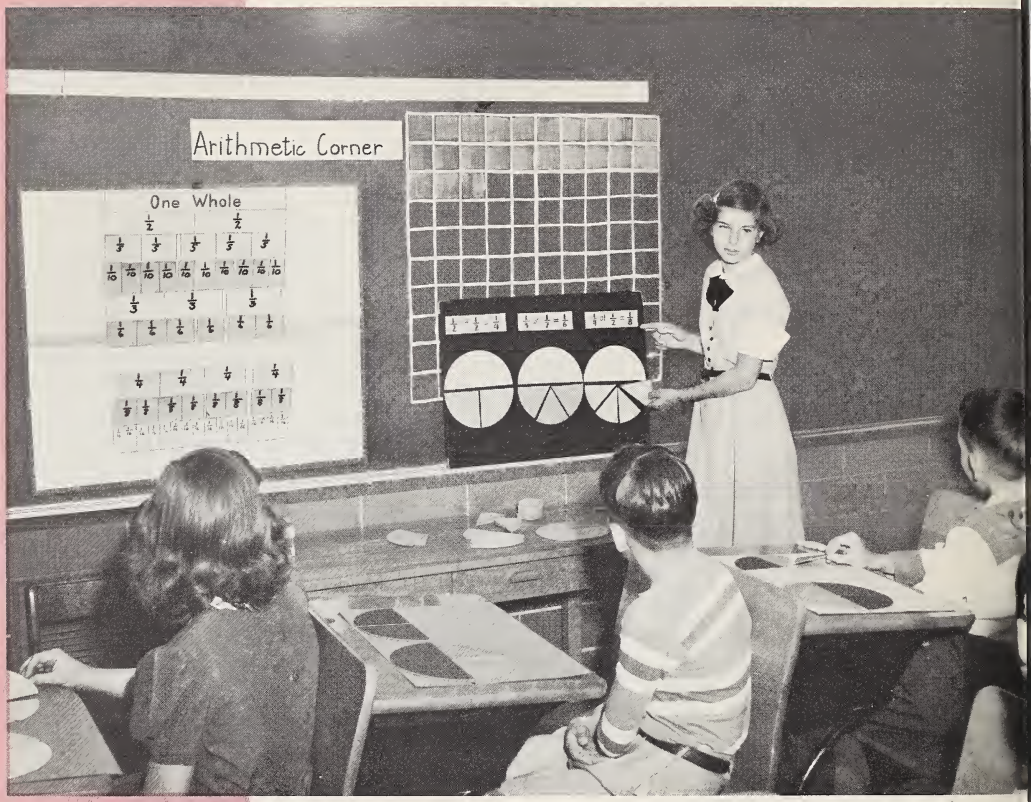
1	2	3	4	5	6	7	8	9
×1	×1	×1	×1	×1	×1	×1	×1	×1
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
×2	×2	×2	×2	×2	×2	×2	×2	×2
2	4	6	8	10	12	14	16	18
1	2	3	4	5	6	7	8	9
×3	×3	×3	×3	×3	×3	×3	×3	×3
3	6	9	12	15	18	21	24	27
1	2	3	4	5	6	7	8	9
×4	×4	×4	×4	×4	×4	×4	×4	×4
4	8	12	16	20	24	28	32	36
1	2	3	4	5	6	7	8	9
×5	×5	×5	×5	×5	×5	×5	×5	×5
5	10	15	20	25	30	35	40	45
1	2	3	4	5	6	7	8	9
×6	×6	×6	×6	×6	×6	×6	×6	×6
6	12	18	24	30	36	42	48	54
1	2	3	4	5	6	7	8	9
×7	×7	×7	×7	×7	×7	×7	×7	×7
7	14	21	28	35	42	49	56	63
1	2	3	4	5	6	7	8	9
×8	×8	×8	×8	×8	×8	×8	×8	×8
8	16	24	32	40	48	56	64	72
1	2	3	4	5	6	7	8	9
×9	×9	×9	×9	×9	×9	×9	×9	×9
9	18	27	36	45	54	63	72	81

The 81 Division Facts

Through $81 \div 9$

1	2	3	4	5	6	7	8	9
1)1	1)2	1)3	1)4	1)5	1)6	1)7	1)8	1)9
1	2	3	4	5	6	7	8	9
2)2	2)4	2)6	2)8	2)10	2)12	2)14	2)16	2)18
1	2	3	4	5	6	7	8	9
3)3	3)6	3)9	3)12	3)15	3)18	3)21	3)24	3)27
1	2	3	4	5	6	7	8	9
4)4	4)8	4)12	4)16	4)20	4)24	4)28	4)32	4)36
1	2	3	4	5	6	7	8	9
5)5	5)10	5)15	5)20	5)25	5)30	5)35	5)40	5)45
1	2	3	4	5	6	7	8	9
6)6	6)12	6)18	6)24	6)30	6)36	6)42	6)48	6)54
1	2	3	4	5	6	7	8	9
7)7	7)14	7)21	7)28	7)35	7)42	7)49	7)56	7)63
1	2	3	4	5	6	7	8	9
8)8	8)16	8)24	8)32	8)40	8)48	8)56	8)64	8)72
1	2	3	4	5	6	7	8	9
9)9	9)18	9)27	9)36	9)45	9)54	9)63	9)72	9)81

NOTES



Detroit Public Schools

It is easy to show the meaning of fractions and to develop understandings when a good supply of arithmetic materials is available.

PART IV · SUPPLEMENTARY HELPS FOR TEACHERS

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Alternate Computation Tests

The next two pages contain seven alternate chapter tests, one for each chapter. Each test parallels the corresponding test in the text in the number and kind of examples and in the level of achievement that each tests. Because of this, you may substitute the appropriate alternate test if you want to use the test in the text for pretest practice or for any other purpose. Of course, this procedure may be reversed, and the alternate test given as practice while the computation test in the text is used for final testing.

If you use an alternate test, you may either write it on the board or mimeograph it. If you write the examples on the board, pupils should copy the examples first and then work them.

Since each alternate test has the same number of items as the corresponding test in the text, the table given at the end of the appropriate chapter in the Manual may be used for deriving per-cent scores for individual test-record cards.

Alternate Computation Test, Chapter 1

	a	b	c	d	e
1.	56 − 43 = **13**	35 + 24 = **59**	\$7.56 × 5 = **\$37.80**	74 − 35 = **39**	687 − 362 = **325**
2.	245 + 397 = **642**	806 − 238 = **568**	640 × 6 = **3,840**	48 × 56 = **2,688**	5,279 − 1,685 = **3,594**
3.	45 − 37 = **8**	549 − 176 = **373**	485 − 59 = **426**	934 + 198 = **1,132**	646 − 257 = **389**
4.	800 − 395 = **405**	764 × 28 = **21,392**	\$7.05 × 54 = **\$380.70**	4,650 × 706 = **3,282,900**	\$73.75 + 16.43 = **\$90.18**

5. 9, 5, 7, 3, 6, 4 = **34**

6. \$32.41, 54.04, 70.23, 12.12 = **\$168.80**

Alternate Computation Test, Chapter 2

	a	b	c	d
1.	**12** R 7)84	**18, R3** 9)165	**245** 6)1,470	**7** 70)490
2.	**9, R4** 14)130	**6** 32)192	**8, R25** 65)545	**3, R23** 43)152
3.	**\$0.85** 34)\$28.90	**7** 800)5,600	**65** 142)9,230	**84** 735)61,740

4. 6 gal. 3 qt. + 3 gal. 1 qt. = **10 gal.**

5. 7 hr. 25 min. − 4 hr. 35 min. = **2 hr. 50 min.**

6. 9 ft. 7 in. × 5 = **47 ft. 11 in.**

7. 3 bu. 2 pk. × 6 = **21 bu.**

8. 6 oz.)4 lb 2 oz. = **11 oz.**

9. $\frac{1}{3}$ of 8 wk. 1 da. = **2 wk. 5 da.**

10. $\frac{1}{10}$ of 6 yd. 2 ft. = **2 ft.**

11. $\frac{1}{4}$ of 2 yr. 4 mo. = **7 mo.**

Alternate Computation Test, Chapter 3

1. Change these numbers to best form:

$7\frac{2}{8}$ $\frac{12}{15}$ $\frac{56}{8}$ $6\frac{12}{9}$ $\frac{29}{4}$ $2\frac{12}{7}$ $5\frac{10}{14}$ $3\frac{12}{8}$

$7\frac{1}{4}$ **$\frac{4}{5}$** **7** **$7\frac{1}{3}$** **$7\frac{1}{4}$** **$3\frac{5}{7}$** **$5\frac{5}{7}$** **$4\frac{1}{2}$**

2. Subtract each of the following fractions from 1:

$\frac{3}{5}$ $\frac{7}{12}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{8}$ $\frac{6}{11}$ $\frac{1}{4}$ $\frac{3}{10}$

$\frac{2}{5}$ **$\frac{5}{12}$** **$\frac{2}{7}$** **$\frac{5}{9}$** **$\frac{1}{8}$** **$\frac{5}{11}$** **$\frac{3}{4}$** **$\frac{7}{10}$**

Copy and work the following:

3. $5\frac{1}{3} + 3\frac{5}{6} = 9\frac{1}{6}$	**4.** $7\frac{3}{4} - 3\frac{1}{8} = 4\frac{5}{8}$	**5.** $7\frac{5}{8} + 3\frac{1}{2} = 11\frac{1}{8}$	**6.** $26\frac{7}{8} - 8\frac{1}{2} = 18\frac{3}{8}$
7. $5\frac{1}{4} + 3\frac{1}{2} = 8\frac{3}{4}$	**8.** $7\frac{1}{6} - 4\frac{1}{2} = 2\frac{2}{3}$	**9.** $32\frac{1}{10} - 9\frac{5}{10} = 22\frac{3}{5}$	**10.** $45\frac{1}{4} + 6\frac{5}{12} = 51\frac{2}{3}$
11. $6\frac{1}{5} + 9\frac{1}{3} = 15\frac{8}{15}$	**12.** $12\frac{3}{8} + 6\frac{3}{4} = 19\frac{1}{8}$	**13.** $15\frac{1}{4} - 3\frac{2}{3} = 11\frac{7}{12}$	**14.** $14\frac{1}{6} - 4\frac{3}{4} = 9\frac{5}{12}$

Alternate Computation Test, Chapter 4

1. 500 − 283 = **217**	**2.** 675 − 168 = **507**	**3.** 4,063 − 1,185 = **2,878**	**4.** 286, 405, 743, 890, + 58 = **2,382**	**5.** 545, 423, 367, 520, + 406 = **2,261**
6. 274 × 26 = **7,124**	**7.** 865 × 400 = **346,000**	**8.** 3,358 × 2,060 = **6,917,480**		

9. a. 56)248 = **4, R24** **b.** 24)212 = **8, R20** **c.** 58)2,942 = **50, R42**

d. 34)2,848 = **83, R26** **e.** 14)6,524 = **466**

	a	b	c	d	e
10.	$3\frac{1}{2} + 5\frac{1}{4} = 8\frac{3}{4}$	$3\frac{1}{2} + \frac{5}{8} = 4\frac{1}{8}$	$5\frac{3}{4} + 3\frac{1}{3} = 9\frac{1}{12}$	$8\frac{5}{8} - 3\frac{2}{3} = 4\frac{23}{24}$	$7\frac{1}{8} - 4\frac{5}{8} = 2\frac{1}{2}$
11.	$5\frac{4}{7} + 6\frac{3}{7} = 12$	$\frac{3}{4} + 5\frac{3}{8} = 6\frac{1}{8}$	$5 - 2\frac{1}{3} = 2\frac{2}{3}$	$6\frac{1}{2} - 4\frac{2}{3} = 1\frac{5}{6}$	$7\frac{1}{2} - \frac{5}{8} = 6\frac{7}{8}$

12.

$7\frac{2}{3}$	$8\frac{3}{5}$	$14\frac{4}{5}$	$9\frac{3}{4}$	$3\frac{1}{3}$
$\times 4$	$+ 7$	$\times 3$	$\times 5$	$\times 6$
$30\frac{2}{3}$	$60\frac{1}{5}$	$44\frac{2}{5}$	$48\frac{3}{4}$	20

13.

27	15	21	26	500
$\times 3\frac{1}{3}$	$\times 4\frac{3}{5}$	$\times 2\frac{2}{3}$	$\times 3\frac{1}{2}$	$\times 1\frac{3}{5}$
90	69	56	91	800

14. **a.** $4 \times 3\frac{7}{8}$ $15\frac{1}{2}$ **b.** $3\frac{2}{3} \times \frac{1}{2}$ $1\frac{5}{6}$ **c.** $5\frac{1}{6} \times 6$ 31
d. $6 \times 3\frac{2}{5}$ $20\frac{2}{5}$ **e.** $7 \times 2\frac{1}{3}$ $16\frac{1}{3}$

15. **a.** $\frac{3}{5} \times 2\frac{1}{3}$ $1\frac{2}{5}$ **b.** $\frac{7}{8} \times \frac{4}{5}$ $\frac{7}{10}$ **c.** $3\frac{3}{4} \times 1\frac{2}{3}$ $6\frac{1}{4}$
d. $\frac{1}{9} \times \frac{3}{5}$ $\frac{1}{15}$ **e.** $\frac{3}{5} \times \frac{1}{6}$ $\frac{1}{10}$

Alternate Computation Test, Chapter 5

Work the examples in rows 1 and 2 by using common denominators.

1. **a.** $4 \div \frac{3}{5}$ $6\frac{2}{3}$ **b.** $3 \div 2\frac{1}{3}$ $1\frac{2}{7}$ **c.** $2 \div 3\frac{1}{2}$ $\frac{4}{7}$
d. $7 \div \frac{2}{3}$ $10\frac{1}{2}$ **e.** $8 \div 3\frac{3}{4}$ $2\frac{2}{15}$

2. **a.** $\frac{3}{5} \div \frac{4}{15}$ $2\frac{1}{4}$ **b.** $\frac{5}{9} \div \frac{7}{12}$ $\frac{20}{21}$ **c.** $4\frac{1}{5} \div \frac{1}{3}$ $12\frac{3}{5}$
d. $\frac{7}{10} \div \frac{3}{5}$ $1\frac{1}{6}$ **e.** $1\frac{3}{5} \div \frac{2}{5}$ 4

For the examples below, divide by inverting the divisor and multiplying.

3. **a.** $\frac{3}{5} \div \frac{5}{6}$ $\frac{18}{25}$ **b.** $\frac{5}{8} \div \frac{1}{4}$ $2\frac{1}{2}$ **c.** $\frac{4}{5} \div \frac{4}{15}$ 3
d. $\frac{3}{10} \div 1\frac{1}{5}$ $\frac{1}{4}$ **e.** $\frac{5}{9} \div 2\frac{1}{3}$ $\frac{5}{21}$

4. **a.** $6\frac{2}{3} \div \frac{3}{5}$ $11\frac{1}{9}$ **b.** $4\frac{2}{3} \div \frac{7}{9}$ 6 **c.** $7\frac{1}{2} \div 1\frac{1}{2}$ 5
d. $4\frac{2}{5} \div 5\frac{1}{5}$ $\frac{11}{13}$ **e.** $8\frac{1}{2} \div 4\frac{1}{4}$ 2

Check your quotients for row 3 by multiplying the divisor by the quotient.

Alternate Computation Test, Chapter 6

1.	**2.**	**3.**	**4.**
23.5	0.67	0.7	94
$+ 6.3$	$- 0.38$	$\times 6$	$\times 0.4$
29.8	0.29	4.2	37.6

5.	**6.**	**7.**	**8.**
7.005	76.5	37.8	0.36
$- 2.269$	$+ 7.84$	$- 6.935$	$\times 5$
4.736	84.34	30.865	1.80

9.	**10.**	**11.**	**12.**
2.315	84.75	6.8	7.2
$\times 6$	$\times 2.5$	$\times 4.6$	$\times 0.215$
13.890	211.875	31.28	1.548

13.	**14.**	**15.**	**16.**
76.9	6.78	27.3	10.04
$\times 3.18$	$\times 0.7$	$\times 0.005$	$- 7.46$
244.542	4.746	0.1365	2.58

17.	**18.**	**19.**	**20.**
53.06	0.07	30.6	6
$+ 7.954$	$\times 0.4$	$+ 8.75$	$- 0.076$
61.014	0.028	39.35	5.924

Alternate Computation Test, Chapter 7

Divide. Round uneven quotients to nearest hundredth.

1. $16\overline{)42}$ → 2.63	**2.** $7\overline{)6}$ → 0.86	**3.** $6\overline{)1.87}$ → 0.31
4. $0.9\overline{)376}$ → 417.78	**5.** $21\overline{)68}$ → 3.24	**6.** $13\overline{)5.33}$ → 0.41
7. $35\overline{)89.37}$ → 2.55	**8.** $6.4\overline{)1.184}$ → 0.19	**9.** $0.4\overline{)0.032}$ → 0.08
10. $5.3\overline{)758}$ → 143.02	**11.** $0.5\overline{)176.95}$ → 353.9	**12.** $7.4\overline{)21.841}$ → 2.95

Term Tests

These term tests consist of three parts, each part designed to test learning outcomes of a different kind. Part I evaluates understanding of important terms, concepts, and relationships; Part II, problem-solving ability; and Part III, computational skill. As they are set up, the three parts are intended to be given at three sittings.

The entire test should be mimeographed or hectographed so that each child may have his own set of questions.

If you wish to keep the set of tests for use with other classes, you may change the directions for Part I and ask pupils to write answers on a sheet of lined paper on which they have numbered from 1–40. For Part I, Ex. 1–20, direct pupils to write "yes" or "no" for each question. For Ex. 21–30, direct pupils to write a, b, or c. For Ex. 31–40, direct pupils to write answers to questions asked. For Parts II and III, pupils show their work for examples and problems in the usual way.

In giving directions, stress the importance of accuracy and allow enough time for all but the slowest 15–20 per cent to finish. If you wish measures of rate of work, you can easily procure them by entering the time taken for each part of the test on each pupil's paper as he hands it to you. Or, you may be satisfied merely to know the rank of each child in rate of work. If so, you need only write consecutive numbers on the answer or work sheets to record the order in which they are returned to you.

NOTE: If your class is not up to standard in reading ability, do not hesitate to read the items in Parts I and II to the children, pausing after each item to allow time for work. Be careful that by inflection or stressing words here and there you do not give your pupils clues to correct answers (Part I) or correct procedure (Part II).

Term Test for the First Portion of Grade 6

(Chapters 1–4 in the text)

PART I

Draw a circle around **Y** for "yes" or **N** for "no" to show your answers, as has been done for you in the sample.

SAMPLE. *Are ten dimes and five nickels equal to $1.50?* Y (N)

1. Does $\frac{1}{2} \times \frac{1}{2}$ equal 1? Y (N)
2. Does the multiplier tell the size of the groups? Y (N)
3. Are $\frac{4}{8}$, $\frac{9}{15}$, and $\frac{8}{5}$ in best form? Y (N)
4. Is LIX larger than LVIII? (Y) N
5. Does an even number times any whole number always give an even product? (Y) N
6. Does $7 + 9 + 15$ equal $15 + 7 + 9$? (Y) N
7. Is 3 miles more than 16,000 feet? Y (N)
8. To check subtraction, can we subtract the minuend from the remainder? Y (N)
9. In $4 \times 2\frac{1}{2}$, is the multiplicand a whole number? Y (N)
10. In 15×475, are there two partial products? (Y) N
11. Is the average of 7, 10, and 4 an even number? Y (N)
12. Are measuring numbers more accurate than counting numbers? Y (N)
13. Are $\frac{2}{3}$, $\frac{1}{5}$, and $\frac{7}{8}$ examples of denominate numbers? Y (N)
14. Is the area of a rectangle always measured in square units? (Y) N
15. Is $\frac{2}{3} = \frac{8}{12}$ an example of the Golden Rule of Fractions? (Y) N
16. Is 1492 in the 14th century? Y (N)
17. Is the smallest common denominator of $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{6}$ an even number? (Y) N
18. Does rounding $5\frac{1}{2}$, $2\frac{8}{17}$, and $3\frac{5}{8}$ produce three even numbers? (Y) N
19. When multiplying by numbers less than 1, is the product always smaller than the multiplicand? (Y) N
20. Does the Golden Rule of Fractions apply in reducing fractions and using cancellation? (Y) N

For Ex. 21–30, three answers are given for each question. Circle the correct answer as shown in the sample.

SAMPLE: *What is the perimeter of a rectangle 12 ft. × 15 ft.:*

a. *180 ft.* **b.** *27 ft.* **c.** *54 ft.*

21. Which example would you work to find the value of n in $275 + n = 750$?

a. $\begin{array}{r} 750 \\ +\ 275 \end{array}$ **b.** $\begin{array}{r} 750 \\ \times\ 275 \end{array}$ **c.** $\begin{array}{r} 750 \\ -\ 275 \end{array}$

22. Which number below correctly shows 2,950 rounded to hundreds?

a. 2,900 **b.** 3,000 **c.** 2,800

23. Which Roman numeral below is read 1956?

a. MCMLVI **b.** MDMLVI **c.** MCMDVI

24. Which example below would you work to find the number of pints in 5 gallons.

a. 4×5 **b.** 8×5 **c.** $5 \div 4$

25. Which year below is in the nineteenth century?

a. 1910 **b.** 1730 **c.** 1805

26. What is the smallest common denominator for the fractions $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{5}{6}$?

a. 15 **b.** 30 **c.** 3

27. Which example below is an application of the Golden Rule of Fractions?

a. $\frac{1}{2} = \frac{5}{10}$ **b.** $\frac{3}{5} = 0.6$ **c.** $\frac{3}{5} \div \frac{5}{3}$

28. If $\frac{1}{12}$ of a class of 36 pupils had all the examples correct in a mathematics test, how many pupils made at least one mistake?

a. 33 **b.** 3 **c.** 24

29. Which number below is the best estimate for the product of 79×198?

a. 1,600 **b.** 14,000 **c.** 16,000

30. After spending $\frac{3}{5}$ of his allowance, Bill had $1.00 left. What is Bill's allowance?

a. $4.00 **b.** $1.50 **c.** $2.50

In Ex. 31–40, follow the directions given.

31. Study the relationship below and write the next three numbers.

1, 2, 4, 8, 16, 32

32. Round each of the following numbers to the nearest hundred:

a. 150 200 **b.** 12,449 12,400 **c.** 999,999 1,000,000

33. Write the missing numbers.

a. 1 bu. = 4 pk.
b. 1 decade = 10 yr.
c. 1 sq. ft. = 144 sq. in.
d. 1 hr. = 3,600 sec.

34. Change each of the following to best form:

a. $\frac{5}{5} = 1$ **b.** $\frac{9}{12} = \frac{3}{4}$ **c.** $\frac{31}{6} = 5\frac{1}{6}$ **d.** $4\frac{9}{5} = 5\frac{4}{5}$

35. Find the smallest common denominator for each group of fractions.

a. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ 6 **b.** $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}$ 12 **c.** $\frac{5}{8}, \frac{3}{4}, \frac{1}{2}$ 8

36. Use the Golden Rule of Fractions to change each of the following to twelfths.

a. $\frac{5}{6} = \frac{10}{12}$ **b.** $1 = \frac{12}{12}$ **c.** $1\frac{1}{2} = \frac{18}{12}$ **d.** $\frac{2}{3} = \frac{8}{12}$

37. Use the example at the right to solve the following examples.

a. $6 \times 425 =$ 2,550
b. $20 \times 425 =$ 8,500
c. $87{,}550 \div 206 =$ 425

$\begin{array}{r} 425 \\ \times\ 206 \\ \hline 2\,550 \\ 85\,00 \\ \hline 87{,}550 \end{array}$

38. Estimate the answer for each of the following:

a. $97 \times 506 =$ 50,000 **b.** $49\overline{)10{,}105} =$ 200
c. $495 + 107 + 1{,}402 =$ 2,000

39. Write in figures, one million ten thousand one hundred one.

1,010,101

40. Round each of the following to the nearest whole number:

a. $3\frac{1}{2}$ 4 **b.** $5\frac{7}{16}$ 5 **c.** $10\frac{2}{3}$ 11 **d.** $1\frac{3}{8}$ 1

PART II

Write your work for the following problems:

1. For his new car Mr. Griffin paid $1,250 down and $50 a month for 2 years. Find the total cost of the car. $2,450

2. Of his 75¢ weekly allowance, Tommy spends 50¢ and saves the rest. How much will he save in a year? $13

3. At $1.75 a square yard for linoleum, how much will it cost to cover a kitchen floor 12 ft. by 18 ft.? $42

4. Mary's marks in the last four mathematics tests were 75, 90, 60, and 95. What was her average mark for the four tests? 80

5. Bobby drinks 1 pt. of milk at each of three daily meals. At this rate, how many quarts will he drink in a week? $10\frac{1}{2}$ qt.

6. Mrs. Potts needed $6\frac{2}{3}$ yd. of material for a new dress. If the clerk cut it from a bolt containing $42\frac{1}{2}$ yd., how much was left? $35\frac{5}{6}$ yd.

7. For the family reunion, Mr. Price bought three turkeys weighing 18 lb. 6 oz., 15 lb. 8 oz., and 22 lb. 9 oz. What was the total weight of the turkeys? 56 lb. 7 oz.

8. A rectangular room is $14\frac{1}{2}$ ft. by $18\frac{1}{4}$ ft. How many feet of molding are needed to go around this room? **$65\frac{1}{2}$ ft.**

9. At \$1.65 a yard, how much will Mrs. Potts pay for $6\frac{2}{3}$ yd. of material? **\$11**

10. Mrs. Williams bought six $2\frac{1}{3}$ oz. tins of sardines. How much more or less is this than one pound? **1 oz. less**

Part III

Work and check these examples:

1. 5,036 − 3,479 = **1,557**

2. \$32.68 × 45 = **\$1,470.60**

3. \$137.45 + 15.26 + 76.19 + 21.38 = **\$250.28**

4. \$525.67 − 273.85 = **\$251.82**

5. 5,070 × 3,005 = **15,235,350**

6. $9\overline{)\$45.36}$ = **\$5.04**

7. $105\overline{)10,710}$ = **102**

8. 5 oz.)4 lb. 1 oz. = **13 oz.**

9. 6 ft. 9 in. × 5 = **33 ft. 9 in.**

10. 8 hr. 15 min. − 5 hr. 45 min. = **2 hr. 30 min., or $2\frac{1}{2}$ hr.**

11. 7,645 + 574 + 3,628 + 5,631 = **17,478**

12. $\frac{1}{5}$ of 2 yr. 11 mo. = **7 mo.**

13. The sum of $7\frac{1}{2}$ and $3\frac{3}{5}$ is **$11\frac{1}{10}$**

14. \$41.85 divided by 27 is **\$1.55**

15. $7\frac{5}{6}$ plus $8\frac{3}{4}$ minus $2\frac{1}{2}$ is **$14\frac{1}{12}$**

16. Multiply 450 by $5\frac{1}{9}$ **2,300**

17. Find n in $6\frac{2}{3} \times 3\frac{3}{4} = n$ **25**

18. Find twice the sum of $5\frac{1}{2}$ plus $3\frac{2}{3}$ **$18\frac{1}{3}$**

19. $2\frac{4}{5} \times \frac{4}{7} \times \frac{5}{8} =$ **1**

20. $3\frac{1}{4} + 5\frac{2}{3} + 8\frac{1}{6} + 1\frac{1}{2} =$ **$18\frac{7}{12}$**

Term Test for the Second Portion of Grade 6

(Chapters 5–7 in the text)

Part I

Draw a circle around **Y** for "yes" or **N** for "no" as in the sample.

Sample. *Are 6 pt. more than 1 gal.:* Y (N)

1. Do the bars in a vertical bar graph run from left to right? Y (N)
2. Does $4 \div \frac{2}{5}$ equal $20 \div 2$? (Y) N
3. Will $\frac{2}{3} \times 450$ give a larger product than 0.75×450? Y (N)
4. If Jim is 3 years older than Bob, is the ratio of his age to Bob's age 3 to 1? Y (N)
5. Is the quotient of $3 \div 15$ a what-part-of number? (Y) N
6. In $\frac{2}{3} \div \frac{3}{8}$, do we invert the $\frac{2}{3}$ to solve? Y (N)
7. If 2 lb. of candy is divided equally among 8 children, will each child get 4 oz.? (Y) N
8. Is the quotient of $1 \div \frac{1}{2}$ less than 1? Y (N)
9. Is the sum of 9 and 15 four times the difference? (Y) N
10. To find $\frac{1}{3}$ of 6, can we write $\frac{1}{3} \div 6 = \frac{1}{3} \times \frac{1}{6}$? Y (N)
11. Using a scale of 1 in. = 1,000 ft., would a line $10\frac{1}{2}$ in. long represent a distance of approximately 2 mi.? (Y) N
12. Are line graphs usually better suited for showing a change than picture graphs? (Y) N
13. In $4\frac{1}{2} \div \frac{3}{5}$, is the dividend a mixed number? (Y) N
14. Are the hundreds and hundredths numbers the same in 327.623? Y (N)
15. Are $\frac{1}{100}$ and 0.001 equal in value? Y (N)
16. Is 0.375 larger than $\frac{1}{3}$? (Y) N
17. In multiplying $1,000 \times 2.5$, do we move the decimal point three places to the left? Y (N)
18. Is the product of 0.1×0.1 larger than the sum of 0.1 and 0.1? Y (N)
19. If Bob has delivered 40 of his 75 papers, has he delivered more than 0.5 of them? (Y) N
20. Is 1,001 correctly read one thousand and one? Y (N)

For Ex. 21–30, three answers are given for each question. Circle the correct answer as shown in the sample.

Sample. *Which combination below will produce the largest product?*

a. [*1.05 × 6.37*] b. *0.927 × 0.825* c. *35.2 × 0.095*

21. In solving division-of-fraction examples by inversion, we invert the:
a. dividend b. [divisor] c. quotient

22. Which method below will *not* correctly solve the example $3\frac{1}{2} \div 1\frac{1}{6}$?
a. [$\frac{2}{7} \times \frac{7}{6}$] b. $\frac{21}{6} \div \frac{7}{6}$ c. $\frac{7}{2} \times \frac{6}{7}$

23. Which example will produce the smallest answer?
a. $650 \times \frac{2}{3}$ b. [650×0.375] c. 650×1.001

24. 0.00725 rounded to thousandths is:
a. [0.007] b. 0.0073 c. 0.0003

25. Which of the following fractions does not have an exact decimal equivalent?

a. $\frac{7}{8}$ **b.** $\frac{2}{3}$ **c.** $\frac{3}{5}$

26. Which example would you use to estimate the product of 4.8×8.095?

a. 4×9 **b.** 5×8 **c.** 4×8

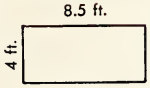
27. In which of the following years did February have 29 days?

a. 1775 **b.** 1931 **c.** 1952

28. Which example below will produce the largest answer?

a. $475 \div \frac{1}{2}$ **b.** $475 \div \frac{7}{8}$ **c.** $475 \div 0.05$

29. What is the area of the rectangle at the right?



a. 25 sq. ft. **b.** 34 sq. ft.
c. 12.5 sq. ft.

30. In which of the following will the product be larger than the multiplicand?

a. $\frac{7}{8} \times 325$ **b.** $2\frac{1}{2} \times \frac{2}{3}$ **c.** $\frac{1}{10} \times 450$

In Ex. 31–40, follow the directions given.

31. Insert the missing numbers below.

$\frac{3}{4}$, $1\frac{1}{2}$, $2\frac{1}{4}$, 3, $3\frac{3}{4}$, $4\frac{1}{2}$

32. Write the missing numerator or denominator.

$\frac{2}{3} = \frac{8}{12} = \frac{24}{36} = \frac{18}{27}$

33. Make the largest possible number by rearranging the digits in 2,075,369. 9,765,320

34. How many pint containers can be filled from a 5-gallon jug of coffee. 40

35. Multiply each of the following numbers by 1,000:

a. 2.5 2,500 **b.** 0.0672 67.2 **c.** 0.0001 0.1

36. Write decimal equivalents for the following:

a. $\frac{7}{8} = 0.875$ **b.** $3\frac{3}{5} = 3.6$ **c.** $\frac{2}{3} = 0.66\frac{2}{3}$

37. Round the number 0.0527 first to tenths, then to hundredths, and then to thousandths.

a. 0.1 **b.** 0.05 **c.** 0.053

38. Write the number which is 1 hundredth less than 2.175. 2.165

39. Write the fraction which expresses the ratio of perimeter to area for a rectangle 8 in. and 4 in. wide. $\frac{3}{4}$

40. Insert the three missing digits in the dividend.

$$\begin{array}{r} 24 \\ 32\overline{)768} \\ \underline{64} \\ 128 \\ \underline{128} \\ 0 \end{array}$$

Part II

Write your work for the following:

1. Mrs. Thomas went on a diet and lost $12\frac{1}{2}$ lb. in 5 weeks. What was her average loss per week? $2\frac{1}{2}$ lb.

2. Bill delivers his papers in $\frac{3}{4}$ hr. and Tom delivers the same number in only $\frac{1}{2}$ hr. Tom's time is what part of Bill's time? $\frac{2}{3}$

3. Mr. Quinn bought a 16 lb. turkey at $0.55 a pound. If he paid for it with a ten-dollar bill, how much change did he receive? $1.20

4. If gasoline costs $0.26 a gallon, how much will it cost to fill a tank which holds 14.5 gallons? $3.77

5. In 435 times at bat, Ted Williams hit safely 156 times. Find his batting average correct to thousandths. (Divide number of hits by times at bat.) 0.359

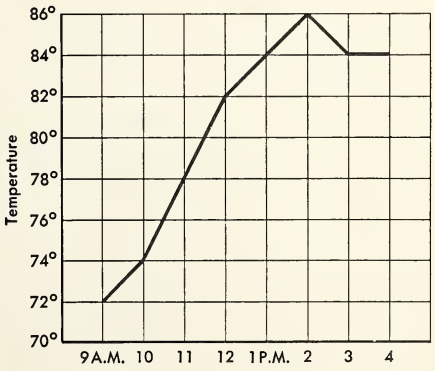
6. Mr. Forest, a salesman, travels about 22,500 miles a year. If he averages 16.5 miles on one gallon of gas, how many gallons (to the nearest whole gallon) will he use in a year? 1,364 gal.

7. Jimmy has read 225 of the 500 pages in his history book. What fractional part of the book is left for him to read? $\frac{11}{20}$

8. A gallon of sap makes 0.20 lb. of maple sugar. How many gallons of sap will it take to make 100 lb. of sugar? 500 gal.

9. To raise money for the Community Chest, the local Scout troops presented a minstrel show. The show was attended by 135 adults at 50 cents each and 100 children at 25 cents each. If expenses amounted to $22.75, how much was raised for the fund? $69.75

10. Using the graph below, find the average temperature between the hours of 9 and 4. $80\frac{1}{2}°$



Outside Temperature between 9 A.M. and 4 P.M. in One Town

Part III

Work and check these examples:

1. $6\frac{7}{8} \div 2\frac{3}{4}$ $2\frac{1}{2}$

2. $3\frac{1}{3} \div \frac{5}{6}$ 4

3. $15 \div 1\frac{1}{4}$ 12

4.
302.600
− 6.375
296.225

5.
47.2
× 0.005
0.2360

6.
50.04
× 30.07
1,504.7028

7. $0.05\overline{)0.875}$ 17.5

8. $35\overline{)1.497}$ 0.042, R27

9. $0.7\overline{)49{,}014}$ 70,020

10. $3\frac{3}{8} \times \frac{4}{9} \times 4\frac{1}{2}$ $6\frac{3}{4}$

11. 1.076 plus 15.4 minus 6.28 = 10.196

12. Divide 0.72 by 90 0.008

13. 1 divided by $3\frac{1}{2} = \frac{2}{7}$

14. From $25\frac{1}{4}$ take $4\frac{7}{8}$ $20\frac{3}{8}$

15. Find the difference between these two numbers: 0.735 and 0.658 0.077

16. $3\frac{3}{4} + 7\frac{1}{8} + 2\frac{5}{12} = 13\frac{7}{24}$

17. Find the product of 92.5 and 20.7 1,914.75

18. Find the average of $20.50, $17.28, and $19.04 $18.94

19. $69.01 ÷ $1.03 = $67.00

20. $\frac{1}{8}$ of 2 hr. 16 min. + $\frac{1}{3}$ of 1 hr. 27 min. = 46 min.

Instructional Aids

Easily Produced Teaching Aids

Modern teaching techniques call for many teaching aids. Of prime importance are those which pupils can make themselves and keep at their desk for independent use. Invaluable, too, are devices which the teacher can use to demonstrate or clarify meanings but which permit pupil participation and discovery. Not to be discounted, either, are games, which, if properly used, can offer real assistance by providing excellent motivation and practice.

This section comes in four parts and is designed to help the teacher by (*a*) mentioning the kinds of visual materials children might construct and keep in an **arithmetic portfolio**; (*b*) describing how certain **devices** may be simply and inexpensively constructed; (*c*) giving the basic ideas and procedures for several **games** that may be used for practice after meanings have been firmly established; (*d*) suggesting **equipment** that may be useful in teaching and enriching arithmetic.

On the pages which follow you will find many suggestions intended to help you develop and use such learning aids. While you will very likely find these pages full of useful ideas, remember that the learning aids that arise in the everyday experiences of your pupils are just as useful and should not be overlooked.

Arithmetic Portfolios

It is generally agreed that one of the most important ways to promote learning in arithmetic is to let pupils construct learning aids for their own use. To encourage activities of this kind, let each pupil build and maintain an "Arithmetic Portfolio" in which all of the charts, tables, devices, newspaper and catalogue clippings, etc., that are made or collected by pupils during the year may be stored. The portfolio might be a folder, plastic bag, box, or other suitable container.

Undoubtedly the contents of the portfolios will vary from class to class, and even from pupil to pupil. Many suggestions for materials which might be included have been made in the teaching lessons on previous pages in this Manual, and a dynamic arithmetic program cannot help but produce a host of other possibilities.

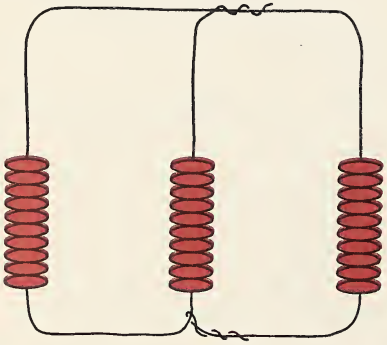
Brief descriptions of some of the various kinds of materials which might be placed in the portfolios are given at the right with page numbers that refer to the occurrences of the suggestions on the preceding pages. Many other suggestions given in Part III of this Manual may be used to devise additional portfolio material.

Portfolio Material	Manual Page
a. Place-value charts	25
b. Tables of local statistics (population, etc.) to help pupils with estimates and rounded numbers.	27–28
c. Roman numerals and Hindu-Arabic equivalents	29
d. Charts showing different types of addition facts, including helpers for each type	31–32
e. Number lines	33–34
f. Examples of measurement division and fractional-part division	69–72
g. Study cards	75
h. Charts for reference measures and standard measures	96
i. Chart showing how some measures of time are related to movements of moon, earth, etc.	106
j. Materials for work with fractions (circles, squares, etc.)	119
k. Fraction-chart on page 102 of text and fraction-strips showing common denominators	121
l. Floor plan of home or room drawn to scale	177
m. Picture graphs based on local statistics	180
n. Graphs (bar, line, or picture) from magazines and papers	184
o. Completed copy of chart on text page 261	271

Devices

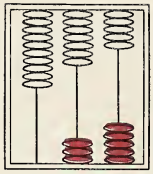
Throughout the Manual mention has been made of devices which may be employed to bring out or clarify meanings. While for the most part these aids may be purchased from commercial sources, almost all of them may be constructed from common and inexpensive materials. You will find below brief instructions for making and using several of the most helpful devices. Consider these as suggestions only, for you may find better methods for constructing them, and you will certainly think of other good ways to employ each device since only one or two of the many possible uses are given.

Abacus. On page 80 in *The Teaching of Arithmetic* (see reference on page 323 of the Manual) Spitzer gives directions for constructing a simple abacus. His plans call for bending a 36-inch length of wire, such as a coat hanger, into the shape and size shown in the diagram below. Beads (see page 321 of the Manual for a possible supply source) should be strung on the wire during the bending process. A half inch of wire must be left at each end in order to secure the frame. Since this abacus is simple to make, you might want to consider making several of them.



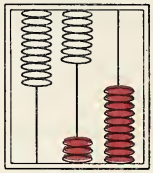
The diagrams in the next two columns, with the accompanying explanations, describe the procedures used on the abacus for adding $35 + 16$ and also for subtracting $36 - 9$. These specific illustrations should help you in adding and subtracting on the abacus with other types of examples.

Multiplication and division are also possible on the abacus, but for multipliers and divisors of more than 1 digit, the procedure becomes quite complicated. Use simple examples and demonstrate solely for the purpose of showing how multiplication is repeated addition of the multiplicand, and how division is repeated subtraction of the divisor.

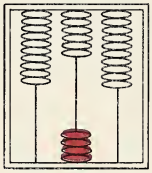


a. Addition: $\begin{array}{r} 35 \\ +\ 16 \\ \hline \end{array}$

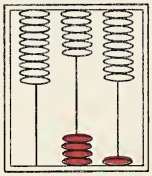
STEP 1. To show the 35 to be added, count and bring down 5 beads on the one's rod and 3 beads on the ten's rod.



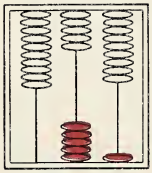
STEP 2. Start to bring down 6 more beads (for the 6 ones in 16), counting "1-2-3-4-5" (there is no bead for counting "6"). Remember the 1 bead not counted.



STEP 3. Move the 10 ones up to the top. Move 1 bead down on ten's rod to stand for the 10 ones moved up.

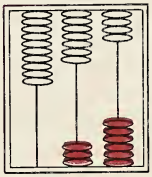


STEP 4. Now bring down on one's rod the 1 one remembered from Step 2.



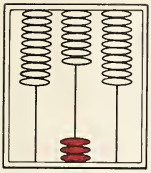
STEP 5. Move one more ten down for the 1 ten in 16.

Answer: $35 + 16 = 5$ tens and 1 one, or 51.

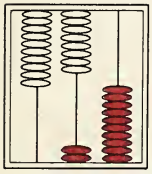


b. Subtraction: $\begin{array}{r} 36 \\ -\ 9 \\ \hline \end{array}$

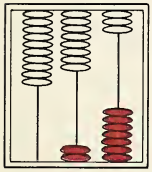
STEP 1. To show the minuend, 36, count and bring down 3 beads on the ten's rod and 6 beads on the one's rod.



STEP 2. To subtract, move beads up counting "1-2-3-4-5-6." Since there are not 9 beads in one's column, you will need more ones.



STEP 3. Move 10 ones down. Move 1 bead up on ten's rod to stand for the 10 ones moved down.



STEP 4. Continue to count ones, moving beads up, "-7-8-9."

Answer: 36 − 9 = 2 tens and 7 ones, or 27.

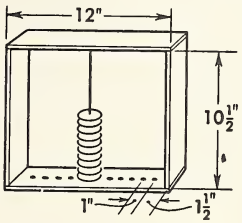
Area Squares. Have cut from oak tag about two hundred inch-squares, ten or so foot-squares, and one or two squares to represent a square yard. At least one of the square-foot pieces should be lined into its 144 square inches, and the back of one of the square-yard cards should be marked into its 9 square feet. For convenience, the latter item ought to be made to fold.

Pupils may use these squares for discovering concretely the area of various surfaces. For example, a sheet of paper $8\frac{1}{2}''$ by 11″ when compared with a square-foot card is seen to be smaller than a square foot. Eleven rows of eight inch-squares each may then be used to cover the paper's surface. Pupils will see quickly that one half of a square inch will be needed to fill out each row, or, to finish covering the sheet, $5\frac{1}{2}$ full square-inch cards. Relying on your pupils' understanding of multiplication as a quick way to add equal groups, the formula for area (*length* times *width* equals *area*) may be developed.

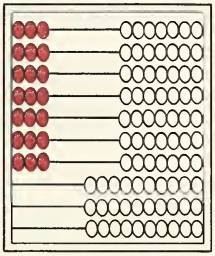
In much the same way, square-inch cards may be used to cover a square-foot card in leading to the idea that one square foot contains 144 square inches. Other such relationships between area measures should be developed in the same way.

Bead Board. Use $\frac{3}{4}''$ stock that is 2″ wide. From it cut 2 pieces 12″ in length and 2 pieces $10\frac{1}{2}''$ in length. From discarded coat hangers, cut 10 straight pieces of wire 11″ in length. One hundred half-inch beads will also be required (see Manual, page 321, for a possible supply source).

Draw a center line on the face of each 12″ piece of wood. One and one-half inches in from the end drill a hole $\frac{1}{4}''$ deep, using a drill slightly larger than the diameter of the wire. From that point on, drill 9 similar holes at 1″ intervals.



Assemble the frame as shown in the diagram. String the beads ten to a wire and insert the wires by placing one end in the bottom hole and bending the wire enough to insert the other end at the top. If desired, 1″ beads may be used to make a larger model.



Since the Bead Board has ten groups of 10 beads each, it can be thought of as representing the numbers from 1 to 100. It can, therefore, be used for a variety of activities. Addition, subtraction, multiplication and division of 1- and 2-place numbers; column addition; adding by endings; and many other procedures may be demonstrated with the board. It can also be used as an abacus. The schematic drawing above shows how $7 \times 3 = 21$ is set up on the Bead Board so that the pupil can see that multiplication is repeated addition of equal groups.

If the hundred beads are thought of as one whole, each one of the beads may be considered to

stand for $\frac{1}{100}$ of the whole. Similarly, each strand of 10 beads would represent $\frac{1}{10}$ of the whole. Hence, work with decimals and per cent is possible. For example, pupils will be able to find $\frac{1}{2}$ of all the beads easily and they will quickly see the 50 as representing $\frac{50}{100}$, or 0.50. When two or more bead boards are used in conjunction with one another, decimals greater than one whole may be handled.

Bead Line. This is a simple device consisting of 100 or more beads (see Manual, page 321, for a supply source) strung together on wire or cord and stretched between two points in the classroom. The Bead Line is in effect a number line.

Simple addition, subtraction, multiplication, and division meanings and examples may be demonstrated. For $43 \div 8$, draw 43 beads over to the left side of the wire. Then show how 5 groups of 8 beads each could be moved one at a time back to the right side of the wire. The 3 remaining beads would make the quotient 5, R3.

For fraction work, $\frac{2}{3}$ of a group of 15 can be demonstrated by finding $\frac{1}{3}$ of 15 beads; then the number in 2 of these groups of five. In a manner similar to the use of the Bead Board described above, the 100 beads may be considered a whole, each bead representing $\frac{1}{100}$ of the whole, for work with decimals.

In general, whenever a number line can be used, the Bead Line also can be used, as easily and effectively but at a more concrete level.

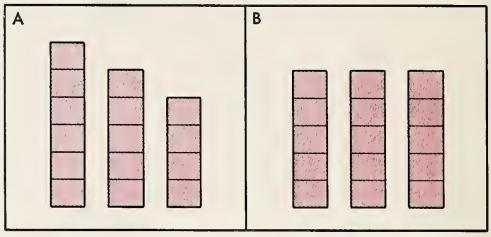
Calculating Blocks and Strips. *a.* Whole Numbers. Obtain a piece of wood 1 inch square and about 6 feet long. Measuring in from one end, cut a piece at the 1-inch line and mark it "1." Make the next piece 2 inches long and mark it "2." Continue in the same way, making each piece 1 inch longer than the previous piece and numbering consecutively to 10. It is a good idea to make several blocks for each of the numbers 1 to 10.

Oak tag or stiff cardboard strips may be used in place of wood if desired. In this case, a flat one-inch square, rather than a cube, will represent the number 1.

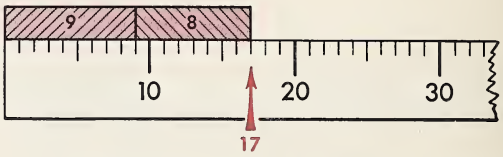
Obtain ten boxes (such as cigar or cheese boxes), label each with a number, and use them to store the blocks or strips.

These blocks or strips may be used to show some of the basic facts. For example, three 2″ blocks laid end to end will equal in length a 6-block ($3 \times 2 = 6$).

These blocks may also be used to show how finding the average is the same as finding what each number would be if all were equal. Box A below shows blocks representing 6, 5, and 4. Box B shows how the top block from the 6 pile may be placed upon the 4 pile, to make all piles equal in height. The average, then, is demonstrated to be 5.



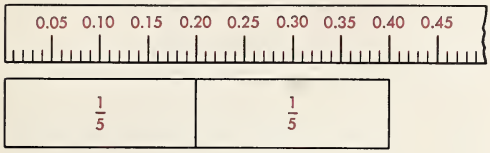
The illustration below shows how the strips may be used with a number line to show the fact $9 + 8 = 17$. If a 6 strip is added to this, the result is column addition in which the last addition requires bridging into the twenty decade.



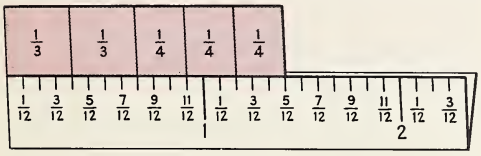
b. Common and Decimal Fractions. It is also a good idea to have on hand calculating blocks or strips to represent the fractions from $\frac{1}{2}$ through $\frac{1}{12}$. Obtain flat material such as oak tag or $\frac{1}{4}$″ plywood, and cut twelve pieces measuring 12″ by 1″. Leave one piece whole (to represent the number 1), and cut the next piece into halves, the next into thirds, continuing in this fashion until the last piece has been cut into 12 equal parts, each representing $\frac{1}{12}$. For work with mixed numbers, you will need to divide and cut two or three 12″ lengths for each different fraction. Be sure also to provide several extra full-length pieces.

Each fractional piece may be labeled with its proper fractional unit. Each $\frac{1}{10}$ piece may be turned over and itself marked off into $\frac{1}{10}$'s. When ten $\frac{1}{10}$ pieces are reversed and placed together, each mark will stand for $\frac{1}{100}$ of the whole. Hence, decimal work with these particular blocks or strips is possible. Some of the other pieces may also be marked or labeled on the back.

For example, the $\frac{1}{2}$ piece could be labeled 0.5, the $\frac{1}{8}$ piece 0.125, and so on. You may also mark off one of the uncut full-length pieces into hundredths. This may be used as a scale for measuring fractional pieces in terms of decimals as shown below (with part of a scale) for measuring $\frac{2}{5}$.



If possible, it is also advisable to make, for each set of fractions, labeled pockets similar to the one below. These could be made of folded oak tag. Notice that in order to add $\frac{2}{3}$ and $\frac{3}{4}$ below, the strips had to be placed in a pocket showing twelfths, since $\frac{1}{12}$ is the largest fractional unit that will measure both $\frac{1}{3}$ and $\frac{1}{4}$ an even number of times. This use of the fraction strips and pockets will help pupils to see the relationship between fractional units and the common denominator.

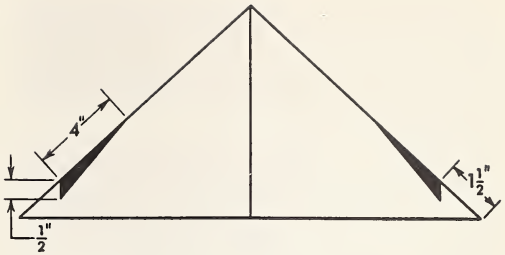


If the pockets are pinned one below the other on a bulletin board, they can be used to bring out fraction, decimal, and per-cent equivalents and at the same time the storage problem for the individual fractional pieces will be solved. It would be wise, however, to obtain and label cigar boxes, or the like, for more permanent storage.

Flannel Board. With wrong sides together, sew 2 rectangular pieces of flannel on three sides and turn right side out. Slide in a rectangular piece of cardboard slightly smaller in size and close the open end of the flannel pocket with snap fasteners.

Glue small pieces of flannel or sandpaper to cutouts of various sizes and shapes so they will adhere to the flannel board.

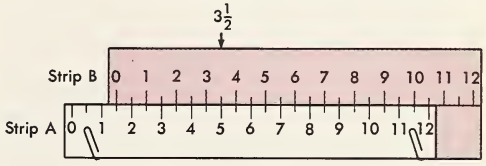
A support for the board may be made by cutting from large, heavy cardboard a piece similar to the one shown in the diagram at the top of the next column. Cut out the blackened area and fold on the middle line.



Pictured and representative objects may be manipulated on the flannel board to show the meaning of addition, subtraction, multiplication, and division, whether it be for whole numbers, fractions, or decimals.

Fraction Slide Rule. Cut oak tag into two 2″ by $12\frac{1}{2}$″ strips. Draw long center lines on each and mark off inch spaces, labeling these with the whole numbers from 1 to 12, and separating all whole-number spaces into eighths.

Fold one strip along the center line with the scale facing outward. As shown below, insert the other strip in the fold thus formed so that the scales align one atop the other. Paper clips placed as in the illustration will help hold the assembly together.



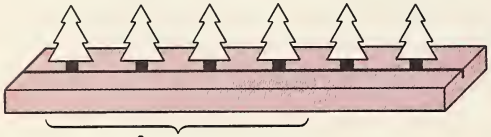
The diagram (which does not show division marks for eighths) also indicates the procedure for adding $3\frac{1}{2}$ to $1\frac{1}{2}$. Place Strip B so that 0 is over $1\frac{1}{2}$ on Strip A. Find $3\frac{1}{2}$, the other addend, on Strip B and read the sum, 5, immediately below it on Strip A. To subtract in the example $5 - 3\frac{1}{2}$, set Strip B so that $3\frac{1}{2}$ is over 5 on Strip A and read the answer, $1\frac{1}{2}$, on Strip A.

Notice that the slide rule consists of two number lines and that one adds by moving to the right and subtracts by moving to the left. Of course, decimal number lines could be used in place of the fraction lines, but it is difficult to mark and label spaces much smaller than tenths unless the whole-number spaces are enlarged considerably. A decimal slide rule held to tenths only would obviously be somewhat limited.

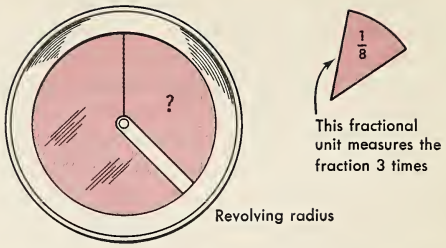
Fractional Pieces. Supplementing the material described under "Calculating Blocks and Strips," above, there should be a further supply of pieces for work with fractional parts of *groups*, whether the fractions under consideration be in common, or decimal-fraction form.

Therefore, cut 100 or so small discs from oak tag (or use 100 of any of the quantity objects listed under "Equipment" on page 319 in the Manual) and about the same number of small squares or rectangles. For display and demonstration purposes, these may be equipped for use with the Flannel Board which was described in more detail on page 313.

In addition to these representative objects, your pupils may draw and cut out pictures of objects to be used for slightly more concrete work. Such pictured objects are shown below in the n-Stand where the bracket shows that $\frac{2}{3}$ of 6 trees are 4 trees. This type of semi-concrete material may be used in a variety of ways to develop the concept of fractional parts of a group.

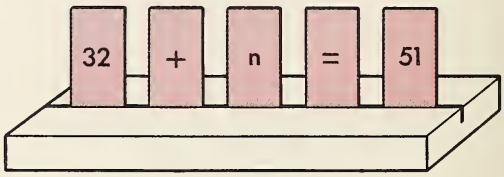


Of course, it is not always wise when working with fractional parts of a *whole* to have the whole piece in the same shape or form as the calculating blocks and strips described on page 312. Therefore, cut circular discs from oak tag, or use paper plates, and separate them into the various common fractional parts. Make some of the discs as large as pie tins which may be used for containers. A revolving radius may be attached to one of the pie tins so that various fractional parts of the whole may be generated. A piece of paper with a radius drawn should be placed in the bottom of the tin. Then, if the circular whole units from which fractional pieces have been cut are the same size as the bottom of the pie tin, they may be used for measuring the generated fractions. The procedure is illustrated at the top of the next column where, after measuring with many fractional units, it was discovered that $\frac{1}{8}$ would measure the fraction exactly 3 times, proving the fraction to be $\frac{3}{8}$ of the whole.



Cut other fractional pieces from smaller discs to make it possible to show that the fractional unit depends upon the size of the whole, i.e., $\frac{1}{2}$ of a large disc is larger than $\frac{1}{2}$ of a smaller disc.

n-Stand. This device is really no more than a display stand constructed by cutting a groove in a piece of wood. It derives its name from the fact that it is most useful for helping children realize that n can represent a missing number in such combinations as $32 + n = 51$, $n \div 18 = 4$, and $12 - n = 5$. For this teaching, small cards are needed. They should be marked with numbers and process signs, with n printed on one card. For $32 + n = 51$, the correct example, $32 + 19 = 51$, would be set up and then, as shown below, the n card inserted in front of the 19 to indicate how n can be made to represent the missing number. Then other examples containing n would be set up and children asked to explain how to work them.



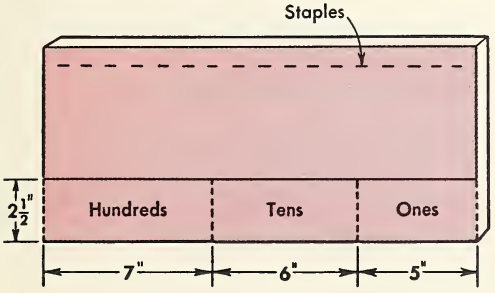
As mentioned under Fractional Pieces above, the stand may be used to display pictures of representative objects. Actually, it can be used for a wide variety of teaching situations where, for better class participation, material should be shown in an upright position.

Peg Board. Use acoustical board and cut it in such a way that there will be 10 columns of 10 holes each. For a more durable board, use a piece of Masonite Peg Board and mount it on a piece of wood for a base.

For pegs, use matchsticks or purchase ready-made pegs from commercial sources.

The Peg Board is based on the same principle (10 groups of 10) as the Bead Board, described on page 311 in the Manual, and hence may be used for similar purposes. Moreover, acoustical board may usually be obtained in quantity sufficient to make 10 Peg Boards. Here, then, is a way of representing the number 1,000, the fraction $\frac{1}{1000}$, or the decimal 0.001, along with all the other numbers that stem from these. A feeling for the size of these numbers may thereby be afforded.

Pocket Chart. *a.* Standard Pocket Chart. Use a piece of oilcloth 8″ by 18″. Folding the long way, turn up a $2\frac{1}{2}$″ flap and secure it in place by sewing those places indicated by the dotted lines in the plan below. Stapling along the upper edge,

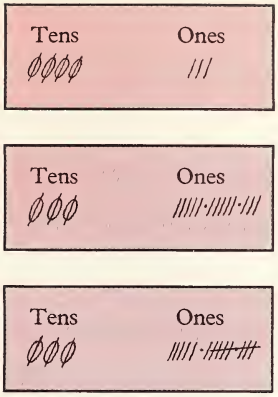


attach the oilcloth to a $5\frac{1}{2}$″ by 18″ piece of wood. Label each pocket with its name as shown below, or with the words "tenths," "hundredths," etc., so pockets may be used for decimal work.

If you wish a more durable pocket chart, design one to be made of plywood. Pocket charts are also used in the reading program, and you may find that these may be adapted for the arithmetic program. Whatever the case, be sure the pockets are large enough to accommodate bundle-numbers.

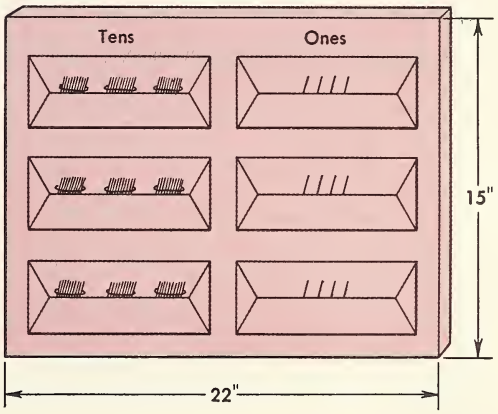
Make from oak tag about five hundred 5″ × 1″ strips. Leave 20 or 30 loose and use elastics to bundle the others in groups of ten. If desired, inexpensive colored splints may be purchased instead (see Manual, page 322, for a possible supply source).

The schematic drawing in the next column shows the three steps to be taken in working the example 43 − 7 = ? The middle step represents the borrowing of a ten and the cancel marks in the last step represent the subtracting of the 7 ones.



b. Multiplication Pocket Chart. Starting at the bottom, paste or staple 6 business-sized envelopes to a piece of 14″ by 18″ oak tag (or plywood). Label as shown in the diagram below, where 3 × 34 is demonstrated. Use the cards or splints mentioned for the Standard Pocket Chart at the left.

Multiplication with bundle-numbers can be shown with a Standard Pocket Chart, but the equality of the groups to be combined and the relationship of multiplication to addition are more easily seen when banks of pockets are used.



Sectioned Foot-Rulers and Yardsticks. Cut rulers at the one-inch, three-inch, four-inch, and six-inch points and mount the pieces on flexible tape. By folding the sectioned parts, the meaning of a foot as twelve inches and as fractional parts of a foot may be shown. Similarly, yardsticks may be cut and the pieces hinged to show the meaning of a yard as three feet.

Games

It is important that the use of games be thoughtfully considered. Almost without exception the following games provide practice only. It is immediately apparent, then, that if games are used too soon, before meanings have been well established, no more good will result than from the discredited practice of assigning continued and relentless drill on abstract facts. If adequate meanings have been established, and you wish to vary the practice, games can be invigorating. Even then, excessive or unwise use may move the point of emphasis away from arithmetic and center it on competition or some other undesired and perhaps even harmful goal.

The basic rules of play for several games that may be adapted to arithmetic are given below. The alert teacher will discover liberties which may be taken with the rules and in constructing the necessary equipment so that greater variety and use than is indicated may be obtained. You may find that some of your pupils will be interested in developing variations of the games or in devising new ones. Encourage this interest when you can.

Old Hat. *a.* Uses: The basic requirement for this game is that cards held in the hand be matched to make books of 2. Therefore, practice may be obtained on any material that can be placed on separate cards and then matched. For example, fractions could be written on one set of cards to be matched with pictures of parts of objects or groups on another set. Some other possibilities include the matching of: (*a*) Arabic numbers with Roman numerals; (*b*) numbers with $\emptyset$ -pictures; (*c*) examples with estimated answers; (*d*) problems with the labels for the answers; (*e*) addition, subtraction, multiplication, or division combinations with their answers; (*f*) pairs of facts; (*g*) addition facts with dot patterns; (*h*) simple measurement examples with labeled answers; (*i*) common and decimal fractions.

b. To play: Withdraw one card, leaving a card that cannot be matched, and deal all cards. Pupils should match cards to make books of 2 cards each. These may be placed face down on the table. Players then take turns drawing cards from one another in the hopes of completing books which can be discarded. The child who holds the odd card at the end of the game is the "Old Hat."

Single Search. *a.* Uses: The basic requirement for "Single Search" is that cards be matched to make books of 2.

The most important difference between this game and "Old Hat," above, is that the player must request (instead of draw) from an opposing player the card he needs to make a book. He must, therefore, know what the matching card would be and he must also be able to call for it by name. A few matchings that may be considered are: (*a*) pairs of facts; (*b*) abstract numbers (such as 12,500,103) and their language forms (twelve million five hundred thousand one hundred three); (*c*) abstract numbers (such as 32) and their equivalents in tens and ones (3 tens and 2 ones); (*d*) two forms for any of the basic facts (such as "5×4" and "five 4's"); (*e*) common and decimal fractions (such as "$\frac{3}{4}$" and "0.75").

Double Draw. *a.* Uses: The main difference between this game and "Single Search," above, is that books of 4 are made instead of books of 2. The difference makes the use of this game limited almost entirely to practice in recognizing and calling for the related addition and subtraction facts and the related multiplication and division facts, or for relating the various forms of common and decimal fractions, as "one fourth," "$\frac{1}{4}$," "0.25," and "$\frac{25}{100}$."

b. Preparation: Make a deck of cards by writing related facts one to a card. If related addition and subtraction facts are used, the deck would be started in the manner indicated below.

$4 + 7 = 11$	$5 + 7 = 12$	$6 + 7 = 13$
$7 + 4 = 11$	$7 + 5 = 12$	$7 + 6 = 13$
$11 - 7 = 4$	$12 - 7 = 5$	$13 - 7 = 6$
$11 - 4 = 7$	$12 - 5 = 7$	$13 - 6 = 7$

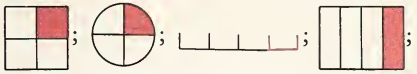
c. To play: Follow the general directions for "Single Search" above. Shuffling and dealing is the same. The difference lies in the fact that related facts are asked for and each book must contain 4 cards. Also, when the requested card is not

obtained, the pupil must draw twice. The player who first runs out of cards is the winner.

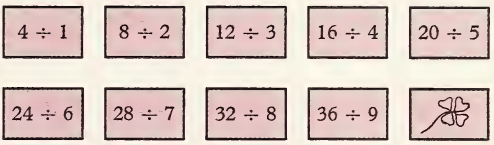
Over Orange. *a.* Uses: For this game, cards in the hand are matched to make books of 3. Therefore, practice may be obtained for any information that can be placed on 3 separate cards and then matched. Consider matchings such as: (*a*) combinations, their reverses, and answers; (*b*) simple examples, estimated answers, and exact answers; (*c*) abstract numbers, $\emptyset$-numbers, and numbers in words; (*d*) 2 forms of addition, subtraction, multiplication or division combinations, and answers (such as $81 \div 9$; $9\overline{)81}$; 9); (*e*) fractions in figures, fractions in language form, and pictures of fractions (such as $\frac{1}{4}$; one fourth;); (*f*) basic facts, their reverses, and dot patterns; (*g*) common fractions in words and figures, and decimal fractions.

b. Preparation: If the matchings in (*a*) above are used, make the deck of cards so that combinations and their reverses are on separate orange cards, and the answers are on blue cards.

c. To play: Shuffle and deal all the blue cards; then all the orange cards. At the signal "over orange," each pupil must pass one orange card to the player on his right. Two orange cards (a combination and its reversal) and the correct blue answer card make a book for which one point is received. Play is continued in this manner until a player goes out, for which an additional point is received.

Clover. *a.* Uses: The basic requirement for this game is that cards be played by following suit. This means that practice can be obtained for any information that belongs distinctively to one set of facts. For example, all higher-decade addition examples that have 7 and 5 in one's place (such as $27 + 5$; $57 + 5$; $37 + 5$) belong to the $7 + 5$ family. Sets of cards containing examples that belong to this family may, therefore, be made and used in this game. Other sets of information that may be used for practice include: (*a*) subtraction-family examples; (*b*) addition, subtraction, multiplication, or division combinations or facts (such as $4 \div 4$; $8 \div 4$; $12 \div 4$; and so on); (*c*) numbers having 3 in ten's place (431, 30, 531, 38, etc.), 0 in one's place (290, 40, 0, 680, etc.), or some such criterion or combinations of criteria; (*d*) measurement numbers that belong to one system (such as 13 oz.; 2 T.; 167 lb.; 55 T.); (*e*) problems that involve the same basic process; (*f*) pictured or written fractions that have the same name ($\frac{1}{4}$; one fourth;  ; etc.); (*g*) examples that have carrying in ten's place or one's place, or borrowing in ten's place; (*h*) decimals that have one, two, or three places.

b. Preparation: (This illustration is for division facts with suits determined by size of quotients.) Obtain 42 cards about $2'' \times 3''$. Write the division facts with quotients 4 on 9 of the cards, as shown below. The other cards might contain combinations with quotients 5, quotients 6, and quotients 7. Facts from other tables could be used, of course. On remaining cards draw "lucky clovers" similar to that shown in the illustration below.



c. To play: Deal 4 cards to each of 2 to 6 players. Place the remaining cards face down in a pile. A player starts by giving the answer to a card (such as $12 \div 3$) which he places face up on the table. If the answer is correct, the card may be left on the table. The next player must show a card of the same family (such as $32 \div 8$). If he cannot, he may play a "clover" card and change the suit by showing another of his cards. Failing both alternatives, he must draw from the center pile of cards. The first person to dispose of all cards is the winner.

Zooks. *a.* Uses: In "Zooks" the attempt is to match 2 cards according to some predetermined criterion. For example, it may be decided that pairs of facts should be matched. The cards would then contain facts and their reverses. Practice may therefore be obtained for any information that can be written on separate cards and matched according to a prescribed criterion. Consider the possibilities for variation given above under "Old Hat," and "Single Search."

b. Preparation: (This illustration is for addition facts giving a sum of 10.) Cut 36 cards about $2'' \times 3''$ and use them to make 4 sets of cards containing one each of the numbers 1 through 9.

c. To play: Deal all cards face down to 2, 3, or 4 pupils. Players should take turns showing one card at a time. When the sum of any 2 exposed cards is 10, the first pupil to call "Zooks" may collect all exposed cards. The player with the most cards wins.

Yay. *a.* Uses: Although this game may be adapted to other kinds of practice, it is probably most useful for reviewing the multiplication and division facts.

b. Preparation: Line $6'' \times 5''$ cards into 30 squares. For the multiplication facts, the top row should contain numbers designating the multiplication tables with multipliers of 2, 3, 4, 5, and 6. Write answers for each table in the appropriate columns. The center square should be marked "Free" (see the illustration below). No two cards should be alike. Write one complete set of multiplication facts on small cards. Counters or squares of paper may be used for markers.

2	3	4	5	6
8	27	16	25	54
6	12	32	15	18
10	21	Free	45	42
4	15	12	30	24
14	6	24	10	48

c. To play: A leader should draw one of the small cards and read the multiplication combination. If a player has the correct answer he should cover it with a marker. The first player to cover 5 squares horizontally, vertically, or diagonally, calls "Yay." He is the winner if, on calling back the numbers, it is found that his play was faultless. A recorder may keep track of the numbers called by putting markers on a sheet prepared like this.

2	2	4	6	8	. . .
3	3	6	9	12	. . .

Mads. *a.* Uses: The playing board for this game contains questions which must be answered by the pupil. Variation may therefore be obtained by changing the playing board. For example, the board shown below might have combinations for a single process rather than mixed combinations for the four processes. The squares might also ask children to (*a*) read abstract numbers; (*b*) tell the abstract number equivalents for $\emptyset$ -numbers; (*c*) give the correct fraction for pictures of parts of objects or groups; (*d*) give estimated answers for examples; (*e*) give a decimal equivalent for a common fraction.

b. Preparation: Use $9'' \times 12''$ construction paper and mark off 20 rectangles (or more if you desire) as shown in the diagram below. Reserve a few rectangles in which directions may be written and write multiplication, addition, division, and subtraction (MADS) combinations in the others. Make a spinner with numbers from 1 to 4. As a substitute, use small cards numbered 1 through 4. Obtain counters, or have pupils make them from oak tag, to be used as markers.

c. To play: Pupils should take turns spinning, or drawing cards, to discover the number of spaces

Start.	$6 + 7 = ?$	$11 - 5 = ?$	Forgot something. Go back to start.	$56 \div 8 = ?$
Red light. Lose 1 turn.	$3 + 8 = ?$	$6 + 7 = ?$	Downhill. Go forward 1.	Short cut. Go to $8 \times 6 = ?$
$9 \div 3 = ?$	Numberland	No gas. Go back 3.	$24 \div 3 = ?$	$9 \times 3 = ?$
Uphill. Go back 1.	$3 - 8 = ?$	$8 \times 6 = ?$	Superhighway. 1 extra turn.	$9 + 8 = ?$

to advance. If the correct answer is not given for a fact, the pupil may receive help. In this case his button stays in the square just as though he had answered correctly, but when his turn comes to advance again, he may not, until he can give the correct answer for the square missed previously. When progress stops in a square where directions are given, these should be observed. The first player to advance his marker to "Numberland" is the winner.

Equipment

Number Boxes. Candy or cigar boxes are ideal for storing items used for individual or independent study. A typical box might contain ten or more sticks (dowels, discs, used matches, etc.), calculating blocks or strips, fractional pieces, a fraction slide rule, and area squares.

Quantity Objects. Frequently large numbers of objects are needed for teacher demonstration or pupil experimentation. The following list suggests some easily obtained items that may be useful for such activities:

acorn cups
beads
blocks
buttons
cardboard strips
clothes pins
coins
discs
dowels
drinking straws
lima beans
marbles
milk-bottle caps
nails
paper clips
soda-bottle caps
splints
spools
tongue depressors
toothpicks
used matches
wooden beads

Measures. Listed in the next column are a few of the many items that are useful in enriching or making more meaningful the study of measures.

a. Capacity. Dry measure containers: quart; peck; bushel. Liquid measure containers: measuring spoons; cup; half-pint; quart; gallon. Comparing capacity: beans, sand, or rice.

b. Length: foot rule and yardstick and perhaps a rod-long stick; tape measure; cyclometer; pedometer.

c. Weight: several ounce, pound, quarter-pound, and half-pound weights in varying size so that weight, not size, can be stressed; lever and spring balances (up to 25 pounds); postal scales.

d. Surface Measure: square-inch, square-foot, and square-yard cards (see "Area Squares" in Devices section above).

e. Temperature: clinical and weather thermometer.

f. Time: calendar; clock; stop watch; timer; hour glass; sundial.

g. Scale Drawings: local and national road maps; simple blueprints and house plans; compass and/or dividers.

Miscellaneous. Since numbers can be associated with almost any activity or object, the following list must be considered merely illustrative of the many miscellaneous aids that are helpful in the classroom:

any of the items mentioned under the Devices section
calculating machine
catalogues (seed, department store, etc.)
colored paper
egg boxes
empty grocery-store boxes and cans
envelopes
graph paper
notebooks or scrapbooks
oak tag or heavy cardboard
old magazines and newspapers
paper plates
pie tins
ribbon and string for fractions
speedometer
toy money

Commercial Teaching Aids

Many excellent teaching materials are produced for sale. Some of these are given below, classified according to the nature of the item. Since prices are subject to some variation, none are given. This should be no deterrent, however, for most companies will be happy to provide such information, along with descriptive literature concerning their products.

It is sometimes assumed that a large supply of teaching materials is necessary for superior teaching, but the reverse of this condition often seems more likely. Select your arithmetic teaching equipment with care and in the light of the whole program. Consider some of the already-mentioned class-made substitutes that might be more useful and instructive. Beyond cost, appraise carefully the durability, ease of handling, storage problems, and teaching efficiency offered by each item. Remember that the simplicity of an inexpensive item may enhance its effectiveness while the wide applicability and extreme durability of some of the costlier materials may just as well make investment in them attractive and worthwhile.

By all means consider the value to you and your class derived from the purchase and use of commercial aids, but be sure to retain and develop that talent so vital to the teacher, the talent for getting the most out of the immediate environment as it stands.

Films. These carefully-selected 16 mm motion picture listings are given in alphabetical order by title. The main source of each film is given immediately following each title. Requests for further information from these producers or distributors will usually bring excellent advertising literature describing the nature and extent of each firm's film offerings including its plans for future film production.

Decimals Are Easy. Coronet Films, 65 East South Water Street, Chicago 1.

Division Is Easy. Coronet Films, 65 East South Water Street, Chicago 1.

Division of Fractions. Knowledge Builders, 625 Madison Avenue, New York 22.

Fractions Series. Johnson Hunt Productions, 6509 De Longpre Avenue, Hollywood 28. These seven titles from a series of eight: *Introduction to Fractions; How to Change Fractions; How to Add Fractions; How to Subtract Fractions; How to Multiply Fractions; How to Divide Fractions; Decimal Fractions.*

Helping Children Discover Arithmetic. Wayne University, Audio-Visual Materials Bureau, 5272 Second Avenue, Detroit 1.

How to Use Decimals. Knowledge Builders, 625 Madison Avenue, New York 22.

Language of Mathematics. Coronet Films, 65 East South Water Street, Chicago 1.

Meaning of Long Division. Encyclopaedia Britannica Films Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

Measuring Simple Areas. Knowledge Builders, 625 Madison Avenue, New York 22.

Multiplication Is Easy. Coronet Films, 65 East South Water Street, Chicago 1.

Multiplying Fractions. Knowledge Builders, 625 Madison Avenue, New York 22.

Number System, The. Encyclopaedia Britannica Films Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

Parts of Things. Young America Films Inc., 18 East Forty-first Street, New York 17.

Simple Fractions. Knowledge Builders, 625 Madison Avenue, New York 22.

We Discover Fractions. Coronet Films, 65 East South Water Street, Chicago 1.

What Are Decimals? Instructional Films Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

What Are Fractions? Instructional Films Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

Filmstrips. (Listed in alphabetical order by series title.) To permit direct communication for further and more complete information, the name and address of the filmstrip producer or distributor is given after each of the following titles:

Adventures with Numbers. Popular Science Publishing Company, Audio-Visual Division, 353 Fourth Avenue, New York 10. A series of six titles: *Making Change; Zero in Multiplication; Two Figure Divisors; Dividing a Whole Number by a Fraction; Meaning of Decimals; Dividing with Decimals.*

ARITHMETIC-KIT 1. The Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11. These three titles from a series of nine: *Multiplication and Division; Addition and Subtraction of Fractions; Multiplication and Division of Fractions.*

DECIMALS AND PERCENTAGE SERIES. Eyegate House Inc., 330 West Forty-second Street, New York 18. These six titles from a series of nine: *Introduction to Decimals; Decimals and Common Fractions; Comparing Decimals; Adding and Subtracting Decimals; Multiplying Decimals; Dividing Decimals.*

FRACTIONS. Curriculum Films Inc., 10 East Fortieth Street, New York 16. A series of nine titles: *What Is a Fraction? Fractions of a Group; How Large Is a Fraction? Writing Fractions; Mixed Numbers; Common Denominators; Adding Fractions; Multiplying Fractions by Fractions; Using Mixed Numbers.*

FRACTION SERIES. Society for Visual Education Inc., 1345 West Diversey Parkway, Chicago 14. A series of seven titles: *Meaning of Fractions; Changing the Terms of Fractions; Adding Like Fractions and Mixed Numbers; Adding Unlike Fractions and Mixed Numbers; Subtracting Unlike Fractions and Mixed Numbers; Multiplying Fractions and Mixed Numbers; Dividing Fractions and Mixed Numbers.*

HISTORY OF MEASURES. Young America Films Inc., 18 East Forty-first Street, New York 17. These five titles from a series of six: *History of Our Number System; History of Telling Time; History of the Calendar; History of Linear Measurement; History of Area Measurement.*

INTRODUCTION TO FRACTIONS. The Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11. A series of five titles: *Fractional Parts of a Whole,* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; *Fractional Parts of Groups,* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; *Fractional Parts of a Whole and Groups,* $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$; *Non-Unit Fractions of a Whole and Groups; Comparing Fractions.*

STUDY OF FRACTIONS SERIES, A. Photo and Sound Productions, 116 Natoma Street, San Francisco 5. A series of ten titles: *Units and Fractional Parts; Reducing and Changing Fractions; Multiple Fractions—Improper Fractions; Improper Fractions (cont.)—Mixed Numbers; Changing Fractions to a Common Denominator (Part 1); Changing Fractions to a Common Denominator (Part 2); Comparing Fractions—Adding and Subtracting; Multiplying Fractions; Dividing Fractions; Reciprocals—The Rule of Division.*

STUDY OF MEASUREMENT, A. Photo and Sound Productions, 116 Natoma Street, San Francisco 5. These two titles from a series of eight: *Linear Measure; Surface Measure.*

Charts and Flash Cards

BULLETIN BOARD CHARTS ON ARITHMETIC. F. A. Owen Publishing Company, Dansville, New York. Charts covering the following subjects: *Measures I; Measures II; A Time Chart; Linear Measure; Roman Numerals; Change for a Dollar; Kinds of Subtraction; Temperature; Weight; Liquid and Dry Measure; Measure by Counting; Decimals; Using Decimals I; Using Decimals II; Fraction Chart; More Fractions; Adding Fractions; Interest Problems; A Long Division Chart.*

Decimal Equivalent Chart. Brown and Sharpe Manufacturing Company, Department 43, Providence, Rhode Island. Decimal equivalents of the fractional parts of an inch. Helps emphasize the importance of arithmetic in industry. Free

Fraction Cards. Steck Company, Austin 61, Texas. Ten cards showing $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ pictured as squares, circles, and triangles.

Group Recognition Cards. Ginn and Company, 72 Fifth Avenue, New York 11; Ideal School Supply, 8312 Bickhoff Avenue, Chicago; Kenworthy Education Service, Buffalo 3, New York; Scott, Foresman and Company, 433 east Erie Street, Chicago; Steck Company, Austin 61, Texas. Number groups showing objects, dots, etc.

History of the Standard Units of Measurement. H. G. Ayre, Western State Teacher's College, Macomb, Illinois. Set of six 8 by 10 pictures for the arithmetic bulletin board.

Pocket-Size, Plastic, Decimal-Equivalent Card. Brown and Sharpe Manufacturing Company, Department 43, Providence, Rhode Island. Decimal equivalents for common fractions. Free

Time Telling. Educational Service Department, Hamilton Watch Company, Lancaster, Pennsylvania. Watchmaking and the importance and measurement of time. Teacher's guide, with chart. Free

Working with Numbers Teaching Aids. Steck Company, Austin 61, Texas. Includes almost 20 sets of flash cards for addition, subtraction, multiplication and division, fractions, and counting; 100 and 200 chart. Manuals for teacher use accompany the various teaching aids.

Devices

Abacounter. The Abacounter Educational Company, 50 Broad Street, New York 4. An arithmetic teaching and training aid built on the idea of an abacus, but much larger.

Beads. J. L. Hammett Company, Kendall Square, Cambridge, Massachusetts. In half-inch and inch diameters for making abacus, bead boards, or the like.

Educational Toy Money. J. L. Hammett Company, Kendall Square, Cambridge, Massachusetts. Simulated coin and paper money representing about $100.

Fraction Trainers. Robinson Howell Company, 641 Mission Street, San Francisco 5. Common fractions cut from plastic circles.

Fraction Wheel. Ideal School Supply Company, 8312 Bickhoff Avenue, Chicago 20. Ten 7-inch circles segregated into equal parts to show the most used fractions. Box makes easel.

Fractional Parts. Visual Specialties Company, 5701 West Vernor Street, Detroit 9. Felt cutouts of circles (Set 140) and squares (Set 141). Felt yardage also obtainable from this company for making fractional pieces.

Fraction-Decimal Comparator. Robinson-Howell Company, 641 Mission Street, San Francisco 5. Simple slide rule for fractions, decimals, and per cents. Made of vulcanized fibre, with transparent acetate slide which moves to left or right.

Fractions Made Easy. Ideal School Supply Company, 8312 Bickhoff Avenue, Chicago 20. Squares, triangles, and circles, on which cutouts fit to show various fractions.

Ideal Colored Sticks. Ideal School Supply Company, 8312 Bickhoff Avenue, Chicago 20. For grouping on ten and to show the addition, subtraction, multiplication, and division facts and processes.

Large Peg Board Outfit. J. L. Hammett Company, Kendall Square, Cambridge, Massachusetts. To be used like a hundred board for showing addition, subtraction, multiplication, and division facts. Colored pegs for use with homemade boards also available.

Math-O-Blocks. D. T. Davis Company, 178 Walnut Street, Lexington, Kentucky. Twenty-three wooden number blocks. The size of each block represents a number so facts may be shown by combinations of blocks.

Number as the Child Sees It. Grossnickle, Metzner, Wade materials, John C. Winston Company, Philadelphia, Pennsylvania. Some concrete and semi-concrete materials (and manual) for use in discovering simple addition and subtraction facts; counting discs; fact finder; modernized abacus; fractional parts; cohere-o-graph (flannel board); fraction chart; hundred board; place-value pockets; and the like.

Number Kit. Houghton Mifflin Company, 2 Park Street, Boston 7. Materials for use with the Catherine Stern approach to arithmetic.

One Hundred Chart and Two Hundred Chart. Steck Company, Austin 61, Texas. Ten strips with ten red dots on each; one square with one hundred red dots; wall chart.

Parts Imparter. Exton-aids, Box MT, Mill Brook, New York. Rotating double discs which can be manipulated to show fractional parts. For teacher and class.

Place Value Chart. L R Learning Aids, 854 Howard Street, Detroit 26. Shows the place value of whole numbers and is extended to the right of the decimal point to show fractional values.

Hundred Chart. J. L. Hammett Company, Kendall Square, Cambridge, Massachusetts. Chart with numbers to 100 in sequence and arranged in ten rows of ten columns each. For use in developing understanding of 2-place numbers.

Teach-A-Number Kit. Teach-A-Number Company, 725 Polydras Street, New Orleans, Louisiana. Ten wooden number blocks. The size of each block represents a number so facts may be shown by combinations of blocks.

Ten-Ten Counting Frame. Milton Bradley Company, Springfield, Massachusetts. Board with ten wires, each with ten beads for counting.

Games

Addi-Fax; Multi-Fax. The Plaway Games, 18 Division Street, Sidney, New York. Addition and multiplication number cards for rummy-like game.

Addit. Maxim Games Company, New York. Addition cards and discs.

Aritho. Psychological Services, 4502 Stanford Street, Chevy Chase, Maryland. Beano-like game for all four processes.

Arith-O-Cards. Arith-O-Card Company, 157 South Mentor Avenue, Pasadena 5, California. Addition and multiplication number cards for rummy-like game.

Attaboy's Number Puzzle Games. The Attaboy Company, Wichita Falls, Texas. Addition, subtraction, multiplication, division.

Chutes and Ladders. Milton Bradley Company, Springfield, Massachusetts. Counting and addition game played with dice.

Fiddlestix. Plaza Manufacturing Company, New York. Addition and multiplication.

Fracti-Fax. The Plaway Games, 18 Division Street, Sidney, New York. Set I contains forty-five cards with fractions printed on each. Object is to make books of equivalent fractions.

Go-to-the-Head-of-the-Class Game. Milton Bradley Company, Springfield, Massachusetts.

Hit. The Plaway Games, 18 Division Street, Sidney, New York. Beano-like game.

Imout. Imout Company, Box 146, Station A, St. Petersburg, Florida. Two beano-like games for fractions and/or multiplication and division.

Let's Go Shopping. Samuel Gabriel Sons and Company, New York. Buying and selling with metal coins. Addition and subtraction.

Multiplication Tables, Arithmetic Game. Miss I. D. Fogelson, 5520 South Short Drive, Chicago 37. For encouraging pupils to learn their multiplication tables. Individual use.

Pla-Pak. The Plaway Games, 18 Division Street, Sidney, New York. Box of colored wooden objects, with directions for activities to teach counting, addition, multiplication, and division.

Say-it Games. Garrard Press, 119 West Park Avenue, Champaign, Illinois. Addition, subtraction, multiplication, and division practice.

Spinno. The John C. Winston Company, Philadelphia. Circular cards fit under a spinner to make drill game for all facts.

Tallit. Maxim Games Company, New York. Multiplication cards, and discs.

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Amazing Story of Measurement, The. The Lufkin Rule Company, Saginaw, Michigan. The story of measurement from the cavemen to precision methods used today is told in comic book form. Free

Beyond the Horizon. National Association of Manufacturers, 14 West Forty-ninth Street, New York 20. A poem-message of blank verse telling the story of American arithmetical progress from pioneer days. Free

Budgeting through School Savings. Educational Section, United States Savings Bonds Division, Treasury Department, Washington 25, D.C. Information and suggestions for the teacher concerning lessons in budgeting. Free

Day of Two Noons. Association of American Railroads, Transportation Building, Washington 6, D.C. The

story of the adoption of Standard Time. Correlates geography, social studies, and arithmetic in an interesting way. Free

Fascinating Figure Puzzles. Burroughs Adding Machine Company, 6071 Second Boulevard, Detroit 32. A collection of puzzles and trick problems for enriching the program. Free

From Abacus to Monroe. Monroe Calculating Machine Company, P.O. Box 540, Orange, New Jersey. Brief history of calculating machines. Discusses the contributions in this area of Napier, Pascal, and Leibnitz. Free

From Og to Googol. Marchant Calculating Machine Co., Oakland 8, California. Brief history of numbers. Good supplementary reading. Free

General Electric Booklets. General Electric Company, Schenectady, New York. Placed on the reading table or bulletin board, these four booklets help to correlate science and arithmetic: *Adventures into the Future; Edison and Electricity; Engineering Tomorrow; Steinmetz, The Computer.* Free

How Long Is a Rod. Ford Motor Company, Dearborn, Michigan. Describes the origin of our standards of length. Excellent enrichment aid in the study of measurements. Free

Just for Fun. Z. L. Loflin and Ida Mae Heard, Southwestern University, Lafayette, Louisiana. Jokes for arithmetic teachers and pupils.

Know Your Money. United States Secret Service, Treasury Department, Washington, D.C. Shows the difference between counterfeit and real money. Free

Lessons in Arithmetic through School Savings. Education Section, United States Savings Bonds Division, Treasury Department, Washington 25, D.C. Emphasizes student participation in the training and practice of thrift and planned spending. Free

Measuring Instruments. Bruce Publishing Company, 424 North Milwaukee Street, Milwaukee 1, Wisconsin. Illustrates and explains the use of many of the more precise measuring devices used in industry today. $0.15

Numbers and Numerals. Bureau of Publications, Teachers College, Columbia University, New York. An interesting history that tells how our number system developed from earliest times. $0.35

Railroad Arithmetic, Book II. Baltimore and Ohio Railroad, Room 2021, Baltimore and Ohio Building, Baltimore 1, Maryland. An excellent source of railroad problems involving all phases of arithmetic. Free

Ranger 'Rithmetic. Forest Service, United States Department of Agriculture, Washington, D.C. Forestry and conservation problems for the sixth grade. Free

Story of Figures, The. Burroughs Adding Machine Company, 6071 Second Boulevard, Detroit 32. Brief history of mechanical computation. Free

Timekeeping through the Ages. United States Department of Commerce, National Bureau of Standards, Washington, D.C. Written in simple narrative language easily understood by elementary school children. Free

Your Money. The Federal Reserve Bank, Minneapolis, Minnesota. Pictures on money and banking in the United States. Free

Source Guides to Teaching Materials

"Aids to Teaching," *Mathematics Teacher.* National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C. Regular feature in the *Mathematics Teacher.* Lists, describes, and appraises the latest materials, devices, games, films, and filmstrips for use in studying mathematics.

"Audio-Visual Supplement," *The Instructor,* January, 1953. Department AV, Dansville, New York. Covers all phases of audio-visual activity, giving uses, sources of films, and so on. $0.05

Blue Book of 16 mm Films. Department of Audio Visual Instruction, The Educational Screen, 64 E. Lake Street, Chicago. Official and annual Journal of National Education Association. Describes more than 7,000 films.

Classroom Aids. E. C. Dent, U.S. Office of Education, Washington, D.C. Good references on pictures, maps, charts, and the like.

Descriptive and Evaluative Bibliography of Mathematics Films, A. Henry W. Syer, Boston University School of Education, Boston. Mimeographed. $0.75

Directory of 16 mm Educational Motion Pictures on Mathematics. Coronet Films, Coronet Building, Chicago 1. Free

Discussion Aids. National Association of Manufacturers, Special Services Department, 14 West 49th Street, New York 20. A catalogue of pamphlets, booklets, posters, and films on money, taxes, profits, careers, research in science, and so on. Free

Educational Film Guide. H. W. Wilson Company, 950 University Avenue, New York 52. An annual annotated compilation of all kinds of 16 mm motion pictures by title, subject, and classification. Includes films other than for arithmetic.

Educators Guide to Free Films. Educators Progress Service, Randolph, Wisconsin. An annually published annotated guide to free films. Gives listings by areas, title and subject and indicates sources and availability.

Elementary Teachers Guide to Free Curriculum Materials. Educators Progress Service, Randolph, Wisconsin. Annually published list of free materials covering all areas and all types of materials except films and filmstrips.

Filmstrip Guide. H. W. Wilson Company, 950 University Avenue, New York 52. The 1950 bound issue and the 1952 Annual Cumulation jointly provide an inclusive alphabetized and classified list of over 4,000 filmstrips.

"Mathematical Plays and Programs," *The Mathematics Teacher,* November, 1951, pp. 526–528. National

Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C. References for mathematics teachers. About 50 to 60 plays and programs. Most have appeared in the *Mathematics Teacher*.

Mathematical Teaching Aids. Chicago Schools Journal, Jan., Feb. Supplement, 1950. Chicago Teachers College, 6800 Stewart Avenue, Chicago 21. Free and inexpensive materials of all kinds for all grades.

Mathematical Visual and Teaching Aids. Teaching Aids Service, New Jersey State Teachers College, Upper Montclair, New Jersey. Bibliography of sources of maps, devices, exhibits, films, slides, and filmstrips, games, pictures, publications, and recordings. $0.75

"Projection Materials," The Fiftieth Yearbook, Part II, pp. 177–185, The National Society for the Study of Education. University of Chicago Press, Chicago, 1951.

Society for Visual Education, Inc. 1345 West Diversey Parkway, Chicago 14. Upon request, this organization furnishes pamphlets describing the use of all types of visual aids. Free

Tables of Measures

Liquid Measure

2 cups (c.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Cooking Measures

3 teaspoons (tsp.) = 1 tablespoon (tbsp.)
16 tablespoons = 1 cup (c.)
2 cups = 1 pint (pt.)
4 cups = 1 quart (qt.)

Measures of Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2,000 pounds = 1 ton (T.)

Measures in Counting

12 things = 1 dozen (doz.)
12 dozen = 1 gross
144 things = 1 gross

Measures of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)
52 weeks = 1 year (yr.)
12 months = 1 year
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

Measures of Length

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
36 inches = 1 yard
$5\frac{1}{2}$ yards = 1 rod (rd.)
$16\frac{1}{2}$ feet = 1 rod
5,280 feet = 1 mile (mi.)
1,760 yards = 1 mile
320 rods = 1 mile

Measures of Surface, or Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet (sq. ft.) = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 A. = 1 square mile (sq. mi.)

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